Traditional and Non-Traditional Optimization Tools
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Lecture – 01
Principle of Optimization

Let me introduce myself I am doctor D. K. Pratihar professor of Mechanical Engineering Department IIT Kharagpur. I am going to teach the course on Traditional and Nontraditional Optimization Tools. I welcome you all to this course. Now for this course the textbook which I am going to follow is this soft computing fundamentals and applications written by D. K. Pratihar.

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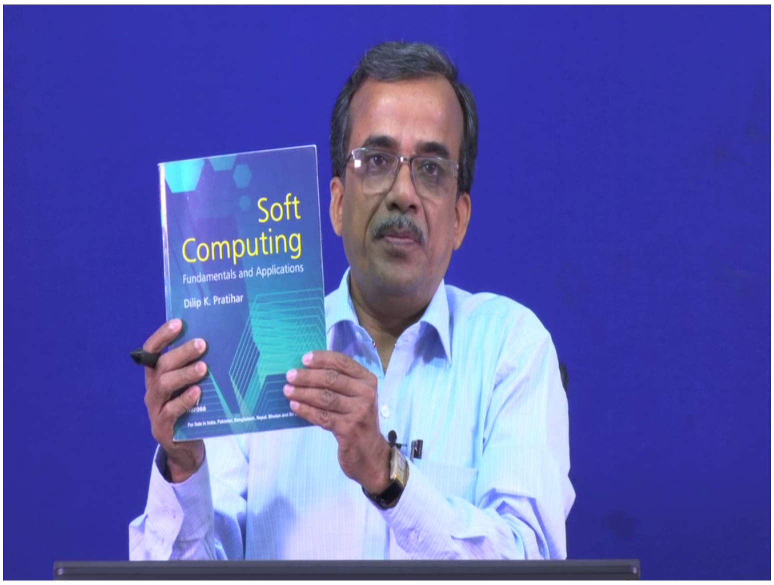

Now, let me start with the first topic that is Principle of Optimization. Now before I start learning this particular course on optimization, several questions may come to our mind these are as follows what is optimization? That is what do you mean by an optimization? Why do we need optimization? The next is how to represent an optimization problem in the mathematical form how do we classify the different optimization problems, how does an optimization tool work while determining the optimal solutions and so on.

Now, I will to answer all these questions one after another.

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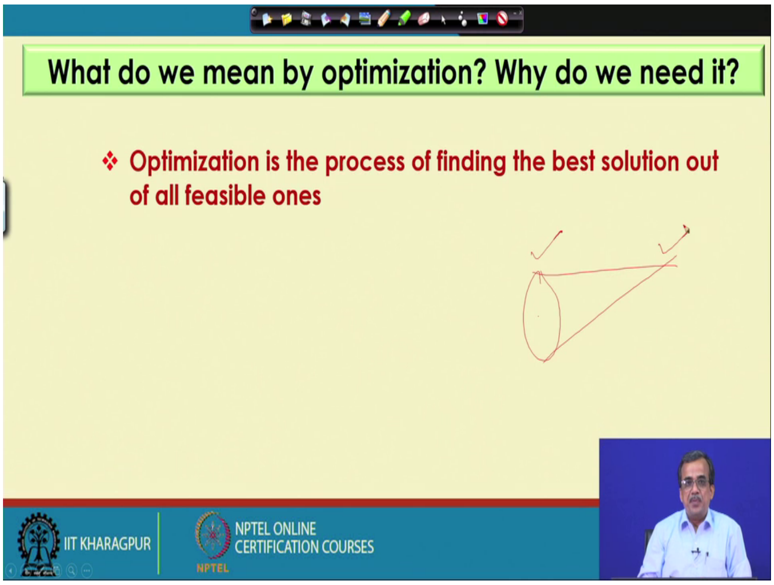

Let me start with the definition of the term optimization that is what do you mean by optimization. Optimization is the process of finding the best solution out of all the feasible ones. Now let me take one very simple example, while delivering the lecture nowadays we use laser pointer, but previously we use to use some sort of wooden pointer.

Now if we see the wooden pointer the designs of wooden pointer, now this wooden pointer is nothing, but the conical shape pointer, it looks like this it is something like this. Now this end is nothing, but the gripping end and this is the pointing end supposing that on a bean or a bucket. So, we have got several designs available of this wooden pointer and now I will have to find out the best one.

Now, this is nothing, but a task of optimization, now the next is why do you need this optimization, we need the principle of optimization because to be in competition in dynamic and competitive market, we will have to produce goods which are cost effective and efficient. Now the principle of optimization is going to help us to ensure the cost effective and efficient product.

