Gear and Gear Unit Design: Theory and Practice
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Lecture – 09
Crossed Helical Gear – I

Module 2 design of spur straight and helical bevel and worm gears and this is lecture number 9 and in this lecture we shall learn about cross helical gear and this is part 1.

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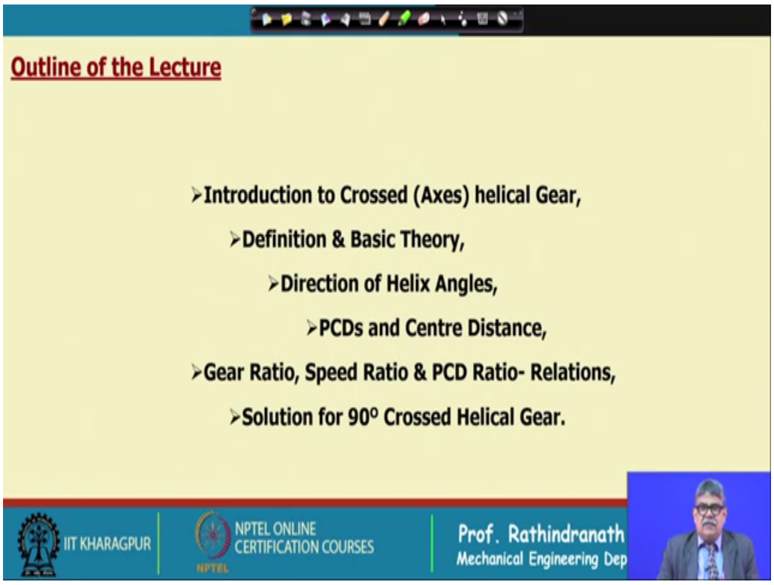

This lecture will cover introduction to crossed axes helical gear, definition and basic theory, direction of helix angles, pitch circle diameters, and center distance, gear ratio, speed ratio, and pitch circle diameter ratio and their relations. And finally, we will solve a problem with 90 degree crossed helical gears; that means, shafts are at 90 degree.

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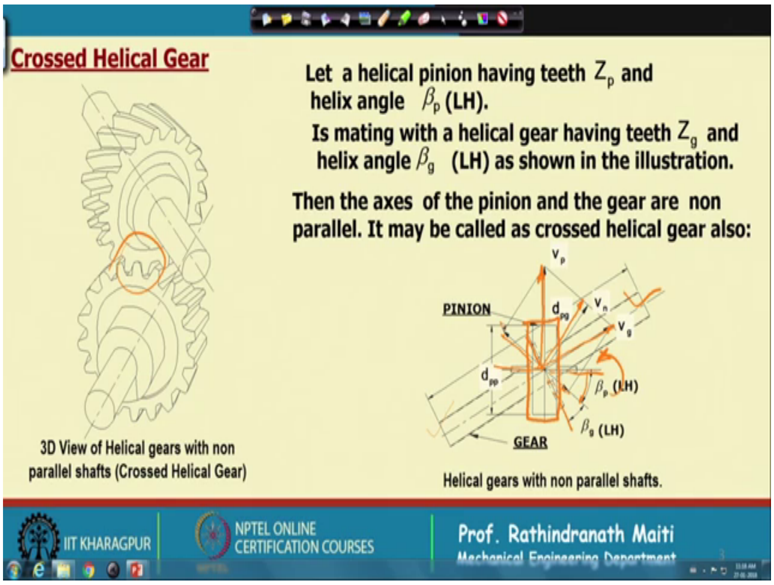

Now, let a helical pinion having teeth number Z P and helix angle beta P, which is left hand as we see in the figure left hand side 3 d view the top one is the pinion and bottom one is the gear. It is mating with the gear, which is at the bottom of having teeth number Z g and helix angle beta g this is also left hand as shown in the illustration..

Now if we look into the right hand side figure that is the geometric view of that in that figure it is shown the pinion at the top. So, this is the pinion.

So, this is the pinion and this is the pinion shaft. And it is having the helix angle beta P left hand of this, this much ok. You see this is this is you have to recognize as that this teeth, whatever the machine teeth is shown at the bottom, if we consider at the top it is in this directions. So, that will be left hand that that visualization, that understanding should be very clear..

Now this is mating with the gear which is this one and it is shaft axis is here and it is having the beta g the helix angle, that is also left hand if we here the tooth mating tooth is shown that is on the gear it is at the top. So, this will be the left hand here if we look into the left hand side one can visualize this.

Now, this let us consider this is rotating the direction is not shown, it is if we consider the pinion it is rotating like this from this side from the right hand side if we look it is rotating in the anti clockwise directions. So, the pitch line velocity will be V P of the opinion and for the gear it is moving in these directions that will be V Z.

Now, if we draw a line perpendicular at the midpoint of the gears and pinion a perpendicular to the teeth in mace that is on that line, then we will get this is the common velocity normal velocity in this directions and as well it will have a velocity also in this directions. Which can be further resolved into the on the on other directions also anyway this is the geometric figure of the crossed helical gear.

