Traditional and Non-Traditional Optimization Tools
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Lecture - 12
Real - Coded GA

Let us start with another topic that is topic 6 Real - Coded GA. Now, the purpose of using the real-coded GA is to carry out optimization in continuous sub space.

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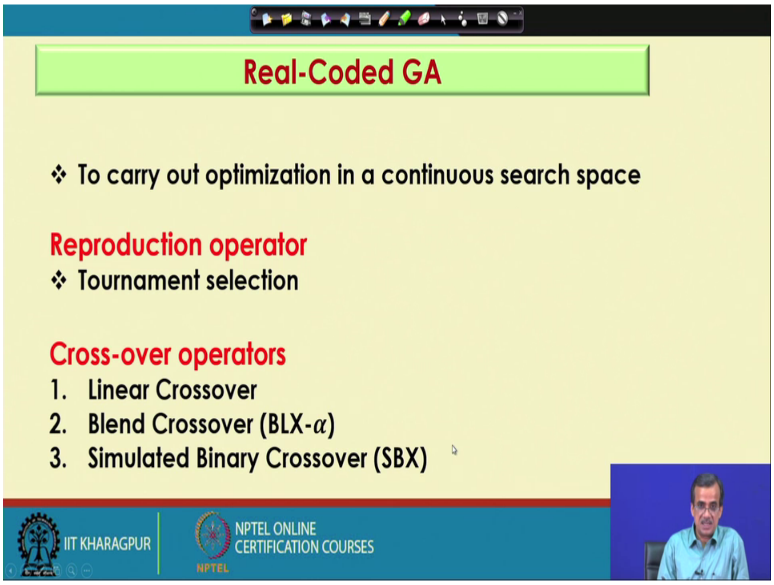

Now, supposing that I have got an optimization problem where the variables are real in nature; that means, they are having fraction. For example, the variable could be 20.65 say 13.56, these variables can be directly coded in the GA solution. Now, let us see how to do that.

So, the reproduction operator which we generally used in genetic algorithm to select the mating pool, we use tournament selection the principle of tournament selection I have already discussed. Now, will have to concentrate on the cross over operators, now if you see the literature in real coded GA we use different types of crossover operators like linear crossover, blend crossover, BLX alpha then comes simulated binary crossover SBX and so on. So, I am just going to discuss the principle of these crossover operators 1 after another.

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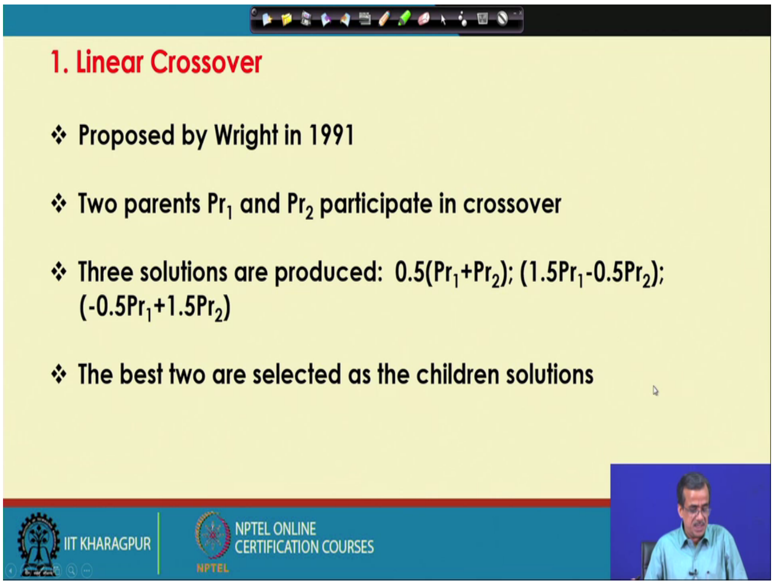

Now, the linear crossover the concept was proposed in the year 1991 by Wright. The principle is very simple, supposing that we have got 2 parents Pr 1 and Pr 2 which are going to participate in linear crossover. Now, using the linear values using sorry the numerical values for this Pr 1 and Pr 2 we calculate the children solutions as follows. For example, say we use this 0.5 multiplied by Pr 1 plus Pr 2, another possibility is 1.5 multiplied by Pr 1 minus 0.5 multiplied by Pr 2 another possibility is minus 0.5 into Pr 1 plus 1.5 into Pr 2. Now, we have got 3 children solutions. So, out of these 3 children solution the better 2 are selected. Now, this is actually a very simple operator.

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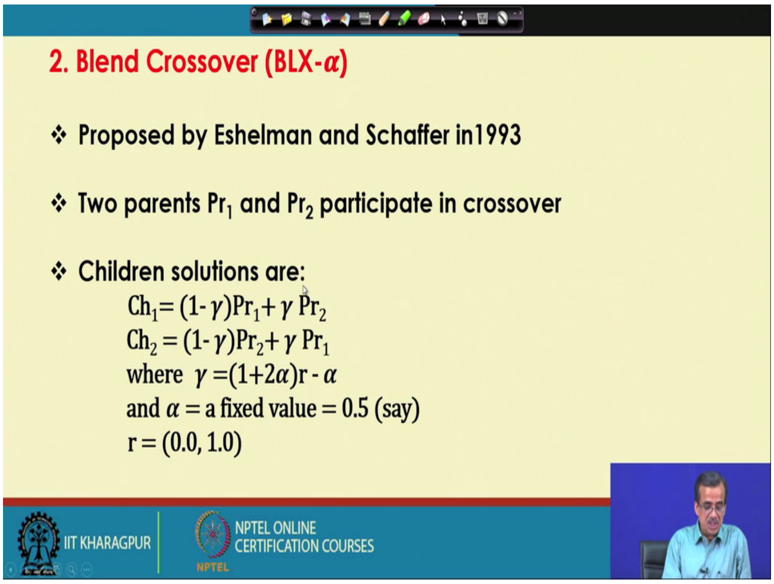

