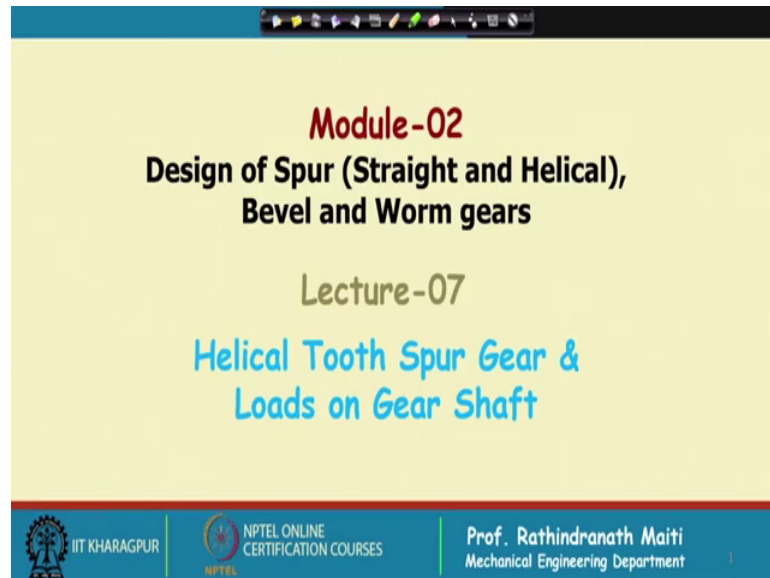


**Gear and Gear Unit Design: Theory and Practice**  
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**Lecture – 07**  
**Helical Tooth Spur Gear & Loads on Gear Shaft**

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**Module-02**  
**Design of Spur (Straight and Helical),  
Bevel and Worm gears**

**Lecture-07**  
**Helical Tooth Spur Gear &  
Loads on Gear Shaft**

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Module 2; we are continuing with design of spur straight and helical bevel and worm gears. This is lecture 7 and in this lecture, I shall discuss about how the helical tooth spur gear is designed and the loads which are coming on shaft gear shaft through the gears.

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**Outline of the Lecture**

- Modification of formula for straight tooth spur gear for helical gear
  - Equivalent Spur Gear & formative number of teeth
- Generalization of formula for designing both straight tooth and helical tooth spur gear
  - Pitch circle diameters of gears and pinion, and centre distance
  - Loads on shaft through gears

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This lecture will recover modification of formula for straight tooth spur gear of for helical gear, this already we have derived the formula for straight tooth spur gear and now, we will see that how it can be transformed to for the helical gears.

Next; equivalent spur gear and formative number of teeth then generalization of formula for designing both straight tooth and helical tooth spur gear and we will also show that pitch circle diameters of gears and pinion and what will be their center distance for helical gear loads on shaft through gears.

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**Strength of gear teeth-Lewis equation (Contd...): Straight tooth spur gear (Rpitulation)**

To estimate module we use:

$$m_n = \sqrt[3]{\frac{2T}{\left(\frac{S_o c_v}{c_w}\right) Z \psi Y}}$$

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Now, in the figure, it is shown a tooth is subjected to load; this means it is transmitting torque the torque gives the tangential load; that means. So, this tangential load is directly derived from the torque divided by the radius. As earlier specified; this tangential load is actually acting at the pitch circle, it is somewhere here at acting at the say here acting at the pitch circle, but while we are designing the teeth; we consider that as if the tooth is acting at the teef.

Also there is another consideration; that usually if the contact ratio is more than one which is which has to be in that case at times the loads are shared by two pair of teeth two pairs of teeth or even more pairs of teeth and also it is when the tip of one teeth; that means, either it is of gear or pinion is subjected to loading at that point at least, there will be another pair in contact, but while, we are considering the gear design we shall consider as if the whole torque is taken by one pair of teeth it is being transmitted through from pinion 2 gears only at contact at one pair as well we consider it is acting at the tip of the teeth.