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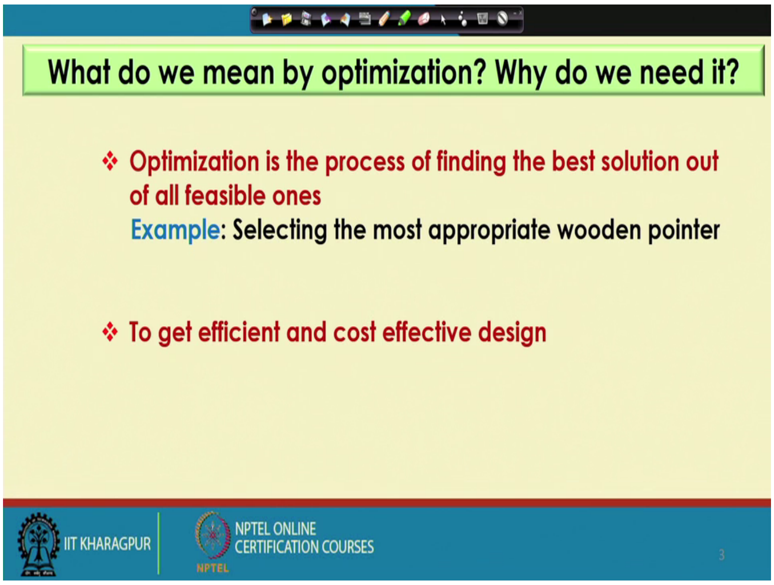

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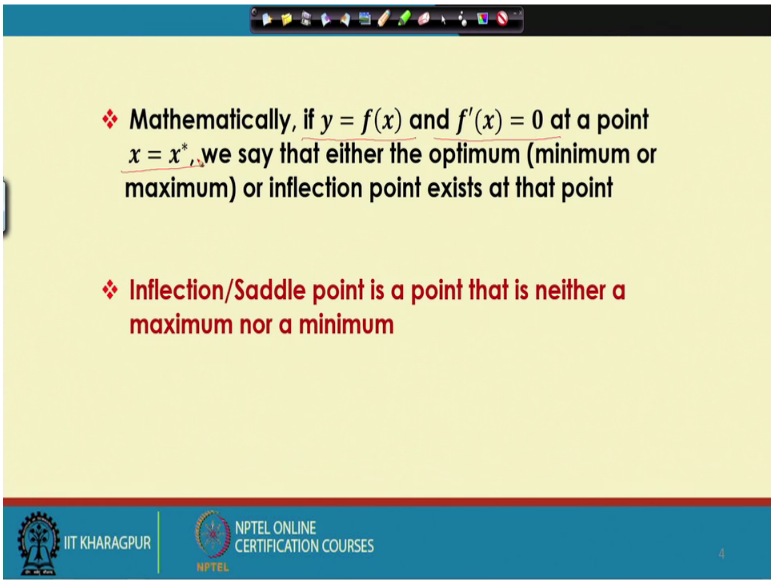

Now, mathematically if you want to represent what do you mean by optimization? Now I will have to take 1 example that is y equals to fx y equals to fx. So, this is a function of only 1 variable, now what you do is we try to find out the first derivative that is a prime x and we put equals to 0. Now we solve for x supposing that we are getting x equals to x star, now at x equals to x star there could be either optimum solution or there could be inflection point. Now this optimum solution means it could be either the maximum or the minimum, now by inflection point we mean this is neither a maximum nor a minimum.

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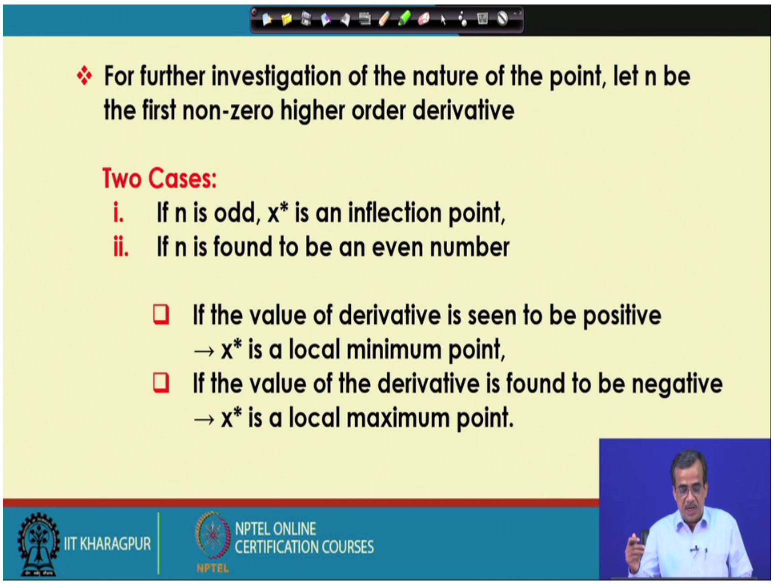