Then comes the concept of the blend crossover. Now, this is popularly known as BLX alpha. So, that was proposed in the year 1993 by Eshelman and Schaffer. Now, the principle is as follows supposing that I have got 2 parents Pr 1 and Pr 2 and they are going to participate in blend crossover, the children solutions are calculated as follows child 1 is actually 1 minus gamma multiplied by Pr 1 plus gamma into Pr 2, then child 2 is equal to 1 minus gamma multiplied by Pr 2 plus gamma multiplied by Pr 1 where gamma is nothing but 1 plus 2 alpha multiplied by r minus alpha. Here r is actually a random number lying between 0 and 1 and alpha is having a fixed value say 0.5. Now, using this particular the blend crossover, very easily I can find out what should be the children solution that is child 1 and child 2.

Now, this particular method actually gained some popularity, but after that some more powerful operator crossover operator was proposed. For example, we have this simulated binary crossover it is popularly known as SBX that was proposed by Deb and Agrawal in the year 1995.

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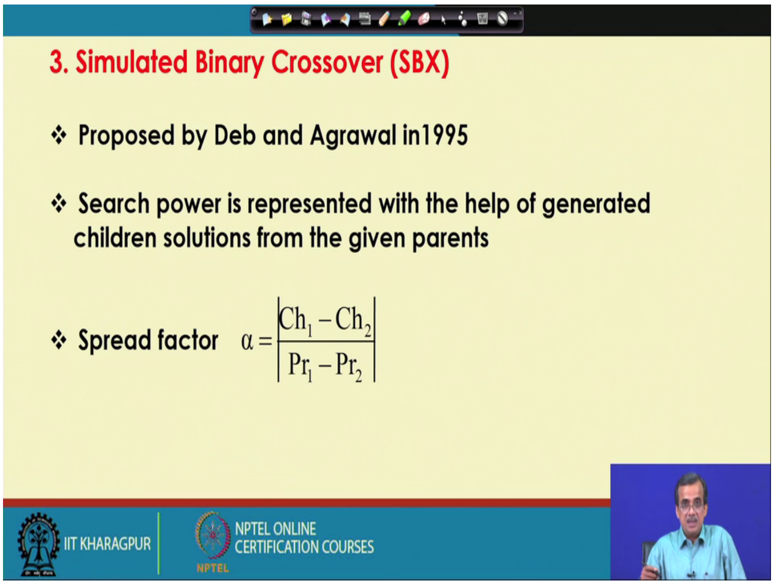

Now, here the search power is represented with the help of a generated children solution from the given the parents. So, the parents are given will have to find out the children solution. Now, we just you want to use one probability distribution function of creating the children solution from the parents and the search power is decided by this particular the probability distribution which I am going to discuss.

Now, before I proceed further. So, I will have to define one parameter that is called spread factor. The spread factor is denoted by alpha and that is nothing but the ratio between the difference between the 2 children solution to the difference between the 2 parents and the mod value of that. So, we try to find out the difference between the 2 children that is Ch 1 minus Ch 2 and the difference between the 2 parents Pr 1 minus Pr 2, so we divide them and we consider the mod value and that is nothing, but the spread factor denoted by alpha.

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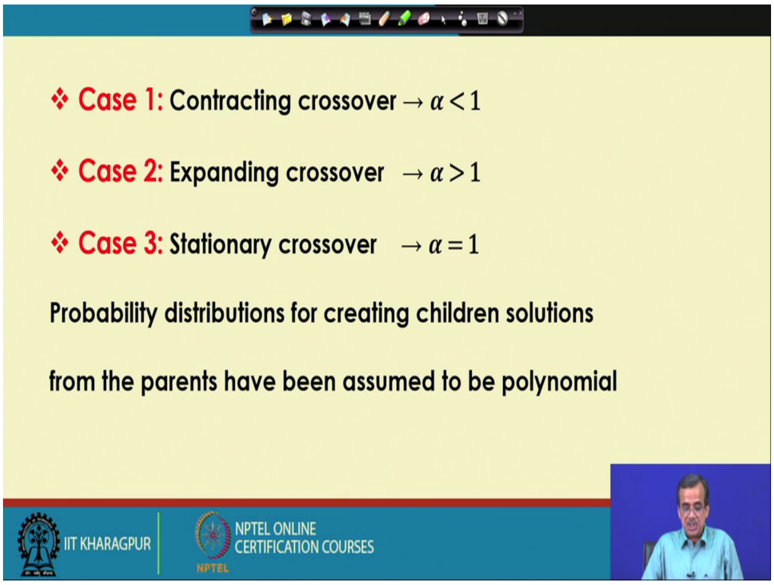

Now, depending on the value of this particular the alpha the crossover operators are called the contracting crossover, expanding crossover or the stationary crossover. Now, if alpha is found to be less than equals to 1 that is called the contracting crossover if alpha is found to be greater than equals to 1m greater than 1 that is called expanding crossover and if alpha is found to be equal to 1 that is called the stationary crossover.

Now, here actually as I told that we are going to take the help of some probability distribution function for creating the children solution from the parents and we assume that. So, this particular probability distribution function is a polynomial.