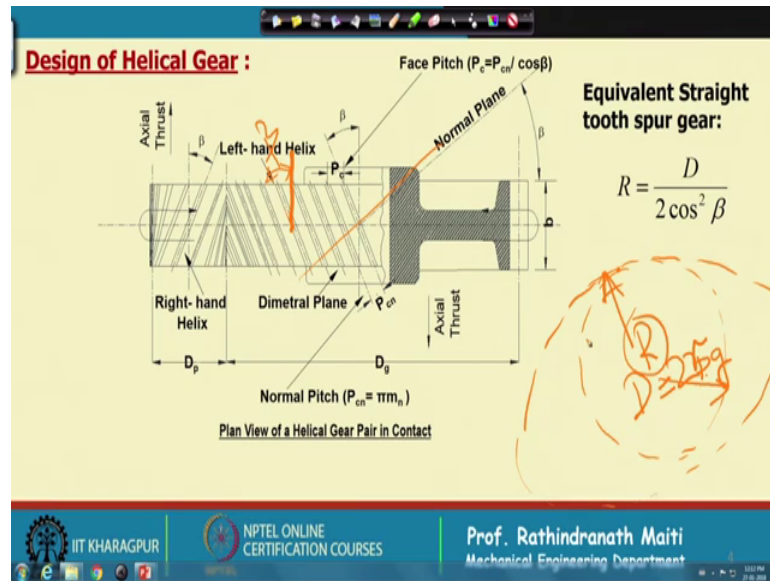
Now, we have also considered a curve like this which may be consider as the curve for ISO stress due to the bending moment and that is use me it is accommodated within the envelop of the teeth that is tooth profile. Now, on the basis already the formula for to estimating the module for straight tooth spur gear is derived in earlier lecture and that formula is a  $m_n$  an earlier we perhaps used only the module is expressed by  $n$ ; that means, in that case module in normal direction and phase directions everything are same.

Here we consider the module in normal direction which for the straight tooth spur gear is given by twice into  $T$  here, the  $T$  is the design torque divided by  $S_0$  allowable strength of material for the gears in a particular operating conditions, then  $c_v$  is the velocity factor which is considered depending on the velocity at which the gears are running and  $c_w$  is the lubrication condition factor  $Z$  is the number of teeth  $\psi$  is the width factor.

That means, the width will be module into this factor and  $y$  is the form factor Lewis form factor it is modified Lewis form factor capital  $y$  which is  $\pi$  into the small  $y$  a small  $y$  is the original Lewis form factor it depends on the geometry of the teeth mainly the root thickness and the height  $h$  in the figure it is shown where the load is acting.

So, these already we have derived we have arrived into this formula earlier.

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Now, if we come to the helical gear this helical gear we may consider the straight tooth spur gear as in they are very thin straight tooth's spur gear, they are put together one after another with a slight rotation. So, if we consider say let us consider this. This is the gear and this is the pinion and these are straight teeth not the helical which is shown in this figure.

Now, make these set into several slices in this directions in the direction of the face I mean parallel to face make the slice thickness are infinitely small and then each section each small section is rotated by certain angle equally then we will get the teeth in this direction in an inclined directions which is shown in this figure.

Now, as per the definitions this speech from one teeth to other teeth along the face this is of course, the circular pitch thickness that is the arc thickness sorry; circular means it is a arc to thickness along the face that is designated by  $P_c$  or sometimes, it is also designated by  $P_f$  that is the it is called face pitch, but the helical how the helical gear will be generated because it would be generated by some standard cutters.

Now, fortunately the hob cutter what we use for generating the straight tooth spur gear that same cutter can be used to generate the helical gear also the hob cutter, it is something like a thread it is like a worm. So, as if their one teeth it is winded off on a shaft and then these are segmented to give the cutter safe. Now that hob cutter now can be put in such a directions that their cutting direction will match with the helix

directions. This means that along this normal plane which is shown here along this normal plane if we cut it like this is as if we are cutting the straight tooth spur gear.

So, while we are considering the design if you would like to consider the formula in that case, we have to first of all we have to consider the equivalent straight spur gear and how it is done how it is estimated we consider say, for example, if we cut in the direction if we cut this gear in this directions then if this is the which circle of diameter of the gear we will get the equivalent this discard section that will look something like this like an ellipse, it will be something like an ellipse.

Now, this radius of curvature at the same point here this radius of curvature  $r$  and say this is the we can consider which circle radius of in this case gear this  $r$  capital  $r$  and  $r$  which circle of gear will be different this will be definitely higher than this. Now if we consider  $d$  diameter of the gear is equal to twice of  $r \cos \beta$ , then this  $r$  can be expressed by  $d$  divided by twice into  $\cos^2$  of this helix angle this angle is  $\beta$ .

So, we now consider that this helix helical gear is equivalent to a straight tooth spur gear of radius  $R$  capital  $R$  radius capital  $R$  and then in the same formula a modified a it is modified slightly and can be used for helical gear also.