Now, if you want to investigate further we try to find out the higher order derivative and supposing that n represents the first non-zero the higher order derivative. Now what I do is we find out the higher order derivative now if this particular n which is nothing, but the first non-zero higher order derivative. If it is found to be odd then it x star is nothing, but an inflection point on the other hand if n is found to be an even number there could be 2 sub cases, if the value of the derivative is seen to be positive then x star is a local minimum point and if the value of the derivative is found to be negative then x star is a local maximum point.

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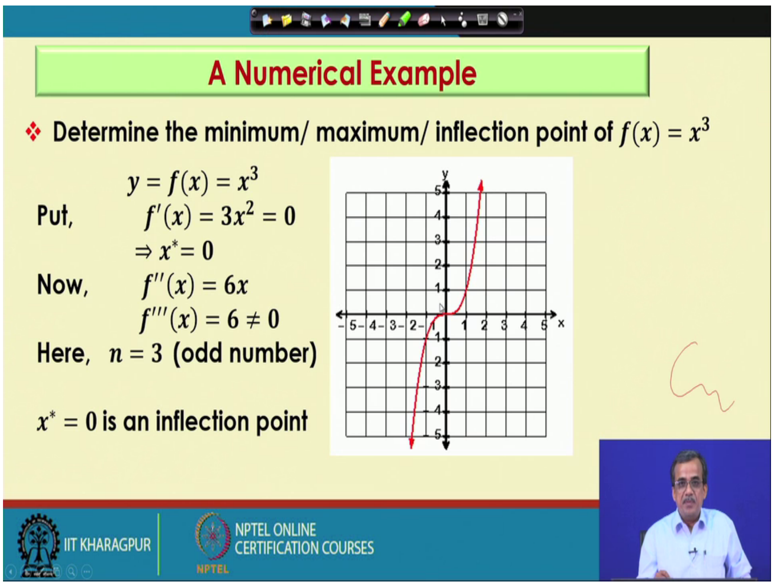

Now, let me try to explain this with the help of 1 numerical example. Now the problem is as follows like we will have to find out the maximum or the minimum or the inflection point of the function f x equals to x cube. Now what we do is we try to find out the derivative of this particular function, we try to find out the derivative of this particular function that is your f prime x and that is nothing, but 3 x square that is nothing, but there x square and we put this equals to 0 we solve for x and supposing that we are getting x star equals to 0.

Now, we try to find out the higher order the derivative. So, f double prime x is equals to 6 x now if I put x equals to 0 this will become equal to 0. So, this is not the non-zero quantity. Next we go for the third order derivative that is f triple prime x and that is equals to 6 and it is not equals to 0; that means, the value of small n that is equals to 3 and it is an odd number.

Now, as n is an odd number. So, this indicates that x star equals to 0 is an inflection point. Now if you see the function plot y is a function of x. So, this shows actually the function plot and here you will find that at x equals to 0 this is neither a maximum nor a minimum and this is nothing, but a saddle point on or an inflection point.

Now, the next question is how can we represent an optimization problem mathematically. Now to represent optimization problem mathematically what we will have to do is.

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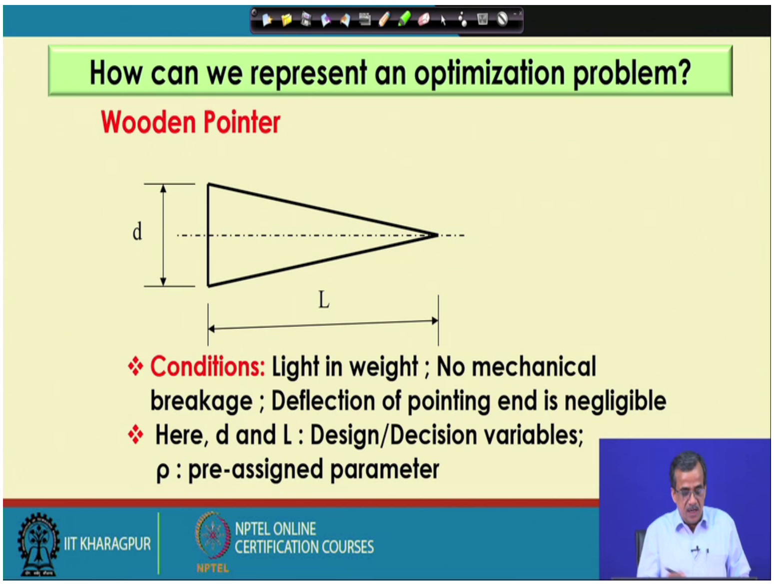

So, I will have to take 1 example and let me try to take the example of the wooden pointer; that means, we will have to find out the optimal design of the wooden pointer and to find out the optimal solution how to represent mathematically that I am going to discuss. So, this is the schematic of 1 wooden pointer having conical shape now this is nothing, but your the d is nothing, but that is the diameter of the gripping end and l is the length of this particular the wooden pointer and this is actually the pointing end.