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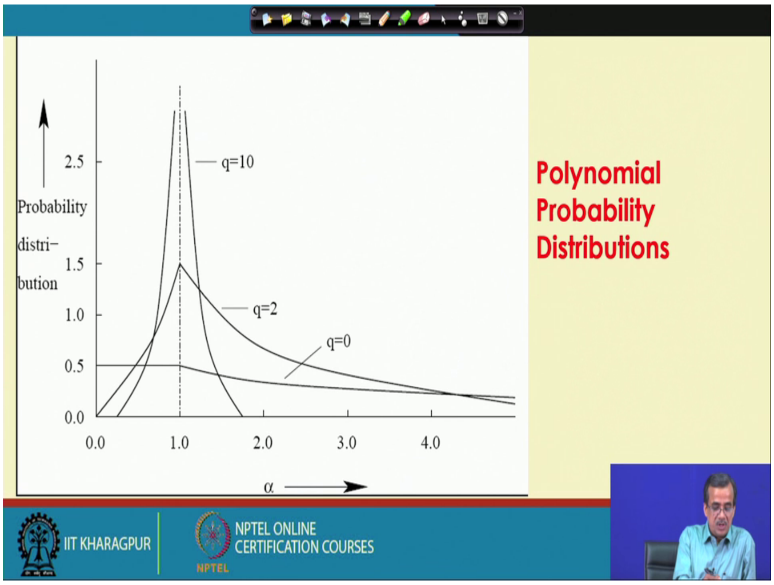

Now, this actually this figure shows the distribution of this polynomial function the probability distribution function and here along this y axis we put probability distribution the factor and along x we have got the alpha and depending on the value of these particular alpha. So, what you do is this is divided into 3 zones. Now, if alpha is found to be less than 1 so this zone is called the contracting zone, if alpha is found to be greater than 1 so this zone is called the expanding zone and when alpha becomes equal to 1 that is called the stationary crossover.

Now, what I do is we try to define this polynomial function in a particular way. And I am just going to show the mathematical expression generally used for this particular the contracting and expanding crossover and once again I will go back to the figure.

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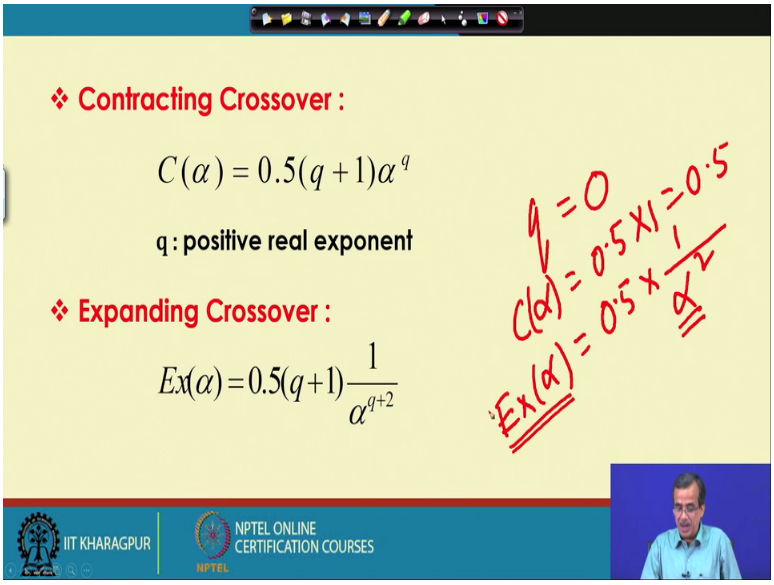

Now, this is the mathematical expression which is generally used for the contracting crossover that is C alpha is nothing, but 0.5 into q plus 1 into alpha raise to the power q here alpha is the spread factor and q is nothing, but a positive exponent. Now, for different values of q, you will be getting the different types of that function distribution these are going to discuss in more details.

Now, here as I told q is a positive real exponent. Now, for the expanding crossover we use Ex alpha is nothing, but 0.5 into q plus 1 multiplied by 1 divided by alpha raised to the power q plus 2. So, this is the mathematical expression further the expanding crossover.

Now, before I proceed further let me discuss what happens if I consider a particular value of q. Now, supposing that I am going to consider say q is equals 2 0. Now, if I put q is equals to 0, this contracting crossover zone, I will be getting it is it is equal to 0.5 some constant value. For example, then that case C alpha will become 0.5 q equals to 0, multiplied by 1 into alpha raise to the power 0 is 1.

So, this is nothing but 0.5 on the other hand for q equals to 0 the Ex alpha will become equal to 0.5, 0.5 multiplied by 1 divided by alpha square because q equals to 0 here, 0.5 into 1 divided by alpha square. Now, as alpha increases the Ex alpha the value of Ex alpha will go on decreasing. Now, let us see what happens to the distribution function.

Now, let me go back to this particular the distribution.

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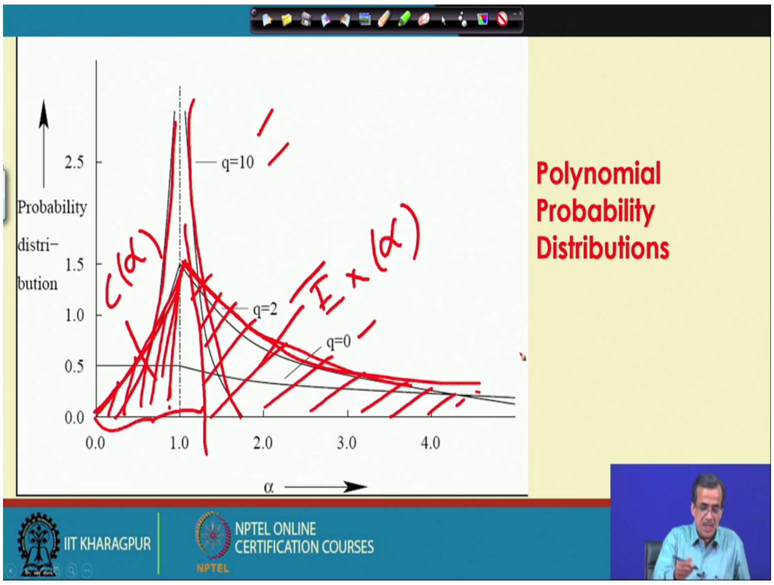

Now, corresponding to q equals to 0. So, if I just consider q equals to 0. So, I will be getting. So, this particular expression like in the in the contracting zone, so q, this C alpha is equals to 0.5 and in the expanding zone. So, this is going to be reduced it is going to follow this particular curve. So, that is corresponding to q equals to 0.