Now, we call crossed it can also be called or it is called that non parallel shaft helix and helical gear if we normally say then we will consider the 2 parallel shaft and the power is being transmitted from one shaft to other through the teeth spur gear teeth, which is helical , but once we call crossed helical gear then we have to consider that these 2 shafts are neither intersecting and not are parallel.

So, here this angle is given it is not even ninety degree it is some angle is there. So, we shall once we mean that crossed helical gear it will mean the non-parallel shaft. And for that the helix angle to be in the same direction of both the pinion and gear.

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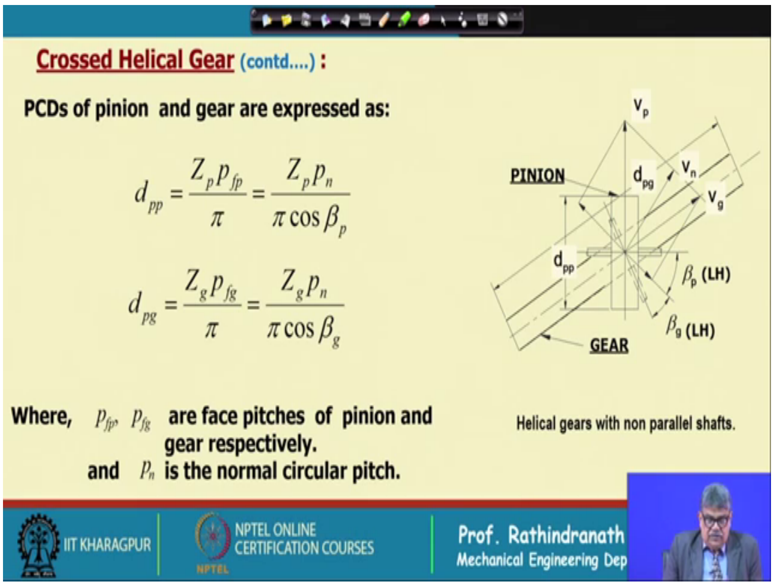

Now, if we calculate their pitch circle diameters, then pinion pitch circle diameters which is designated as d P P. The subscript P first P stands for the pitch circle, second P stands for the pinion that is the number of teeth into P f P by pi whereas, P f P is the face pitch.

Face pitch means if we consider the pitch along the surface of the gear or surface of the pinion in this case P f P is the face pitch of the pinion, that is that will be number of teeth into the normal pitch divided by cos of the helix angle.

So, this relation becomes d P P is equal to Z P P n by pi cos beta P. Similarly the pitch circle diameter of the gear is expressed by Z j into normal pitch divided by pi into cos of helix angle of the gear. Here as already told that P f P and P f P f g are the face pitches of pinion and gear respectively and P n is the normal pitch in the normal directions. So, one pitch we are measuring say for example, in this case.

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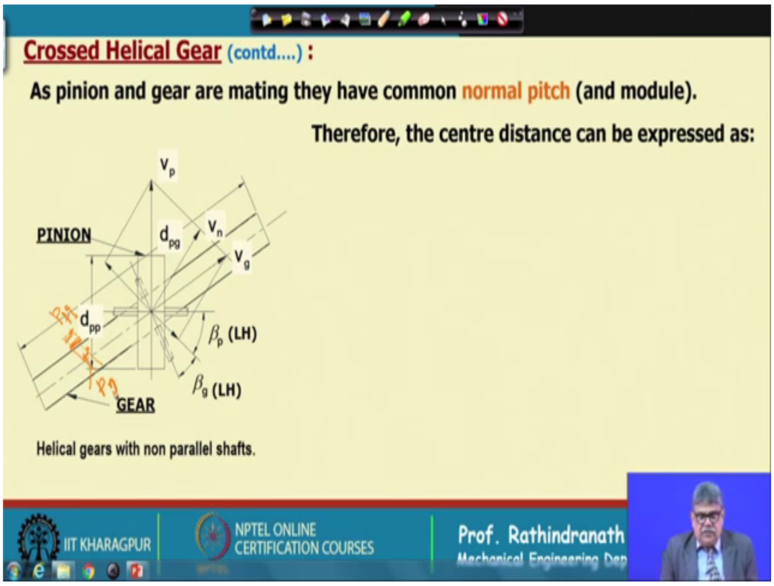

The pitch is like this if we measure the pitch say this helix is shown in this directions on the gears. So, this is our face pitch and normal pitch will be in these directions sorry this will be the face pitch. And if we consider the normal that will be the normal pitch P n n normal pitch which is given by the relations what we have already discussed.

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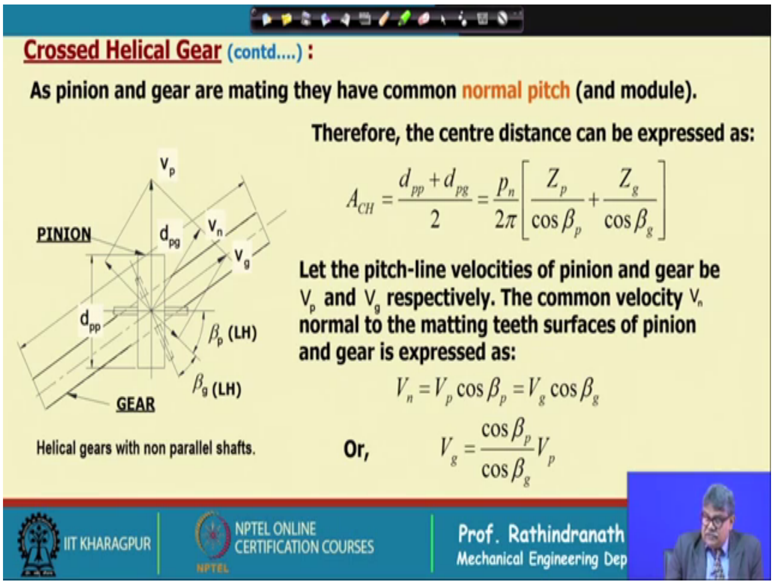