Now, what I do is we try to find out the optimal design based on the condition that it should be light in weight. So, that we can handle it very easily and at the same time there should not be any mechanical breakage and the deflection of the pointing end should be as minimum as possible. Now here this diameter of the gripping end and the length of this particular the pointer. So, these are nothing, but the design or the decision variables and row is nothing, but the density that is the preassigned parameter and that is a fixed quantity. Now using this so what you can do is we can find out the mathematical formulation of this particular the problem.

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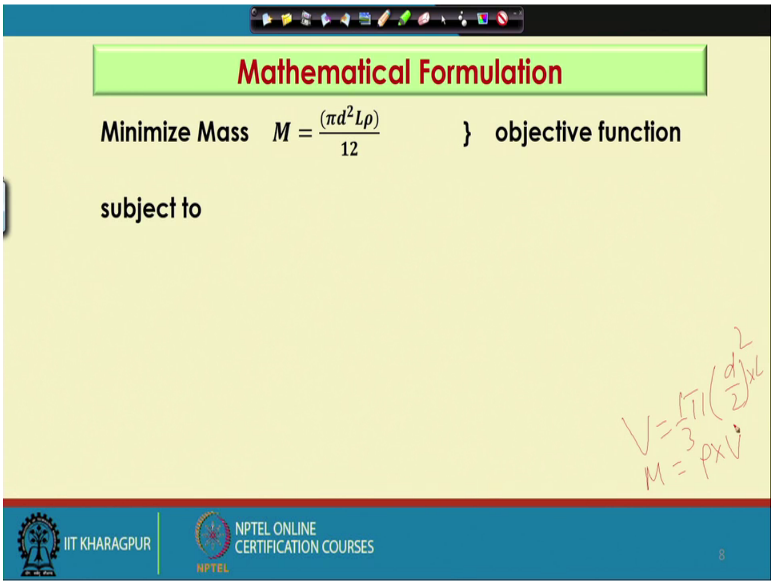

Now, what we do is we try to find out the volume of this conical shaped wooden pointer and the volume is nothing, but your that V is nothing, but one third pi d by 2 square multiplied by L and mass is nothing, but is the row is the density multiplied by the volume and very easily we can find out this particular the expression and once you have got this particular expression. So, I can find out.

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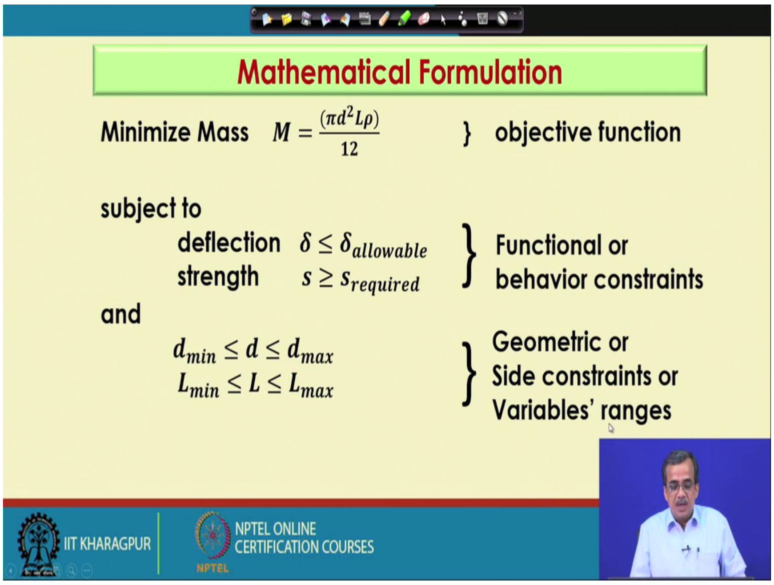

So, this is actually the mass; mass of this wooden pointer and it is subjected to 2 conditions the deflection of the pointing end it should be less than equal to the allowable deflection and the strength should be greater than equal to the s minimum required.