Now, if I take a higher value of q say q equals to 2. So, I will be getting, this type of distribution. Similarly if I take q equals to say 10 higher value, in that case I will be getting this type of steeper distribution for this particular polynomial. Now, why do you select this type of polynomial distribution? There is actually the reason behind the selection of this type of polynomial distribution.

The reason is as follows. If I take q equals to 0 or a small value the children solutions will be distributed widely distributed on the other hand if I take a higher value of q the children solution will be very close to the parents. Now, if I want to more diversification I will have to use less value of q and if I want less diversification; that means, the children solutions should be very close to the parents, so I will have to use the higher value of q.

Now, the way this particular the polynomial distribution has been selected there is a philosophy that is as follows. Now, let me considered a particular value for this q let me consider q equals to 2. So, this is the distribution for q equals to 2 and up to this we have got the contracting zone. Now, here the function is C alpha and here it is Ex alpha. Now, the way it has been selected the area under this particular come in the contracting zone will become equal to 0.5. Similarly, the area under this particular curve in the expanding zone will become equal to 0.5 so that the total probability becomes equal to 1.

Now, this particular thing actually I have written in the slides.

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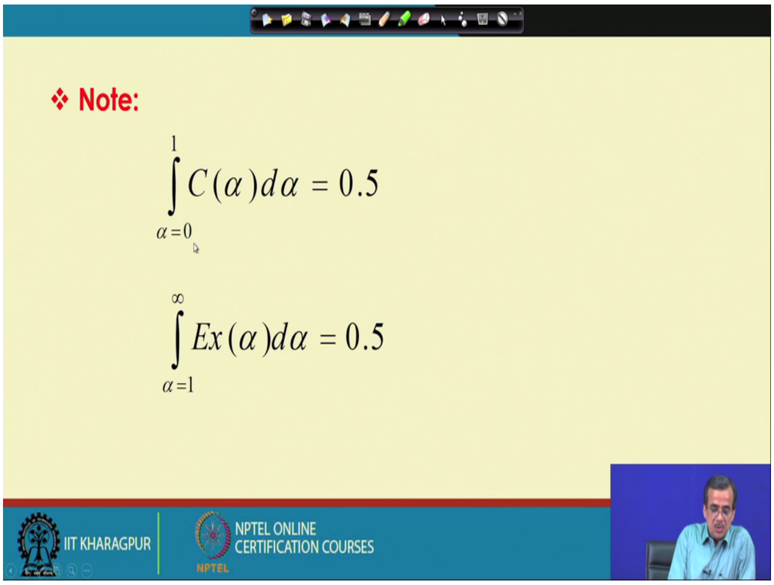

So, integration alpha equals to 0 to 1; that means, I am in the contracting zone C alpha d alpha is equals to 0.5. Similarly integration alpha equals to 1 to infinity Ex alpha d alpha is equals to 0.5. So, these 2 conditions are to be fulfilled.

Now, here I just want to give one note that this selection of the polynomial distribution function the way we have selected the way we have already discussed is not the unique one. Now, we can design some other form of this particular the function for the contracting zone and expanding zone and in fact, this particular selection of the probability distribution function is not the unique one.

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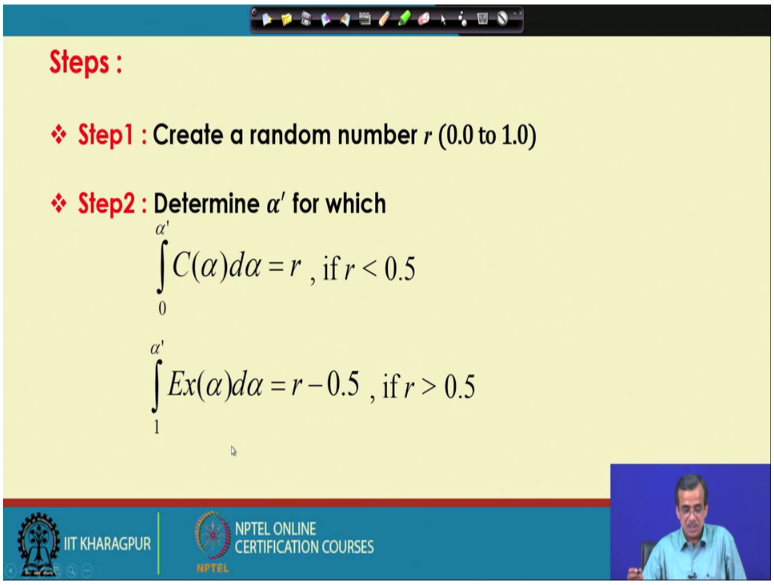

Now, let us see the steps, step one we create a random number r lying between 0 and 1 we determine alpha prime for which integration 0 2 alpha prime C alpha d alpha equals to r, r is the random number lying between 0 and 1 if r is found to be less than 0.5. And if r is found to be greater than 0.5 we use integration 1 2 alpha prime, Ex alpha, d alpha and that is equals to r minus 0.5 and we try to calculate what should be the appropriate value for this particular alpha prime. Now, here I just want to mention that if r is found to be exactly equal to 0.5 then alpha prime will become equal to 1 and will be getting that particular the stationary crossover.