As pinion gear are mating they have common normal pitch has to be and therefore, the module also same therefore, the center distance can be expressed as A C H the center distance the C H stands for crossed helical is pitch circle diameter of pinion and pitch circle diameter of gear summation of that divided by 2, which can be expressed as normal pitch divided by 2 pi whole into whole the Z P, by teeth number of pinion divided by cos of helix angle of the pinion plus, teeth number of gear divided by cos of helix angle of the gear.

Let the pitch line velocity of pinion and gear be V P and V g what already explained respectively, the common velocity V n normal to the mating teeth surface of pinion and gear is expressed as V n is equal to V P cos beta P, and that is equal to V g cos beta g this can be visualized from the figure the left hand side figure. Therefore, V g can be expressed as cos of helix angle of pinion divided by cos of helix angle of gear into pitch line velocity of the pinion that is from these relations.

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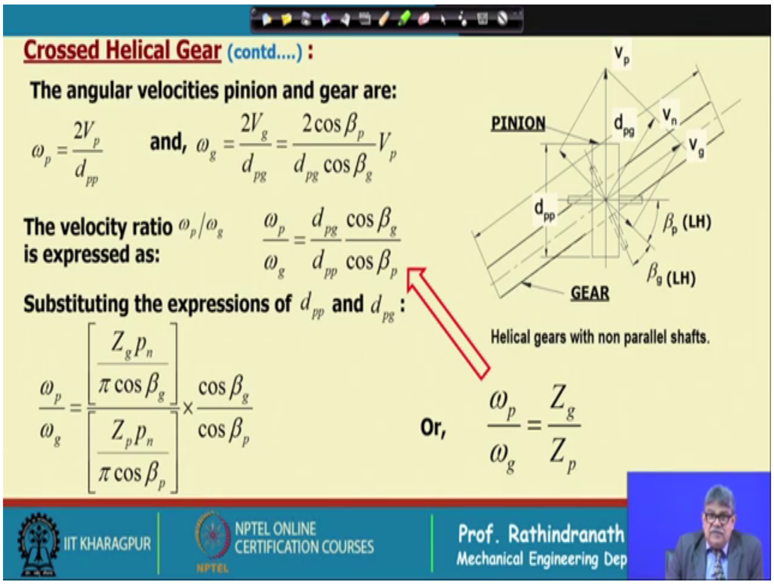

Therefore, in next page what we do the we estimate the angular velocity, omega P is the angular speed of the pinion which is expressed as twice in to pitch line velocity of the pinion, divided by the pitch circle diameter of the pinion..

Similarly omega g that is the angular speed of the gear can be expressed by twice into each line velocity of the gear divided by each circle diameter of the gear, which is equal to if we substitute the values twice into cos beta P divided by d P g into cos beta g into V P.

So, from there we can find the velocity ratio that is in this case we have considered the velocity ratio is equal to the angular speed of the pinion divided by angular speed of the gear, which can be expressed by d P g cos beta g divided by d P p cos beta P that is from the relations already we have derived.

Now, if we substitute the expressions of d P P and d P g what we already we have derived, then the relation becomes omega P by omega g is equal to teeth number of gear into normal pitch divided by pi into cos of helix angle of gear, whole divided by teeth number opinion into normal pitch divided by pi into cos of helix angle of pinion, whole into that ratio into cos of helix angle of gear divided by cos of helix angle of pinion..

If we equate further which will become omega P by omega g is equal to teeth number of gears divided by teeth number of pinion, just keep in mind that pinion is smaller than gear in most of the cases then omega P will be higher, than the omega g and as well the teeth number of gears will be higher than teeth number of pinion. So, these relations you can easily understands.

Now, in case of parallel shaft and in case of crossed helical shaft the speed ratio is directly the ratio of teeth numbers. Whether it is speed of gear divided by speed of pinion or vice versa that will be always the respective teeth number of the gears reciprocate of that ok..

However, if we express this in terms of diameter of the gears, then the relations will be omega P divided by omega g is equal to diameter of pitch circle diameter of gears into cos of helix angle of the gear, divided by pitch circle diameter of the pinion into cos of helix angle of beta P. That means, this helix angle must be expressed, now if we consider these are parallel shaft then definitely cos beta g is equal to cost beta P.

So, this relation we will then directly will become the ratio of the diameters cos terms cos of (Refer Time: 15:17) angle term will simply vanish. Here again I repeat that in case of crossed helical gear or non parallel sacked gears, the angles are different helix angle are difference not only that they are directions are also same. If it is pinion is left hand, then it will be left hand for gear also whereas, in case of parallel shaft this helix angle if one is right hand other will be left hand and vice versa ok. That we should keep in mind their magnitude will be same also in case of parallel shaft.

