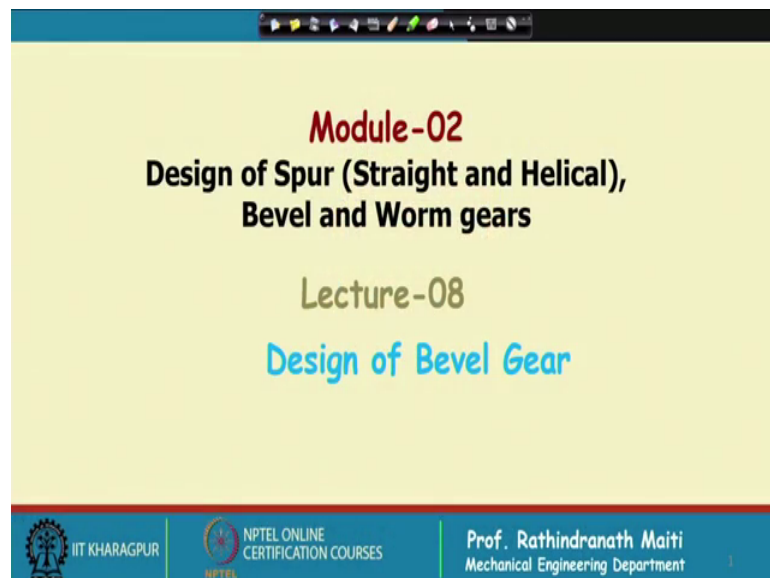


Gear and Gear Unit Design: Theory and Practice
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Lecture – 08
Design of Bevel Gear

Gear and gear unit design theory and practice, we are in module 2 which is design of spur, straight and helical, bevel and one gear. And this is this lecture is on design of Bevel gear.

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The slide content is as follows:

Module-02
**Design of Spur (Straight and Helical),
Bevel and Worm gears**

Lecture-08
Design of Bevel Gear

The slide also features a footer with the following information:

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Outline of the Lecture

- **Concept and Feature of Bevel Gears**
- **Nomenclature and Geometry of Straight Bevel Gear**
 - **Formative Number of Teeth of Straight Bevel Gear**
 - **Typical Gear Unit with Bevel and Helical Gears**
- **Bending Strength basis Design of Straight Bevel Gear**
- **Wear load Capacity and Probable Dynamic Load in Straight Bevel Gear**
- **Tooth Load in Straight Bevel Gear**

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In this lecture, first of all we will go through the concept and feature of bevel gears. Next nomenclature and geometry of straight bevel gear, formative number of teeth of straight bevel gear, typical gear unit with bevel and helical gears, bending strength basis design of straight bevel gear that is you can say Lewis formula, wear load capacity and probable dynamic load in straight bevel gear and finally, tooth load in straight bevel gear.

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Concept of Bevel Gear

Bevel gear
Hypoid gear
Worm gear
Straight Bevel
Spiral Bevel
Worm & Worm Wheel
Hypoid Bevel
Hemisphere Bevel

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Now, bevel gear; we need when the shafts are not only not parallel normally they are at 90 degree. And in the simplest form axis of the pinion and axis of the gear they are

intersecting as shown in 3 d view. Now this angle so, this is the axis and this axis in this case what we have shown this, they are intersecting this is a straight bevel we have shown state to the bevel and this angle may be 90 degree or may be different it might be more than 90 degree or less than 90 degree also, but in common cases you will find this is 90 degree.

Now, bevel gear again it might have also the shafts are not intersecting it is 90 degree less than 90 degree or more than 90 degree, but they are not again intersecting. So, that is possible with offsetting the inclining tooth in a directions and those are usually not straight inclination that is in sort of some spiral.

So, with that we have different kind of bevel gear also. So, this is a spiral bevel gear is shown in spiral bevel gear as you see the tooth are spiral form, it is same as in case of helical gear in case of spur gear straight tooth Spur gear or helical gear parallel shafts here in spiral bevel essentially the axis are intersecting although these are spiral, but these are intersecting as you see, but there is another as I told that this is offset this offset is shown both in this left hand figure and also in this figure.