And moreover in the contracting zone the alpha prime will be less than 1.0 and in the expanding zone the alpha prime will be more than 1.0.

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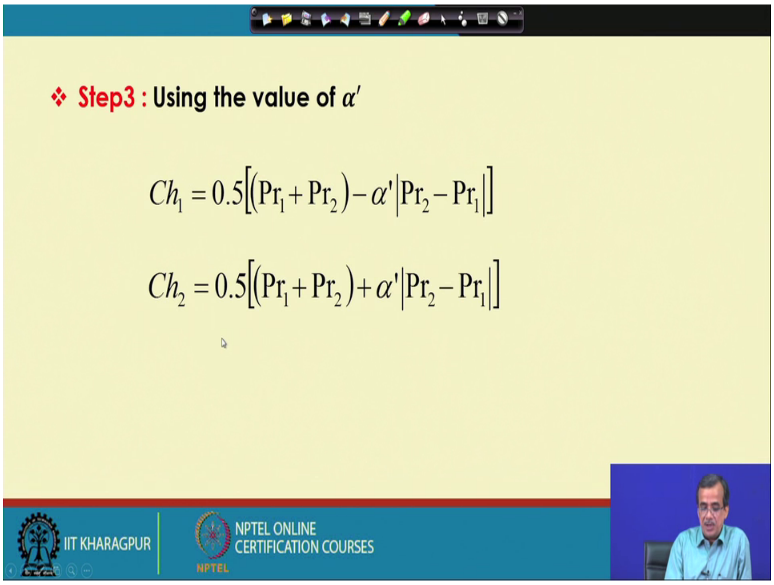

Now, using the value of these particular alpha prime now I can find out the children solution. To find out the 2 children actually what I do is we try to consider the average of the 2 parents that is parent 1 plus parent 2 divided by 2 that is the average, and then alpha prime multiplied by the mod value of the difference between the 2 parents Pr 2 and Pr 1.

So, this particular amount 50 percent of that particular amount we subtract here to get child 1, we add here to get child 2 and accordingly I will be getting child 1 like this and child 2 like this. So, child 1 is nothing but 0.5 multiplied by Pr 1 plus Pr 2 minus alpha prime the mod value of the difference between the 2 parents and child 2 is 0.5 multiplied by Pr 1 plus Pr 2 plus alpha prime multiplied by the mode value of the difference between the 2 parents and whole thing is multiplied by 0.5.

So, this is the way actually we can find out child 1 and child 2. Now so, this particular principle I am going to explain with the help of one numerical example.

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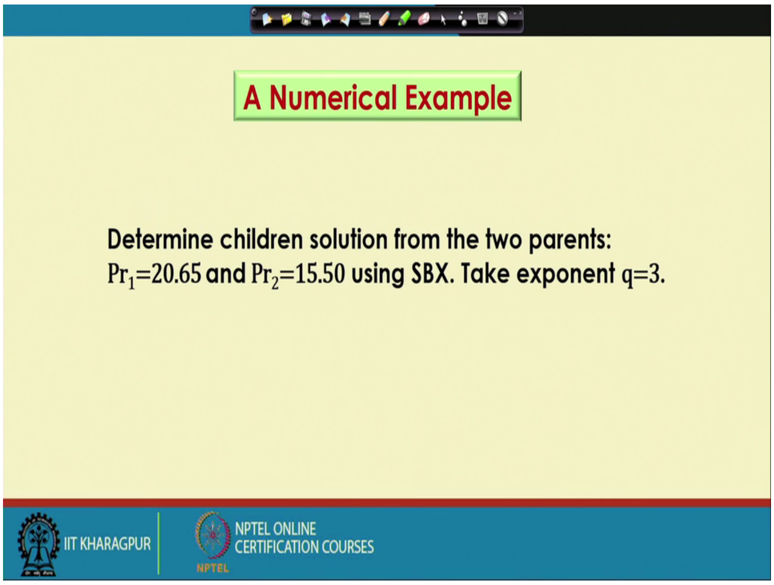

Now, supposing that I have got 2 parents Pr 1 is 20.65 and Pr 2 is 15.50 and I am going to use SBX and our aim is to find out the 2 children solution we take the value of exponent that is q equals to 3 and let us consider a random number r is equal to 0.4.

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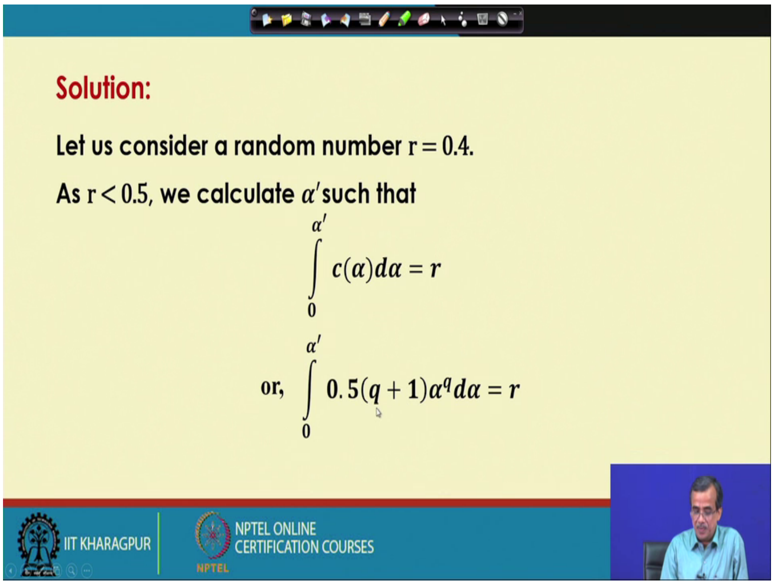

Now, this corresponding to this r 0.4 that r is less than 0.5. So, we are in the contracting zone. So, we will have to find out the alpha prime. So, integration 0 to alpha prime C alpha d alpha equals to r because we are in the contracting zone and we can find out, we can put the expression of C alpha that is 0.5 into q plus 1 alpha raise to the power q d alpha and that is equals to r. Now, you substitute the numerical values for q and carry out this particular integration and r is equals to 0.4 and if we carry out, we will be getting this particular form and very easily you can find out the integration. And if you just simplify this. So, will be getting alpha prime equals to 0.94.