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**Design of Helical Gear :**

**Equivalent Straight tooth spur gear:**

$$R = \frac{D}{2 \cos^2 \beta}$$

$$2R = \frac{Z m_n / \cos \beta}{\cos^2 \beta}$$

$$Z' m_n = \frac{Z m_n / \cos \beta}{\cos^2 \beta}$$

$$Z' = \frac{Z}{\cos^3 \beta}$$

**Z' Formative number of teeth is expressed as:**

Plan View of a Helical Gear Pair in Contact

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So, we will consider twice  $R$  is equal to number of teeth into module divided  $\cos \beta$  that is the pitch circle diameter of this gear because this is this pitch subtle will be if we

take that R at the point where we have considered the capital R; this will become  $Z_m n$  divided by  $\cos \beta$  by 2 and here automatically this  $\cos^2 \beta$  value is there and then we can have that this is nothing, but a teeth number  $Z_d$  into module.

$Z_d$  is the teeth number of equivalent straight tooth spur gear, which is this  $Z_d$  is called formative number of teeth, this is called formative number of teeth and that can be expressed by the actual number of teeth of the helical gear into the module standard module which we have consider divided by  $\cos$  of the helix angle and then finally, divided by  $\cos^2 \beta$  which reduces to  $Z_d$  which is called formative number of teeth is equal to actual number of teeth divided by  $\cos^3 \beta$ .

So, we can now in the formula we can use that  $Z_d$  to find out the form factor Lewis form factor we will come to that before that another thing I would like to mention here itself in parallel axis the helix angle of the gear and pinion the direction of helix angle will be just opposite in this pinion if we consider this pinion, what is the direction of helix angle here as shown in the figure this is the as if this figure is shown. So, this is right hand helix.

So, for the gear it would be left hand helix that we should keep in mind because when we will come later on the non parallel set we will find their helix angle are defined that both are having the helix angle in the same directions, but in case of parallel sub you should always remember that direction of helix angle will be opposite.

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**Design of Helical Gear (Contd...):**  
**Pitch Diameter of Helical Gear and Centre Distance of Mating Helical Gear Pair**

Face Pitch ( $P_c = P_{cn} / \cos \beta$ )

Normal Pitch ( $P_{cn} = \pi m_n$ )

Normal Plane

Dimetral Plane

Right-hand Helix

Left-hand Helix

Axial Thrust

Axial Thrust

Plan View of a Helical Gear Pair in Contact

$$D_p = \frac{Z_p P_c}{\pi} = \frac{Z_p P_{cn}}{\pi \cos \beta}$$

$$= \frac{Z_p m_n}{\cos \beta}$$

$$D_g = \frac{Z_g P_c}{\pi} = \frac{Z_g P_{cn}}{\pi \cos \beta}$$

$$= \frac{Z_g m_n}{\cos \beta}$$

$$A = (D_p + D_g) / 2$$

$$A = \frac{m_n}{2 \cos \beta} (Z_p + Z_g)$$

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Now pitch circle diameter of the pinion is expressed as number of teeth of the pinion  $Z_p$  into the face pitch of the pinion that is  $p_c$  divided by  $p_i$  and as you see this from figure the face pitch of both will be same because if you take a small slice there it will be a straight tooth spur gear. So, it will be  $Z_p$  into  $p_c$  by  $p_i$  and that will be again  $Z_p$  into  $p_c$  that is the normal pitch divided by  $p_i$  into  $\cos \beta$  this is very clear because if we take distance here say this is sorry it is not coming exactly. So, this is you can consider the base and this is perpendicular and this is the.

So, it is simply that  $p_c$  will be equal to  $p_{cn}$  divided by  $\cos$  of this angle this angle is nothing, but  $\beta$  ok, therefore,  $D_p$  can be expressed as equal to  $Z_p$  number of teeth of the pinion into module normal module divided by  $\cos$  of helix angle here we use the symbol for helix angle simply  $\beta$  because they are having equal magnitude all the directions are different, but when we will come into non parallel shaft we have to define the helix angle separately for gear and separately for pinion because they will not be equal if they are equal that might be a unique case, but in any case their direction will be same.

Now, the pitch circle diameter of the gears similarly in the same way fine can be expressed as number of teeth into the normal module that is the standard divided by  $\cos \beta$  therefore, the center distance which is designated by  $a$  which is again the summation of the pitch circle diameter of the gear and pinion deep  $d_g D_p$  plus  $D_g$  divided by 2 and which can be written as  $a$  into summation of teeth number into module divided by twice into  $\cos \beta$ .