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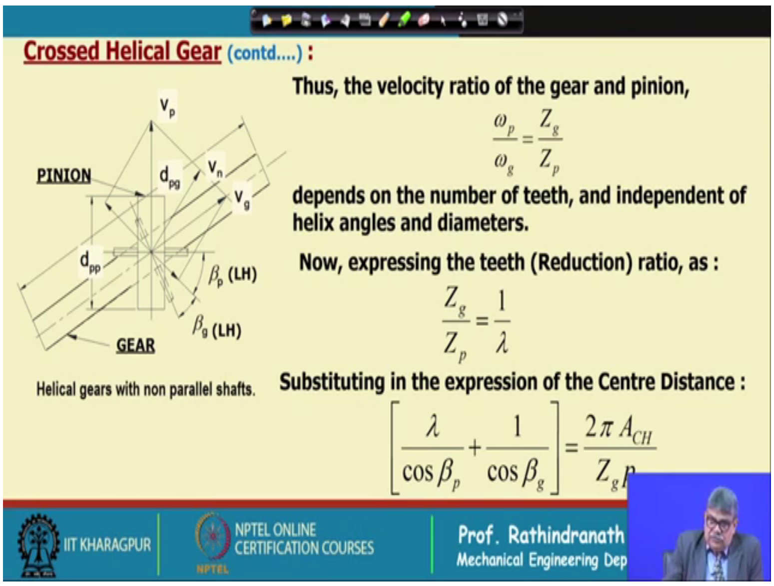

Now the velocity ratio of the gear and pinion what already we have derived w P by w g is teeth number of gear divided by teeth number of pinion, depends on the number of teeth and independent of helix angles and diameters.

Now, expressing the teeth or reduction ratio, now here one thing we should remember when we mean the gear ratio that might be either the ratio of opinion by gear or gear by pinion we have to mention it , but a normal case if he mention a reduction ratio which is also commonly used term in that case, normally the it is the ratio of which is speed is being reduced torque being increased it is normal case and in that case the ratio e will be number of teeth of the gear divided by the number of teeth of the pinion.

Now, that we have taken as one by lambda and this means that lambda is equal to number of teeth divided by number of teeth of pinion divided by number teeth of gears this ratio we should remember. Then we substitute this ratio in the formula already which you which we have derived for the center distance A C H.

So, this becomes lambda by cos of helix angle of pinion plus 1 by cos of helix angle of the gear that must be equal to twice pi a into C H that is the center distance, divided by teeth number of gear into the circular pitch; that means, Z P we have eliminated by the ratio lambda.

So, now if this is this formula is expressed in terms of number of teeth of the gear and the gear ratio lambda and the helix angle.

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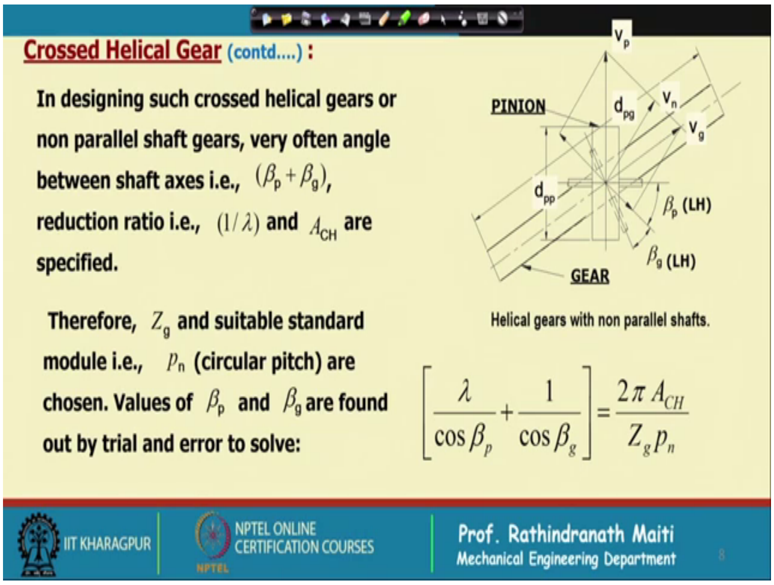

Then, if we would like to design such gears normally what we will find that the angle of beta P I mean helix angle of gear plus helix angle of pinion that is mentions. See for example, if we look into the right side figure say this angle will be defined, because we have the drive the prime over input from one side output from the other sides, their angle is such that we have to define this angle that is summation of beta P and beta g according to their position.

So, this will be defined. And then also what will be the reduction ratio in this case reduction ratio will be one by lambda that will be given and also center distance, because center distance will mean that the distance between the input and output shaft axis

So, this will be mentioned and then Z g the number of teeth of the gear and suitable standard module circular pitch, we have to choose. And finally, values of the helix angle of pinion and the helixum angle of gear are found out by trial and error to solve the equations, because if you look into this equation that which is given in 2 right hand side, that we do not know what should be the angle of beat what the angle is beta P and what the angle is beta g although we know their summation, center distance is given and this P n we will select; that means, module we will select and this ratio is given..