This is called hypoid bevel and this hypoid bevel are widely used and it is having better performance and tooth load carrying capacity also as this is an offset that gives many advantage say for an example, in case of differential gearing; that tail shaft is coming at a point, but the whole unit can be lifted depending on the diameter of the wheel, etcetera and there will be more under clearance below the differential unit this is just to say this is one advantage.

But. In fact, if we think the strength point of view smooth operation where are all such point of view hypoid bevels will be better than straight bevel and as well as, then spiral bevel gear, but spiral bevel gear also very close to hypoid bevel gears only thing in, we if we need that shafts are intersecting we have to go for spiral bevel gear.

Now, this design of hypoid and spiral bevel gears are relatively complicated and in many cases, these tooth generation also complicated and they that is a some special methods are used which are recommended by the manufacturer of such bevel gear cutting machines and they also suggests, how to calculate the strength and other part.

design that is the width of this teeth and this length from the cone apex or pitch apex to the backside of the gears.

Now, again another important issue is here that this is we consider a mean value of this speech circle which is called which is this radius is called r_m which is r_1 plus r_2 by 2 or d_1 by d_2 by 4. Now, this is will be slightly shifted. It is at the midpoint. So, at the midpoint this sometimes all references are given to that mean radius we have followed in this design in the design procedure that radius as the reference radius where if we cut that surface along a and look into from the vertical directions we will look the standard straight tooth's spur profile.

If we take section along a I repeat and if we look from the perpendicular direction we will see this cut section has a like this, they straight Spur gear and this then in that case we can consider this angle is the standard pressure angle, suppose, if we take 20 degree that is this α will be 20 degree as well edge if we measure here.

So, this will be a circular pitch from this point to this point it will be the circular pitch this is πm and also at that section the module we can consider the module is standard; that means, if the teeth number is say 50 or Z we consider and module is m the $Z m$ will be r_m by 2, sorry, 2 into $j m$ divided by 2 will be r_m .

Now, like the helical gear this bevel gear is also having the formative number of teeth that I already I have described it this is given by $Z \text{ dash } Z \cos \gamma$; how it has come if we consider this surface this circle definitely the radius is the more from this point to this point which is $L \tan \gamma$ and then in that circle.

If we consider the equivalent number of teeth of straight for spur gear of the same module that number will be more which can be derived by $Z \text{ dash } Z \cos \gamma$ is equal to Z by $\cos \gamma$ and when we consider the Lewis form factor or any other design issues with respect to the equivalent number of teeth of spur gear straight tooth spur gear, we will consider this formative number of teeth.

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Straight Bevel Gear Design (90° Shafts) (Contd.....) :

Assembled plan view
(Not of the same one as below)

of 2-stage (Bevel-Helical) gear box.
(Top cover open)

Design of a Bevel- Helical Two Stage Gear Box:

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Now in this slides what I have shown that there is a gearbox, we have considered a gearbox in this if we look into the top open view in this case, what we find here as if there are two sets of gear bevel gear it is this is just to manufacturer the draw in this form this first one is one a pair of bevel pinion and bevel gear of one ratio and this and.

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Straight Bevel Gear Design (90° Shafts):

Normal Module (m_{bevel} , in meter),

$$m_{bevel} = \sqrt[3]{\frac{2T}{S_o C_w ZY \psi (1 - \psi_o)}}$$

Straight Tooth Bevel Gear

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So, this is one pair with one ratio and these two is other pair with other ratio, it is shown that in the same gearbox we can put these two sets of gear now next stage. So, this is the

input shaft and this is the intermediate shaft and then intermediate shaft other set of gears are helical or straight spur in case of helical gears are shown.

Now view will be something like this is the housing for this input pinion and there here is the bevel gear bevel here is not exactly straight spur gear and it is some sort of spiral gear or like helical bevel you can say and then what we find that this is having the helical gear helical gear set.

So, the basic purpose is that of using bevel gear that we are transmitting the power not parallelly, we are at an angle, this shafts may be intersecting and shaft may be nonintersecting and offset might be there in that case we have to use the high powered gear only. So, I have given the idea that what is the different type of gears and etcetera, but in this lecture or in this course we shall consider all the straight bevel gear and design of that gears and they again that is for 90 degree shafts.