Now, these are nothing, but the functional or the behavior constraints and the objective function is nothing, but to minimize the mass and of course, this will be subjected to some geometric or the side constraint or the variable bounds. Now here we can see that this d is lying between d minimum and d maximum and one is a lying between L minimum and L maximum these are nothing, but the variable bounds or the geometric constraints.

Now, here in this mathematical formulation so minimize mass is nothing, but the objective function subjected to 2 conditions those are nothing, but the functional or the behavior constraints and the variables are having some range d and L those are nothing, but the geometric or the side constraints. Now for any optimization problem the first thing we will have to do is we will have to identify the design variables. So, mathematically you will have to express the objective function and you will have to find out the functional or the behavior constraints if any and of course, there should be some variable bonds.

Now, the presence of this functional constraint or the behavior constraint is not must for all the optimization problem, but the geometric constraint and the or the side constraint are to be there for all the optimization problems. So, this is the way actually mathematically we can express the optimization problem.

Now, the next is how to classify the different optimization problems.

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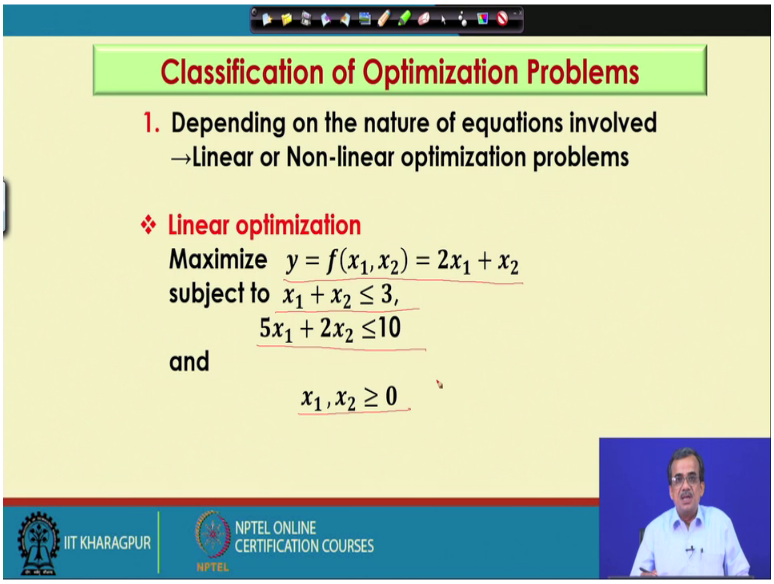

Now optimization problems have been classified in different ways for example, the first 1 is depending on the nature of the equation involved. So, depending on the nature of the equations involved the optimization problem could be classified into 2 groups. Now 1 is called the linear optimization problem, another is called the non-linear optimization problem. Now by linear optimization problem we mean that both the objective function as well as the functional constraint should be the linear function of the design variables.

For example here so our aim is to maximize y is a function of x 1 and x 2 that is nothing, but 2 x 1 plus x 2. So, this is actually the objective function this is actually the objective function and it is subjected to 2 conditions x 1 plus x 2 is less than equals to 3 5 x 1 plus 2 x 2 is less than equals to 10. So, these 2 are nothing, but the functional constraint and of course, there should be variable bounce x 1 x 2 is greater than equals to 0. So, this is nothing, but the side constraint and here both the objective functions as well as the functional constraints are linear function of the design variables so this is a linear optimization problem.

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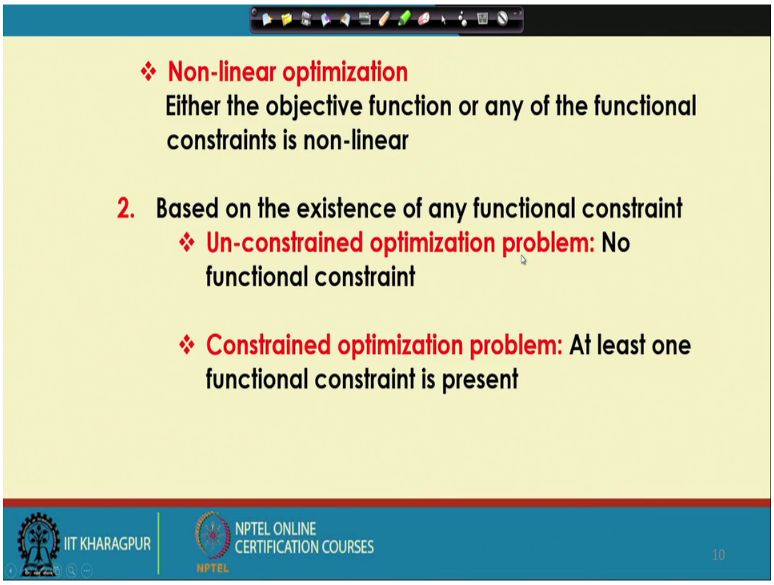

Now, the next is non-linear optimization problem now for this non-linear optimization problem either the objective function or any one of the functional constraints or both will be non-linear function of the design variables, then it is called the non-linear optimization problem. Now the classification is based on the presence of the functional constraints now as I told that the presence of functional constraint is not a must.