So, this from a single equation we cannot solve this. So, we have to use some trial and error method.

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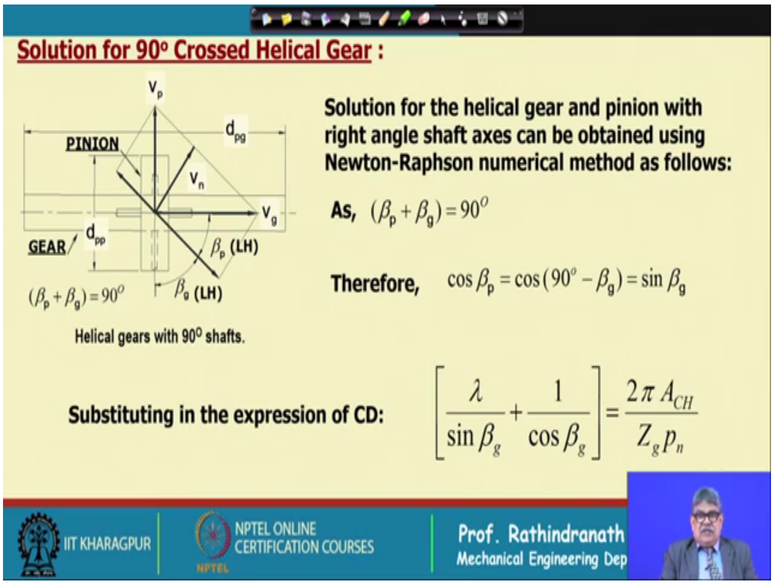

Now, obviously, if this angle is not 90 degree in that case, we have to start from the wild we have to take some value we have to adjust and then, we have to see that whether it is matching and again we have to retake this value. So, it is cumbersome and tedious, but if it is 90 degree; that means, if one shaft is perpendicular to the other although they are not intersecting in that case there might have solution that we will try.

Now, the figure left hand side what we have shown that the pinion at the top and gear at the bottom they are at 90 degree. So, this means that input axis that is the axis of the pinion is parallel to the surface of the gear and axis of the gear shaft that is parallel to the surface of the pinion surface parallel to the pinion surface.

And this velocity diagram it will be same as before, but in this case beta P plus beta g summation of the helix angle of gear and pinion is equal to 90 degree.

Let us see whether it can be solved analytically or some other methods easily. Now here I would like to mention that these also give us the idea of warm gearing, this is exactly same as axes wise same as the warm gearing. So, we will later see that warm gearing is nothing, but the 90 degree crossed helical gearing and solution goes in the same direction.

Now, solution for the helical gear and pinion with right angle shaft axes can be obtained using Newton-Raphson numerical method as follows. Now again I would say that although, if we put 90 degree that final equation expression of the center distance will be further reduced to a single helix angle that can be done, but still there we cannot solve it directly analytical solution is not available.

So, we can use Newton-Raphson numerical method and we can find the magnitude of the angles and other relations now as beta P plus beta Z is equal to 90 degree. Therefore, cos beta b P will be equal to cos of 90 degree minus beta g and which is nothing, but sine beta g. Substituting in the expression of center distance we get lambda by sign of helix angle of gear plus 1 by cos of helix angle of gear, whole equal to twice pi into center distance divided by tooth number of gear into circular pitch circular normal pitch.

Now, here lambda is only related to the teeth number of gears and beta P is equal to beta g, but what we find that from this equation there is a possibility we can calculate the beta g still this equation cannot be solved analytically.

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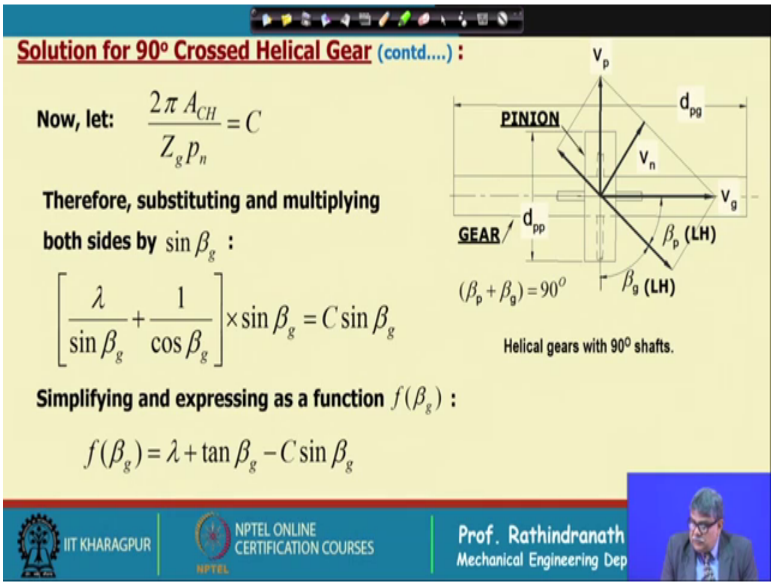

Now what we consider let this expression in the expression twice pi into center distance, divided by teeth number of gear into circular pitch normal circular pitch is equal to each expressed A by C and then the we also multiply both side by sine beta g, then the equation becomes lambda plus 10 of helix angle of beta g minus C into sine beta g, and then as we are trying to solve this then this we will express as a function of theta g.