Now, in this case this module we have written as m bevel that is the because the if we consider the whole teeth of the bevel then in that case it is inside it is of small teeth and outside, it is a bigger teeth and if you would like to compare with a standard one, we will find that if inner is matching with the standard one outer may not match because this is with the as that is gradually the tooth shaft has increased, we have in this design we have considered at the middle section that would be standard; that means, if we take the section through that vertical section we will get the standard state spur gear.

Now, that module will we have considered m bevel which is given by the formula twice T divided by S_0 is the allowable strength into the c vs the velocity vector divided by c_w and we can also sometimes the whole can be defined as S_d ; that means, that is the allowable strength we have considered designing this gear box or the designing this bevel gear.

Considering the necessary velocity factor necessary bevel factor etcetera and Z is the actual number of teeth y is the modified form factor Lewis form factor which is against the formative number of teeth; that means, actual number of teeth divided by \cos of the cone angle \cos of cone angle of that respective gear or pinion and then the ψ is the width factor if we multiply ψ with m we will get the actual width b t is the torque we have considered for the design it is it is maybe the nominal torque or maybe some multiple with some multiplied factor depending on the dynamic condition of the load

Now, ψ_0 is another factor to relate the.

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Straight Bevel Gear Design (90° Shafts):


Normal Module (m_{bevel} , in meter),
 For straight tooth bevel gear, can be estimated as :

$$m_{bevel} = \sqrt[3]{\frac{2T}{S_o c_v Z Y \psi (1 - \psi_0) c_w}}$$

Where:
 T = Torque (Nm),
 Z = Number of teeth,
 S_o = Allowable design strength (Pas),
 $S_o = \text{Yield strength} / 2.5 \text{ to } 3$

c_v = Velocity factor,
 = 1 for precision gears & no shock,
 1.2 for general purpose gear.

c_w = Wear load / Lubrication factor,
 = 1 for force lubrication &
 1.5 for sump/splash lubrication



Straight Tooth Bevel Gear

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The cone length T is the torque Z is the number of teeth. Now c_v is 1 for precision gears no shock 1.2 for general purpose gears. So, for bevel gear we can simplify that 1 or 1.2 c_w is there wear load and lubrication factor which we can take one for force lubrication 1.5 for sump and splash lubrications.

Here it can be mentioned that in comparison to helical or straight spur gear bevel gear needs more gear for lubrication because of their teeth and as the shafts are intersecting sometimes the contact is very poor usually bevel gear checked for contact after manufacturing after putting into the actual gear box using the contacts are tested.

If the contact is not proper then tooth is some correction is given on the teeth. So, that they come in contacts anyway for that if the lubrication say for examples for the same condition of lubrications where we can consider 1.25 for helical gear we should consider 1.5 for sump and splash lubrications S_d is allowable design strength in pascals that is d already I have shown. So, which can be given yield strength by sorry in this case I would say this is not S_d we would put here better that is 0 which is taken yield strength divided by 2.5 or 3 whereas, S_d as I told that whole part can be considered as a S_d .

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Straight Bevel Gear Design (90° Shafts):

Normal Module (m_{bevel} , in meter),
 For straight tooth bevel gear, can be estimated as: :

$$m_{bevel} = \sqrt[3]{\frac{2T}{\frac{S_o c_v}{c_w} ZY \psi (1 - \psi_o)}}$$

ψ = Width factor [active width (b) of gear/module],
 $\psi_o = b/l$ for bevel gear (See Fig. -2), usually 1/3 or less,

Modified Lewis form factor
 $Y = \pi(0.154 - 0.912/Z')$
 and for straight bevel gear,
 Formative number of teeth-
 $Z' = Z / \cos \gamma$



Straight Tooth Bevel Gear

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Now, we are continuing with the designs methods now this psi is width factor active width b ah divided by the module whereas, size 0 is give taken as v by L b is the width L is that cone angle and initial design as psi also we have to assume something.


So, v by L is assumed 1 by 3 later after the first calculation of the module first estimation of the module what you can do we can find out then cone angle etcetera cone length and then again we can calculate b by L that is psi 0 value and we can re check whether model is satisfactory or not if necessary that value can be increased or decreased a little bit.