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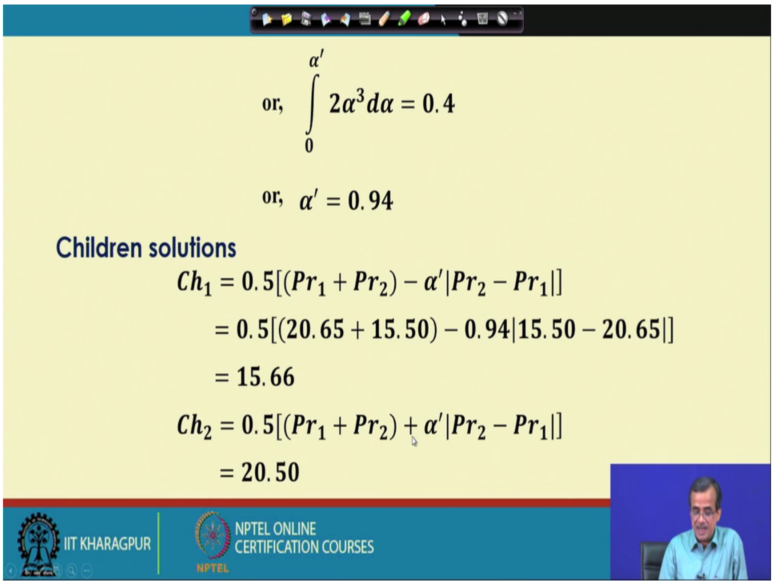

So, once you have got this particular the value for the spread factor that is alpha prime. Now, very easily we can find out the children solution like Ch 1 the first child is 0.5 multiplied by Pr 1 plus Pr 2 minus alpha prime mod value Pr 2 minus Pr 1, you just substitute the numerical values for Pr 1 Pr 2 alpha prime and if you calculate you will be getting 15.6 as child 1. Similarly we can find out child 2 only thing I will have to put plus sign here and will be getting, so the child 2 has 20.50.

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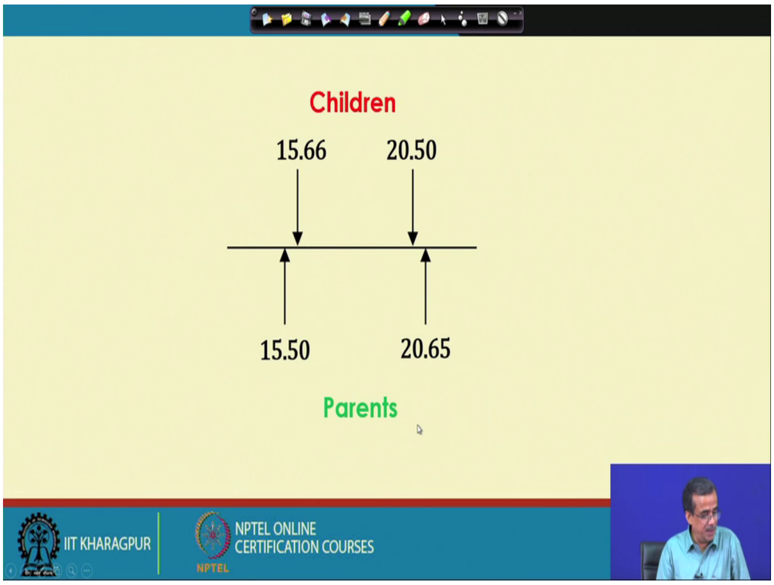

Now, if I just plot on this scale the parent solutions where the parent solutions where 15.50 and 20.65 and the children solution will be 15.66 and 20.50 and this is the case of contacting crossover. So, the difference between the 2 children will be less than the difference between the 2 parents because alpha prime is found to be 0.94 which is less than 1 and that is why we get that the 2 children solutions allying within the 2 parent solutions.

So, this is the way actually we can calculate the children solution starting from the parents using the principle of simulated binary crossover that is SBX.

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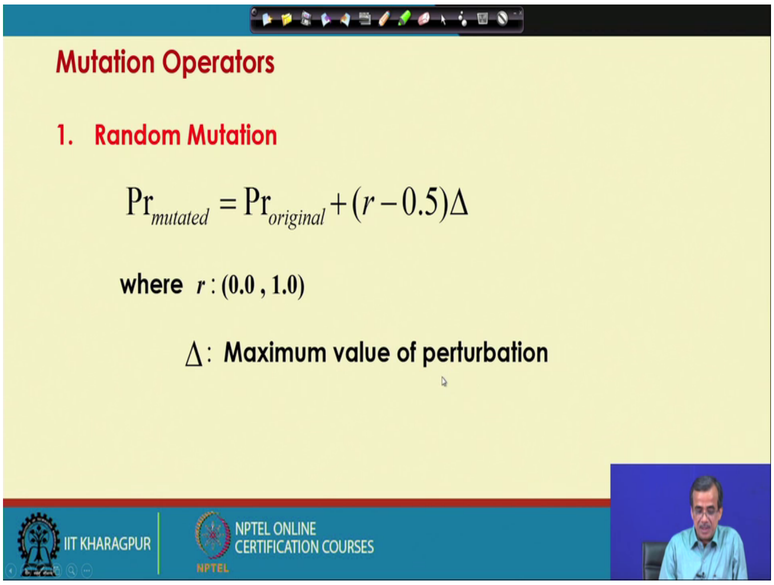

Now, I am just going to discuss the mutation operators. Now, if you see the literature the concept of random mutation came first and that principle is very simple. Here the Pr mutated there is the mutated solution is nothing but the original solution plus r minus 0.5 into delta, r is nothing but the random number lying between the 0 and 1, and delta is nothing, but the maximum value of perturbation.