Now, here itself I can express that if the  $\beta$  is equal to 0; that means, helix angle is equal to 0, then  $p_c$  will be  $p_{cn}$  that is that a face pitch will be circular pitch obvious because the relation  $\cos \beta$  will become 1, now as  $\cos \beta$  is equal to 1, then  $D_p$  and  $D_g$  also simply will be expressed by the number of teeth of the pinion into module and number of teeth of the gear into module respectively center distance also will be expressed as number summation of number of teeth into module divided by 2 because  $\cos \beta$  is equal to 1.

So, this means that these relations also holds good for straight tooth spur gear so; that means, a single relation what we use for helical tooth spur gear that can be used also straight tooth spur gear substituting  $\beta$  is equal to 0.

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**Design of Involute Helical Spur Gear**  
**Module on the basis of bending strength:**

The Lewis Formula for module calculation.

$$m_n = \frac{\sqrt[3]{2T \cos \beta}}{S_o c_v \psi Y Z c_w}$$

**After selecting module (rounded up to standard) tooth's dynamic load capacity and wear load capacity are verified.**

**If necessary module is increased.**

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Now, module on the basis of bending strength; the Lewis formula for module calculation that we use; now can be the a formula which can be used for both straight tooth spur gear and helical gear is expressed by cube root of twice into design torque into cos of helix angle this is for the parallel shaft and divided by allowed strength of the material into  $c_v$  divided by  $c_w$  width factor.

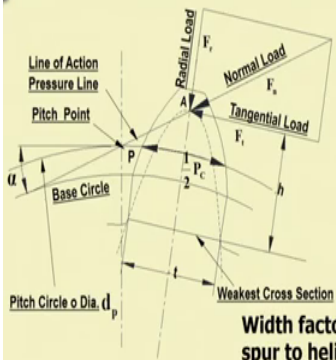
Now, here we should keep in mind we will take the width directly into directly as the this factor into the module no cos beta term will come over there. Now this y form factor is with respect to the formative number of teeth and Z is the actual number of teeth. So, one when we convert the formula for from helical gear sorry straight tooth spur gear to helical gear the cos beta term has come due to that the change in into the torque, etcetera, etcetera, so; however, this same formula. Now we can use for the straight tooth spur gear also the in case of straight tooth spur gear beta will be 0. So, cos beta will be one. So, formula will simply will be reduced to twice T by a 0  $c_v$  by  $c_w$  into psi into Y into Z and Y in that case that is against the formative number of teeth which is now actual number of teeth divided by cos cube beta means 1.

So, after selecting the module rounded up to standard tooth's dynamic load capacity and wear load capacity are verified if necessary module is increased this we are talking about that which already.



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**Strength of gear teeth-Lewis equation (Contd...): Helical tooth spur gear**



To estimate module we use:

$$m_n = \sqrt[3]{\frac{2T \cos \beta}{S_o c_v \psi Y Z c_w}}$$

Lubrication /wear load factor  $c_w$  is taken between 1 to 1.25 for force lubrication to splash lubrication respectively.

Width factor  $\psi$  may be as high as 15 & 20 for straight tooth spur to helical gear respectively.

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In expressed and lubrication and wear load factor  $c_w$  is taken between 1 to 1.25 as in case of straight tooth's spur gear also and for this is 1 is for the force lubrication; that means, the while is directly injected at the point of contact and splash lubrication means the some part of the gear in is merged in the while which is kept in the gear box. So, for that we can go for 1.25 and  $c_v$  factor depending on the accuracy of the gears and dynamic loading conditions a different  $c_v$  factors which already prescribed from the straight tooth spur gear can be used here also.

Width factors  $\psi$  may be as high as 15 to 24 straight tooth spur to helical gear respectively; that means, in case of straight tooth spur gear this width factor this width to be taken less because the what I discussed earlier again I will discuss here in case of the as the sacks are parallel, then in case of straight tooth spur gear expected deformed contact zone should be a perfect rectangle is it not.

Now, if the axis has slightly mismatched they are not exactly parallel or inclined to each other in that case this contact may become I mean after the contact deformation it may become like this. So, therefore, if we take very large width then a certain portion which may be 20, 30 percent of the tip are not at all in contact that is why this restrictions for tests tooth's spur gear it is usually taken 50 whereas, for general practice it can be taken as high as 20; however, in special cases this also precision gears it can be taken more

Now, this width factor 10 to 12 for straight tooth spur and 14 to 16 for helical gear is very common.