Then the Y is same formula is used as in case of straights spur gear and considering the formative number of teeth Z dash which is given here.

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Straight Bevel Gear Design (90° Shafts):

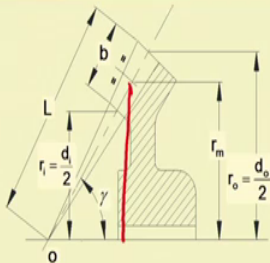
Normal Module (m_{bevel} , in meter),
For straight tooth bevel gear:

$$m_{bevel} = \sqrt[3]{\frac{2T}{\frac{S_o c_v}{c_w} ZY \psi (1 - \psi_o)}}$$


Mean PCD (Straight Bevel)
 $= 2 \times \text{mean } r = Z \times m_{bevel}$

Other relations.
 $\gamma_p + \gamma_g = 90^\circ$

And, $\tan \gamma_p / \tan \gamma_g = Z_p / Z_g$



Formative number of teeth for straight bevel gear,
 $Z' = Z / \cos \gamma$

$\gamma = \text{pitch cone angle.}$

Straight Tooth Bevel Gear

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Now mean PCD is r_m or d_m you can say that is given by Z into module of bevel we have considered, in this way in some design it is outside pitch circle radius is considered in this case we have considered the mean radius and γ_p is the cone angle for pinion and γ_g is equal to cone angle of the gear and we have considered in this design $\gamma_p + \gamma_g = 90^\circ$ and if the teeth number are same then γ_p is equal to γ_g , but that is very rare which is called metering gear.

γ is pitch cone angle and we should remember that this relation actually if I consider this radius and other side this is the radius a the ratio of this must be equal to in terms of this is the common. So, there will come $\tan \gamma_p$ and $\tan \gamma_g$ which is Z_p by Z_g . So, with this relations, we can further proceed into design.

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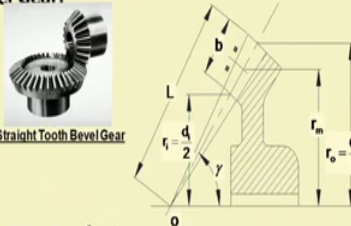
Straight Bevel Gear Design (90° Shafts):

Wear Load Capacity of Straight Tooth Bevel Gear and Probable Dynamic Load:

Wear Load Capacity of Straight Tooth Bevel Gear:

$$F_w = \frac{d_{mp}}{\cos \gamma_p} b K Q_g = \frac{d_{mg}}{\cos \gamma_g} b K Q_p$$

Where,

$$Q_g = \frac{2Z'_g}{Z'_g + Z'_p} \quad \text{and} \quad Q_p = \frac{2Z'_p}{Z'_g + Z'_p}$$


K Is same as defined in case of spur gear against gear and pinion teeth hardness.

Z'_g, Z'_p Formative number of teeth of gear and pinion

And Finally :

$$F_w \geq 1.15 F_d$$

Probable Dynamic Load is Estimated as In Spur Gear

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Now so, we are load capacity of straight bevel gear. So, we have already given I have shown the formula for estimating the module this is in this case only there is another size 0 factor is coming who if we consider we can assume that what might be the value of that then we can arrived into design we should keep in mind that be here this psi factor is not as high as in case of helical gear usually this psi factor is kept very close to 10; that means, width is equal to module that m model into 10 or plus minus 1 or 2 something like that it is kept like that and L b by L is taken 1 by 3 for any cell design and later in finalizing, we can of course, vary a little bit those things.

Now, the wear load capacity of the bevel gear tooth; that means, at the contact surface can be derived as DMP that is mean diameter of the pinion we will get divided by cos gamma p into be the width which you have taken k is the same that factor wear load factor and Q Z which also can be written d m g that is diameter of the gears divided by cos gamma g b into k in that case q p just look into this if here in the first case if you use the diameter of the pinion then relation will be Q g and if we use the diameter of the gear then that will be Q p and this Q g and Q p Q g is written two into number of teeth of gears formative number of teeth of gears divided by formative number of teeth of gear plus formative number teeth of pinion in case of Q p the numerator in numerator Z dash g will be replaced by Z dash p.