So, it is f beta g is equal to lambda plus 10 beta g minus C sine beta g ok.

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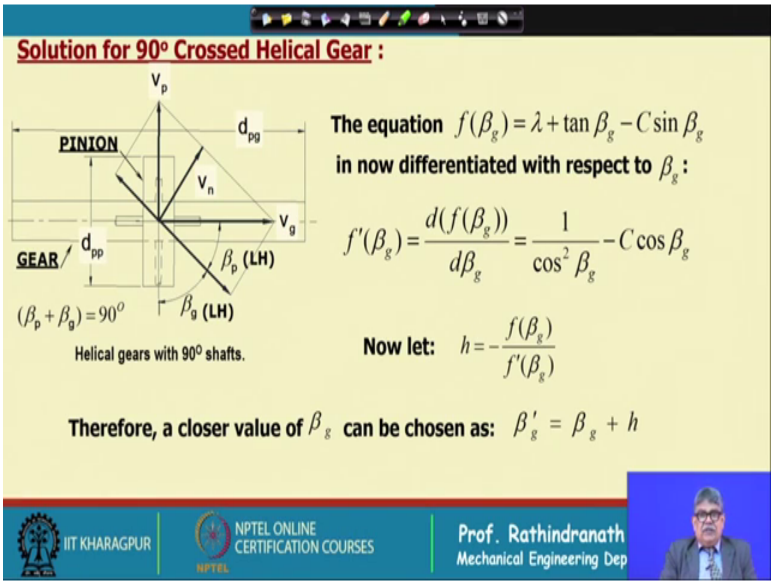

Then we differentiate with respect to beta g that is the helix angle of gear. This we express as f dash of f f dash beta g which is nothing, but d by f beta g that function divided by d beta g, which becomes 1 by cos square beta g minus C into cos beta g.

So, this is a standard differential differentiating process. So, this I think you understand. And then we consider h a incremental value is equal to what is the Newton Raphson methods is equal to minus function of beta g divided by that differential differentiated function of beta g.

Therefore a closure value of now beta g ; that means, the first trial value after the second trial value beta g can now be improved to beta dash g, which is equal to beta g plus h that incremental value.

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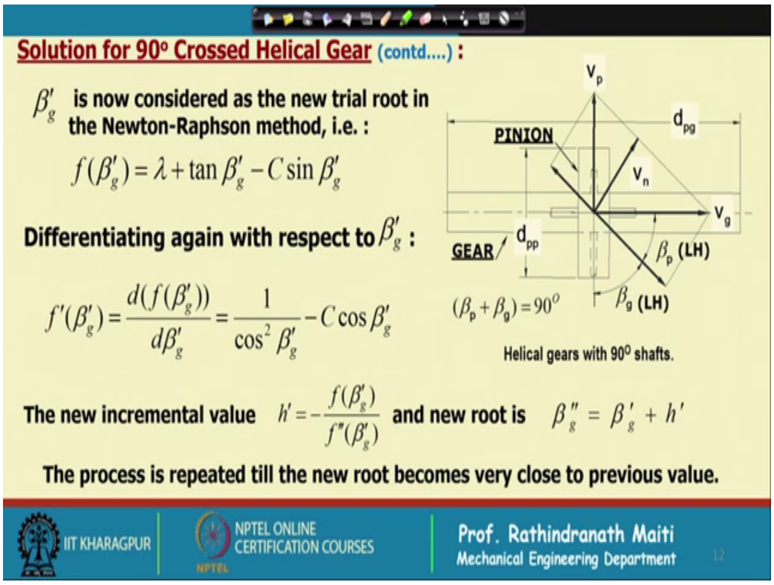

And beta dash g which is the new root which considered as the new trial root for Newton Raphson method, then we define a function of beta dash g is equal to lambda into tan beta g beta dash g minus C into sine beta g and for which the after differentiating it becomes f dash beta dash g is equal to d of function of beta dash g divided by d of beta dash g, which becomes one by cos square beta g dash minus C into cos beta dash g the same way.

And then we defined the next incremental value h dash is equal to f beta dash g divided by f double dash beta g on this only single dash not a double dash. And ah; however, this we next consider a new angle beta double dash g is equal to beta dash g plus h dash.

So, this is the new root which we are considering the process is repeated till the new root value becomes very close to the previous value.