Now, for the optimization problem if there is no such functional constraint that is called unconstrained optimization problem and if there is at least one functional constraint that is called the constrained optimization problem. And it is very important that this side constraint or the geometric bound has to be there for all the optimization problem otherwise it cannot be defined.

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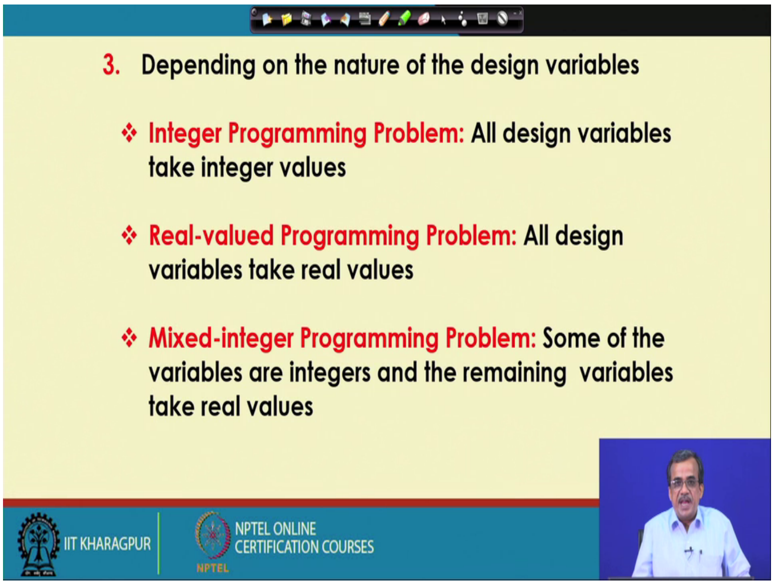

Now, the third classification is based on the nature of the design variables now based on the nature of the design variables optimization problem can be classified into 3 groups. The first one is the linear programming problem for all the design variables or the decision variables are integer; that means the whole number like 10 11 12 like this. Now for the real valued programming problem the design variables are to be fraction or the real number like 10.5 12.6 and so on.

Now, the third type that is called the mixed integer programming problem, here some of the variables will be integer and the other variables will be real. So, this is a combination.

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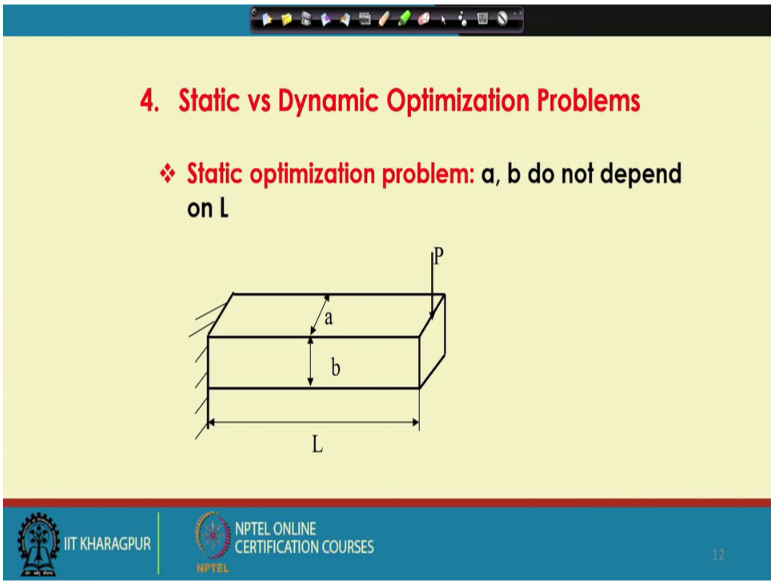

The next classification is either it is static optimization problem or dynamic optimization problem. Now let us try to explain what do you mean by the static optimization problem now let me take 1 very simple example, now this example is supposing that I have got one cantilever beam and here there is one concentrated load acting that is p and this beam is having the length L and cross section having the dimensions a and b and here the cross section has been kept constant throughout the length.