Now, perturbation means dissimilarity; that means, how much dissimilarity you are going to consider in the mutated solution with respect to the original solution that is supplied by the user with the help of this particular the delta. Now, if you supply the delta and r will be generated using the random number generator, so I can find out the mutated solution.

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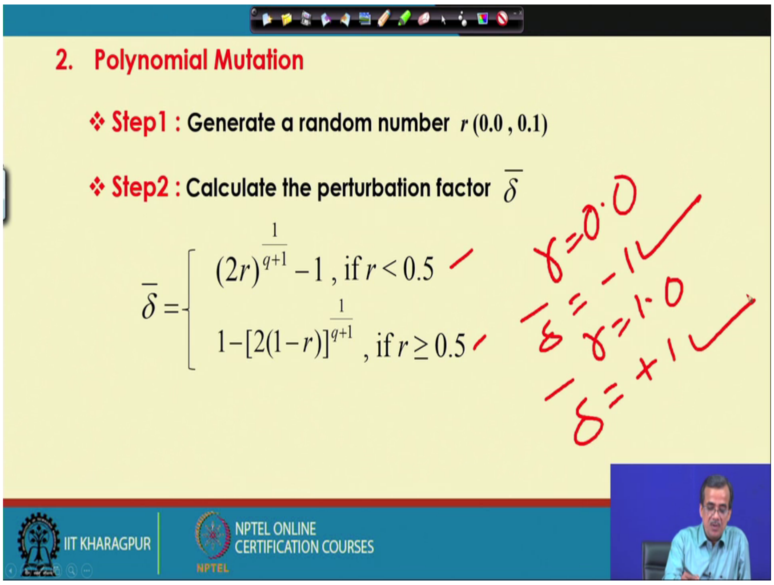

Then comes the concept of the polynomial mutation. Now, here in polynomial mutation actually what I do is we generate a random number r lying between 0 and 1. Step 2 we calculate the perturbation factor that is the dissimilarity factor denoted by delta bar. Now, delta bar is nothing but 2 r raised to the power 1 divided by q plus 1 minus 1, if r is found to be less than 0. On the other hand if r is found to be greater than equals to 0.5. So, delta bar is nothing, but 1 minus 2 into 1 minus r raise to the power 1 divided by q plus 1 if r is found to be greater than equals to 0.5.

Now, the way this particular expressions have been chosen there is a valid reason. Now, if I just put r equals to 0. So, if I consider r equals to 0.0. So, I am here because r is less than 0.5, I will be getting delta bar is equals to minus 1. And if I put r equals to 1.0, I am here and here if I put r equals to 1.0, this will become 0. So, I will be getting delta bar is equals to positive 1. So, this particular perturbation factor it varies from minus 1 to plus 1.

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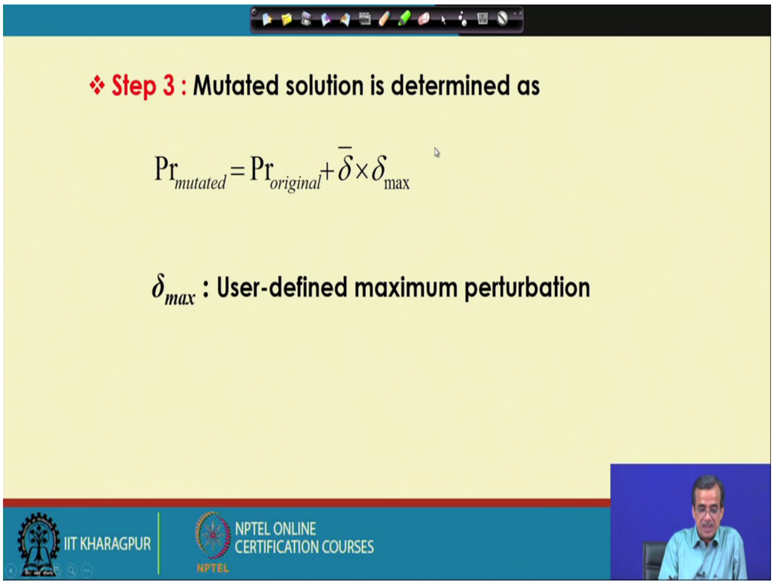

Now, using this particular the value for this perturbation factor I can find out what should be the mutated solution. So, Pr mutated is nothing but Pr original plus delta bar delta bar is nothing, but that perturbation factor multiplied by delta max. Now, delta max is actually the user defined maximum value of perturbation; that means, how much dissimilarity you are going to allow in the value of a variable that is denoted by delta max. And once you have this delta bar and delta max and p original very easily we can find out Pr mutated.

So, using this we can find out what should be the numerical value for the mutated solution.