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**Design of Involute Helical Spur Gear (load on shaft)**

**Tooth Loads**

**Forces on Helical Gear Tooth**

$F_t = \frac{2T \cos \beta}{d_p Z m_n}$  (Nominal Torque)

$F_m = \frac{F_t}{\cos \beta}$

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Now, we shall consider what is the tooth, what is the load coming on the shaft from the gears? Now first of all we will calculate the tangential load  $F_t$  is equal to twice  $T$  divided by pitch circle diameter which is twice  $T \cos \beta$  divided by  $Z$  into  $m_n$ .

Now, in this case I would like to mention that; while we are designing the gears then what the torque value you are considering in the formula for module that torque value we should not consider the nominal torque, it should be a design torque; that means, what may be with the probably increase in torque that might be due to the initial starting torque that might be due to the dynamic load. So, there we need a good judgment recommendations are given by the gearbox manufacturer also who are using the gear, but in this case while we are considering while we are calculating the load we would say that these are the nominal load and this  $T$  this  $T$  is the nominal torque this one is the nominal torque.

So, on that basis we have calculated  $F_t$  now this  $F_t$  is acting in the phase directions of the gears that is this means that if  $T$  is this one  $F_t$ . So, it is shown here this is  $F_t$  now we shall first calculate a value that is in the in this direction. So, we will consider if  $T_n$  which will be  $F_t$  by  $\cos \beta$ . So, this will become the  $F_t$   $n$  the  $\beta$  is the helix angle.

So, this what we calculated in the phase directions. Now we have calculated in the norm converted into normal directions and then.

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**Design of Involute Helical Spur Gear (load on shaft)**

**Tooth Loads**

Forces on Helical Gear Tooth

$$F_t = \frac{2T}{d_p} = \frac{2T \cos \beta}{Z m_n}$$

$$F_m = \frac{F_t}{\cos \beta}$$

$$F_n = \frac{F_m}{\cos \alpha_n} = F_t \sec \beta \sec \alpha_n$$

$$F_r = F_n \sin \alpha_n = F_t \sec \beta \sec \alpha_n \sin \alpha_n$$

$$F_r = F_t \sec \beta \tan \alpha_n = F_t \tan \phi$$

$$F_a = F_n \sin \beta = F_t \tan \beta$$

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This normal load becomes; the further we have to consider this cause of this value  $F_n$  the normal this is the standard, this is the standard pressure angle.

So, it will now become  $F_t$  that what we have calculated from the torque into sack of helix angle into sack of pressure angle standard pressure angle. So, we will find the normal load, then the from this normal load we consider that the radial load is at the sine  $\alpha_n$  sin component of this is the radial load because this radial load will be in this directions.

So, this is the radial load and this can be defined by  $F_t$  into  $\sec \beta$  into  $\sec \alpha_n$  into  $\sin \alpha_n$ . So, this will become the  $F_t$  into  $\tan \alpha_n$  where this angle is this one; however, it is better to express in terms of this one because this is a known value this is also known value and  $F_t$  we have calculated. So,  $F_r$  is calculated here and then  $F_a$  is also calculated that is this axial load is calculated  $F_a$ ; that means, this one is  $F_n$  into sine of  $\beta$  simply. So, this is  $F_t \tan \beta$  this will become  $F_t \tan \beta$ .

Now, we can use this tool formula these 2 formula for the straight spur gear also because in that case this relation  $F_r$  will become  $F_t$  into  $\tan \alpha_n$  because this part will become one and here this  $\tan \beta$  will become 0. So, this part will be 0 in case of straight tooth

spur gear this means that whatever the formula we have derived here to find out the load that can also be used for the straight tooth spur gear.

This means that by the set of the single set of formula we can go for designing both straight tooth spur gear and helical gear only thing we have to remember the width factor can be taken more for the helical gear then the formative number of teeth due to the change in formative number of teeth we have to consider we will have more value for the form factor so; that means, this helical gear is having more strength and also wear is a  $\cos \beta$  term is coming in the module calculations.

So, with this it is possible now to calculate the both straight tooth's spur gear and helical gear and as such they after that the dynamic load what we have to calculate or the load carrying capacity the due to the contact that is  $c_w$  and  $c_d$  that also can be calculated in the same way what we have calculated for the straight tooth spur gear

So, this is the end of this lecture. So, in the in the this is lecture number 7 lecture number sixth we have learned how to design this spur gear for the strength basis designed considering their wear load capacity considering the dynamic load capacity and here we have learned that how the ah initial formula for calculating estimating module that can be used for helical gear or another way what we have derived from the helical gear that can be used for straight tooth spur gear also.

So, at this stage I expect that you have learned how to make a basic design of straight tooth spur gear and helical spur gear next we shall learn how to design the bevel gear and one gear.

Thank you.