Now, what is our aim is to design one optimal cantilever beam. So, that there should not be any mechanical breakage during it is working and our aim is to find out a beam which is having the minimum weight now if this is the optimization problem. So, I can formulate as an optimization problem mathematically and here a and b does not depend on L and it they are not varying with L. So, this type of optimization problem is known as your the static optimization problem.

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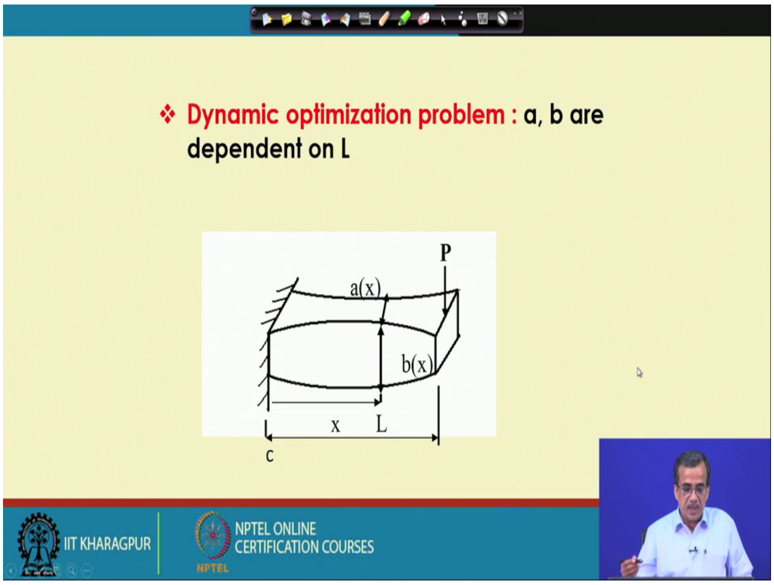

Now, the next is the dynamic optimization problem now here I am taking the same example of the cantilever beam, but here this particular section is varying a long this length of the beam a and b are varying with L that is the length of the beam; that means, your cross section is not kept fixed along the length and this type of optimization problem is known as the dynamic optimization problem here are some of the variables design variables are dependent on, some other design variables and this type of optimization problem is known as the your the dynamic optimization problem now this is how to classify the optimization problems.

Now, I am going to discuss how does an optimization tool work? So, what is the working principle of this particular the optimization tool.

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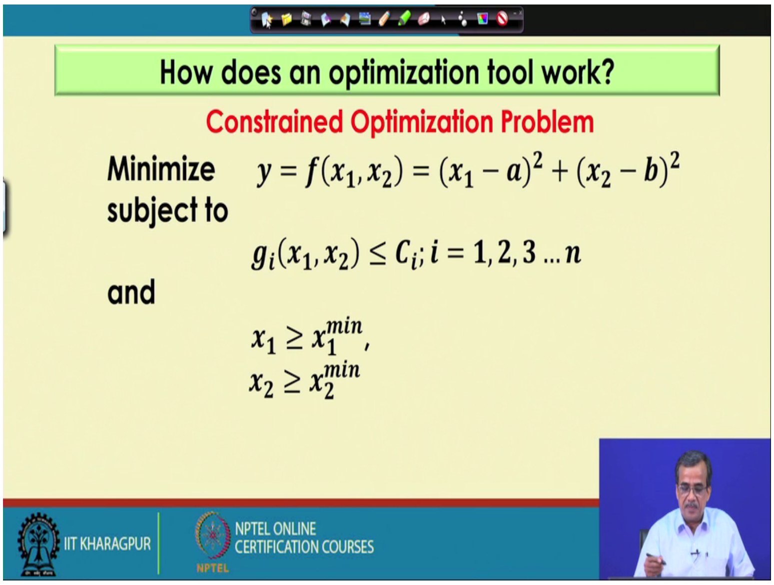

Now to explain this let me try to take the example of a constrained optimization problem; that means, there will be objective function that is y is a function of 2 variables x 1 and x 2 that is this particular mathematical expression I am just going to consider for the objective function subject to subject to the condition subject to the condition the gi x 1 x 2 is less than equals to c I or I is 1 2 up to n.

So, this is nothing, but the functional constraint and there are n number of functional constraint and x 1 is greater than equals to x 1 minimum and x 2 is greater than equals to x 2 minimum. So, these 2 are the variable bounds. So, this is nothing, but a constrained optimization problem. Now let us see how to solve this particular the constrained optimization problem, now to solve this constrained optimization problem.