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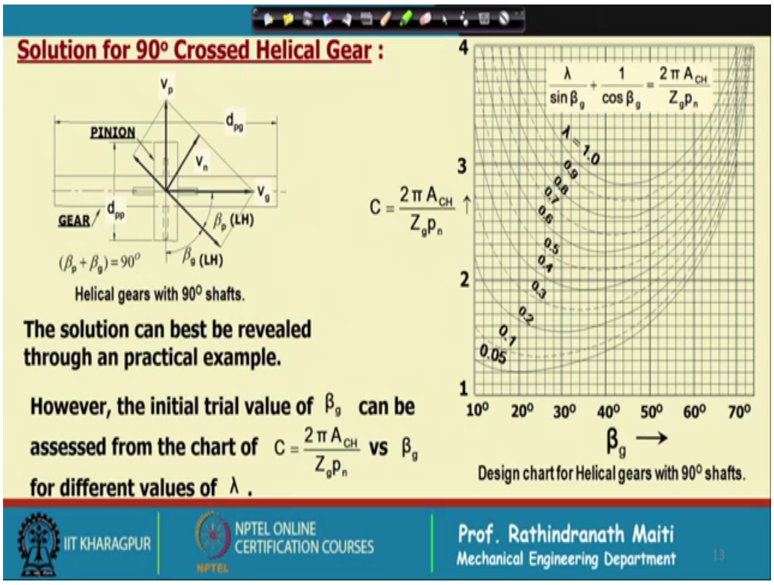

Usually it has been seen 1 or 2 iteration 1 or 2 trial we will arrived into A and acceptable values for beta g and beta P. Now the solution can be can best be revealed through an practical example.

However the initial trial value of beta g can be assessed from the chart of the C which is twice pi into center distance divided by teeth number of gear into circular pitch versus the helix angle of the gear. For different value of lambda, now the graph will look like this in this graph what we have expressed in the y directions C is equal to twice by pi into center distance, divided by teeth number of gear into the circular normal pitch, which is maybe 1 2 3 4 usually it will within this limit and helix angle we have considered 10 to 70, because totally if this is for 90 degree crossed helical gear only.

So, this chart in this chart beta g plus beta P is equal to ninety degree and normally this angle beta g will be within 70 degree or so, and we have plotted these lines for different lambda, lambda we may be 1; where the teeth number of pinion and teeth number of gear are same and it may be as low as point not 5 which is at the bottom.

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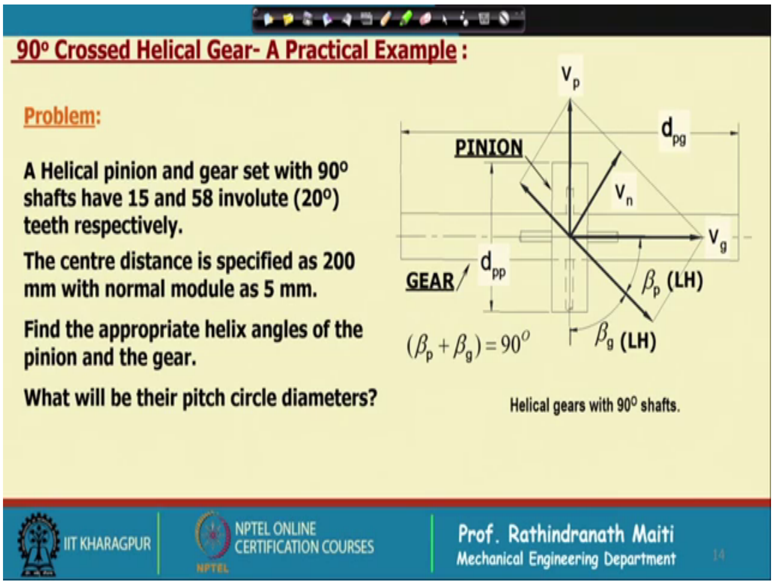

Now, the problem what we have chosen a Helical pinion and gear set with 90 degrees shafts have 15 and 58 involute teeth respectively; that means, pinion is having 15 teeth and gear is having 58 involute teeth 20 degree involute. This center distance is specified as 200 millimeter with normal module as 5 millimeter. Once we are expressing the normal module; that means, circular pitch normal circular pitch also defined, because it is pi into module, find the appropriate helix angle of the pinion and the gear and also what will be their pitch circle diameters that we have to calculate.

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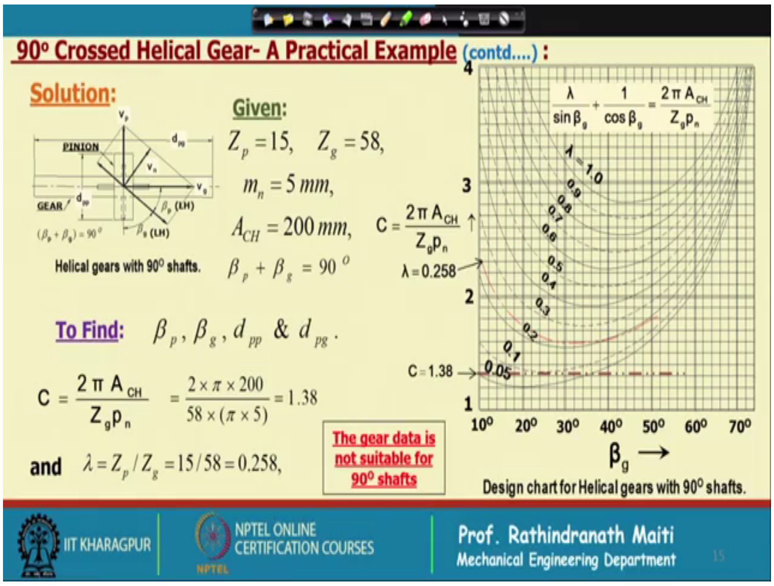

Now, what we have taken that we have we first calculate the required parameters say given parameters are death number of pinion is 15 teeth number of gear is 58, module is 5 millimeter, center distance is 200 millimeter and V P plus V g that summation of helix angle is equal to 90 degree. And we have to find out the helix angle size as as well as pitch circle diameters.