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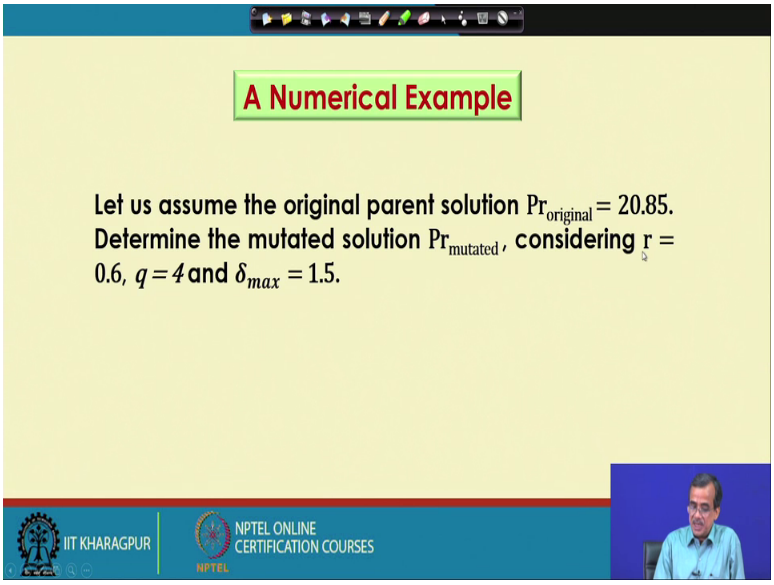

Now, I am just going to solve one numerical example and this is related to polynomial mutation. Now, supposing that the parent solution Pr original is equals to 20.85 and we are going to consider say the random number r is equals to 0.6, q equals to 4 and delta max equals to 1.5.

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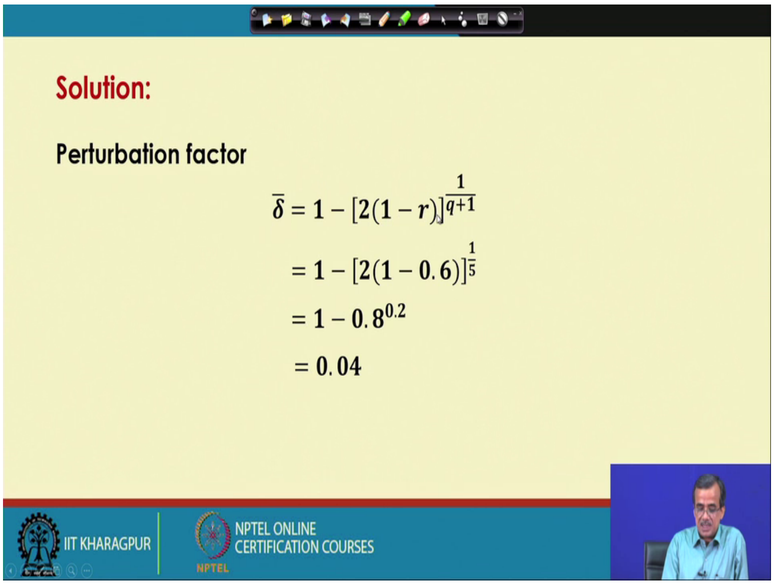

Now, if use this numerical values, very easily I can find out the perturbation factor delta bar. Now, here r is actually 0.6; that means, greater than 0.5. So, I will have to use this particular expression for delta bar and that is 1 minus 2 into 1 minus r, r is 0.6 1 divided by q plus 1 q equals to 4. So, 1 divided by 5 and if you solve it will be getting the numerical value that is 0.04.

So, this is the value for the perturbation factor which varies in a scale of minus 1 to plus 1. And once I have got the value for this particular delta bar , I can find out Pr mutated is equal to Pr original plus delta bar multiplied by delta max and all the numerical values are known and if you substitute then I will be getting the mutated solution that is 20.91 this real coded GA works.

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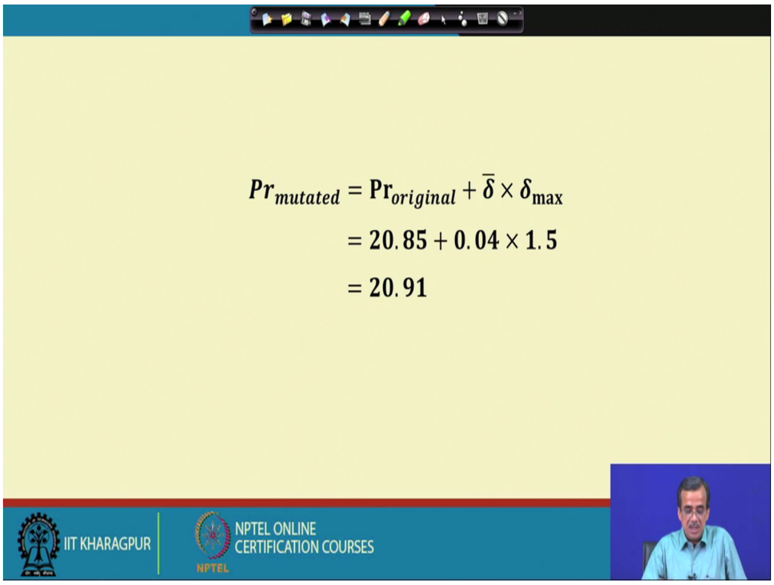

Now, here I just want to tell one merit, the merit is actually bit easy to understand that if the optimization problem is having real variables. So, directly we can handle and there will be no such precision problem the problem which we are we were going to face with the binary coded GA. But here there is one demerit the demerit in the sense, like this your binary coded GA where the convergence could be proved using the concept of schema theorem. Here for this real coded GA there is no concept of that type of indirect proof for convergence till today, but we have seen that this particular real coded GA can solve a variety of complex real world optimization problem very efficiently.

Thank you.