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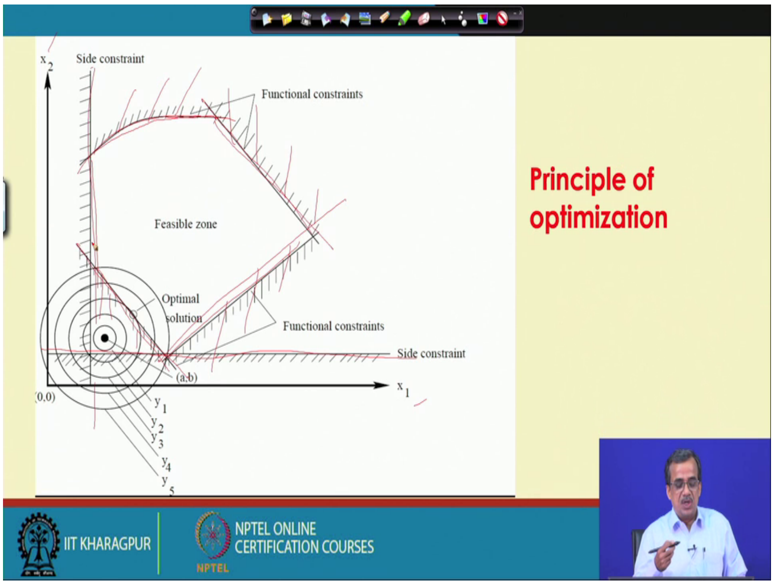

What we do is we try to make a plot of the design variables like x 1 and x 2 and here actually what we do is first we try to plot the side constraint the first side constraint was x 1 is greater than equals to x 1 minimum. So, this is actually the side constraint and the left side is infeasible right side is the feasible 1.

The second side constraint was x 2 should be greater than equals x 2 minimum. So, bottom is infeasible top is feasible. So, I am getting the feasible zone like this now I am just going to draw the functional constraint supposing that this is the first functional constraint. So, this side is the infeasible and this side is the feasible 1 might be the second functional constraint is this. So, this side is infeasible other side is feasible the third functional constraint is something like this. So, this side is infeasible this side is feasible and the fourth functional constraint is this this side is infeasible and this side is feasible.

So, I am getting one feasible zone it is something like this. So, this is nothing, but the feasible zone for this particular your optimization problem. Now any point which are lying inside the feasible zone are known as the free points and the points, which are lying on the boundary of this particular feasible zone are known as the bound points. Now optimal solution could be either the free points lying inside the feasible zone or the bound points which are lying on the boundary of the feasible zone.

Now, how to find out the optimal solution how does an optimization algorithm tries to locate the optimal solution. So, what you do is. So, we have got we have got. So, we have got this objective function that is your this y is f f of x 1 x 2. So, that is nothing, but x 1 minus a square plus x 2 minus b square and this is a minimization problem. Now all of us we know that the minimum value of this particular function is nothing, but 0 because this is the sum of 2 squares and corresponding to the solution 0 the value of x 1 will be equal to 1 and value of x 2 will be equal to b.

Now, so if I consider a particular value for this particular y that is say 0, now what we can do is. So, corresponding to this y equals to 0. So, I can draw 1 circle and that will be nothing, but this point circle. So, that will be the point circle now similarly what we can do is we can take some other value for this particular y say y 1. So, corresponding to this y 1 so I will be getting 1 circle here corresponding to y 2 I will be getting another circle here. Similarly for y 3 another circle, y 4 another circle, y 5 another circle, we will be getting and this is nothing, but the contour plots for this particular the objective function.

Now, the moment I consider y equals to zero; that means I am here. So, this particular point is actually the optimal solution if I consider the unconstrained optimization problem; that means, there is no such functional constraint. The moment we consider the functional constraint this will not remain as the optimal solution because this is lying in the infeasible zone. So, what we will have to do is we will have to consider the next higher value for this y say y 1. Now if I consider y 1 still it this particular circle is there within the infeasible zone, if I take y 2 that corresponds to a circle which is also lying in the infeasible zone, but if I consider y equals to y 3. So, that particular circle is going to touch the feasible zone at this particular point.

That is why. So, this particular point is called the optimal solution for the constrained optimization problem; however, for the unconstrained optimization problem this is the optimal solution; that means, if I put some constraints in terms of quality. So, I will have to quality of the solution in terms of the objective function value and let me repeat once again for the unconstrained optimization problem. So, this is the optimal solution and for the constrained optimization problem. So, this is the optimal solution.

So, this is the way actually one optimization tool works and for any optimization tool actually what we will have to do is we will have to find out the search direction and we will have to find out the step length for each of the iterations. Now the way 1 optimization tool determine the search direction and the step length the that way actually varies from tool to tool techniques to techniques and that is why we have got a large number of traditional tools for optimization.