Now, first we calculate C which becomes 1.3 8 for the just mind it this is for the module 5. We would like to restrict the center distance 200 module 5 is good, but it can be varied that will check. So, we get it is 1.38 now this we have the C 1.38 it is plotted here ok. So, this line will come over here because this ratio is 1.38 whereas, lambda is 0.25 8, which is not available in this chart, but we can extrapolate or interpolate.

Now, this is not properly drawn, but still what we find that we are not getting any idea about the beta Z because this 2 curve is not intersecting, one is straight line and this curve is not intersecting; that means, gear data is not suitable for 90 degree shaft this means that if we would like to take pinion is 15 teeth gear is 58 teeth module is 5 200 millimeter and 90 degree cos shaft helical gear the solution is not possible.

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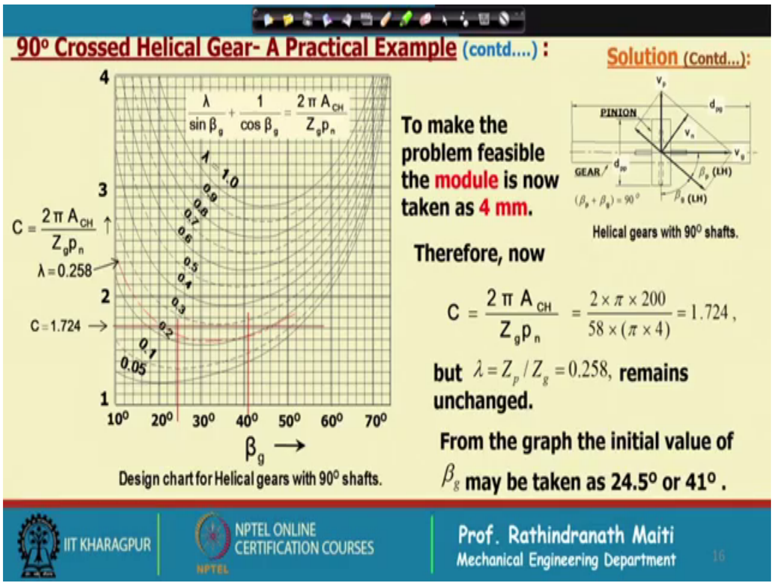

So, what we do now we take the module 4; that means, for the from the strength point of view maybe we have to take a better material, but it is still possible we can have the solution with 4 millimeter module. Now in that case the C we calculate 1.7 2 4, but C is not because it is changing due to the change in module in this case module have changed 4 millimeter.

So, this value has increased to 1.7 to 4 which gives the straight lines here and the lambda curve was as it is, because the gear ratio is not changed and what we find that there will be 2 points this curve is somehow a not placed properly it is somewhere here, it will be if we draw this curve it will come something like this for the value 0.2 5 8. And we get from this 2 points; that means, from this graph ultimately we will get that beta g may be taken as 24.5 degree or 41 degree.

So, what we find the initial value of the beta g is 2.4.5 that is for the initial value for starting with the Newton Raphson methods. So, instead of guessing any value from this chart we can have it is 24.5 or it will be 41 degree it has 2 solutions and then this means that ultimately beta g either will be very close to 24.5. I mean there will be 2 sets of solution in one set the value of beta g will be very close to 24.5 in other case it will be very close to 41 degree. And we shall in the next lecture wish we will see this how the solution is available.

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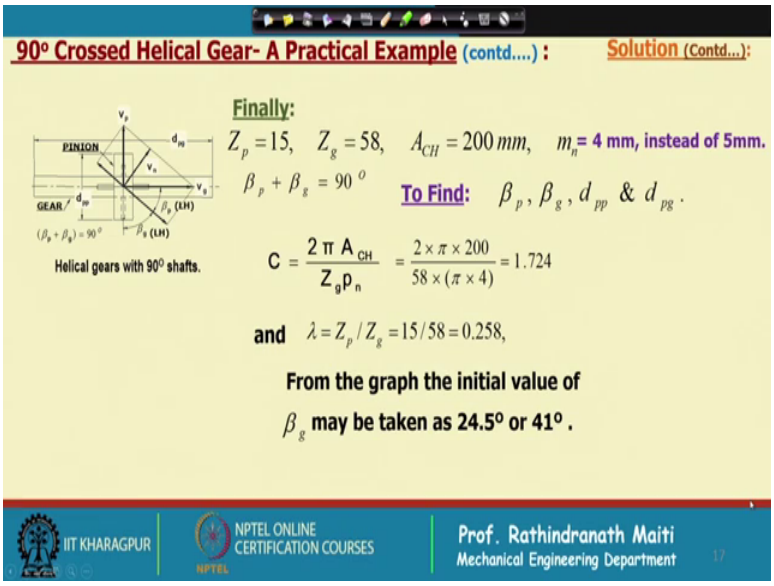

So, ultimately this 5 millimeter was not possible we have taken 4 millimeter and we will try the Newton Raphson method with 1 value 24.5 and other value 41 degree oh.

Thank you.