So, this calculation of Q_p is not a problem and k is same as defined in case of spur gear against gear and pinion teeth. Now hardness. So, when we are considering this hardness for the helical and straight spur gear in the same way we can consider for the bevel gear also only thing, we have to consider the actual the pressure angle standard pressure angle we have taken this means that first, we calculate we are load and for the dynamic load we consider the normal load acting at that contact point multiplied by some factor velocity factor we had considered multiplied or divided by velocity factor depending on how we are using this velocity vector.

Then we are considering that F_w is must be greater than equal to $1.15 F_d$ this means against I repeat that whatever the wear load capacity at the contact point dynamic load should be less than that it is usually 1.15 times less than that. So, the life will be infinity; that means, there will not be easily pitting or wear if the dynamic load is more the first possibility is that the; there will be pitting there will be material removal at the contact points if not there is the tooth breakage.

So, for bevel gear design the care is taken same as we are designing straight tooth spur gear or helical gear only difference is that the formula is slightly different, but that too is not difficult we have to assess b and L at the first level and we need to go for the second level calculations because we need to modify this cone radius L capital L and each cone radius as well as also the b the weight.

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Straight Bevel Gear Design (90° Shafts) (Contd.....) :

Tooth Loads of Straight Tooth Bevel Gear:

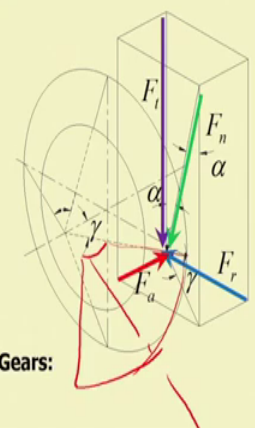
Tangential Load $F_t = \frac{2T}{r_m}$

Normal Load $F_n = F_t \sec \alpha$

Radial Load $F_r = F_t \tan \alpha \cos \gamma$

Axial Load $F_a = F_t \tan \alpha \sin \gamma$

It is to be noted that for 90° Shafts Straight Bevel Gears:
 $F_{ig} = F_{tp}$, $F_{ag} = F_{rp}$ and $F_{rg} = F_{ap}$



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now if we consider the load on the gear teeth as you find we have taken the load at the midpoint we know the radius there which is Z bevel into m by 2 and torque divided by 2 will give the tangential load F_t one F_t is calculated then F_n can be taken as F_t into \sec of α is the standard pressure angle.

So, F_n is acting like this from this corner to the opposite corner at the tooth point and F_r straight way from these geometric lessons it can be taken as F_t into $\tan \alpha$ into $\cos \gamma$ $\cos \gamma$ is the cone pitch angle, we are calculating on this gears and this axial load $F_t \tan \alpha$ into $\sin \gamma$ ok. This is the axial load which is acting along the axis and interestingly in case of 90 degrees shafts straight bevel gears what is the tangential load acting on the pinion that is that will be also acting the same load acting on the pinion of course, it will be in the opposite directions we have shown only the magnitude nor the directions here and axial load of on gear is nothing, but the radial load in the pinion because suppose this is the gear then pinion must be somewhere here.

So, whatever the axial load is there that will become the radial load on the pinion and what is the radial load on this gear that will be the radial axial load on the pinion. So, that is to be remember and if we calculate considering any one of that other can be easily calculated and these all these loads depending on the point of acting that is the midpoint that is directly going on this shaft and usually bevel gears particularly bevel pinion will be a cantilever one.

So, considering this shaft loads from this configuration from this load calculations we can now calculate the what are the load coming on the bearing and as well as we can also find that is what will be the shaft bending moment of the shaft calculation everything can be done and usually bearing spacing, there you can say that in case of pinion that is one thumb rule is three times the diameter larger diameter of on which we have put one bearing.

So, in this lecture we have learnt about the design of bevel gear and we have all the basic idea how to estimate the module of the state bevel gear and how the loads are acting on that and from which we can find out the loads on the shaft and um in this series of lectures there is not scope of detail designing the bevel gears; however, we will solve some problem on bevel gears that will just to learn that what might be might be the modules for a same amount of load for which we go for helical gears and maybe we will

learn about to a little more little about the helical gears and from there we will have an idea how to design a bevel gear unit also.

Thank you.