Gear and Gear Unit Design: Theory and Practice
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Lecture – 06
Involute Straight Tooth Spur Gear

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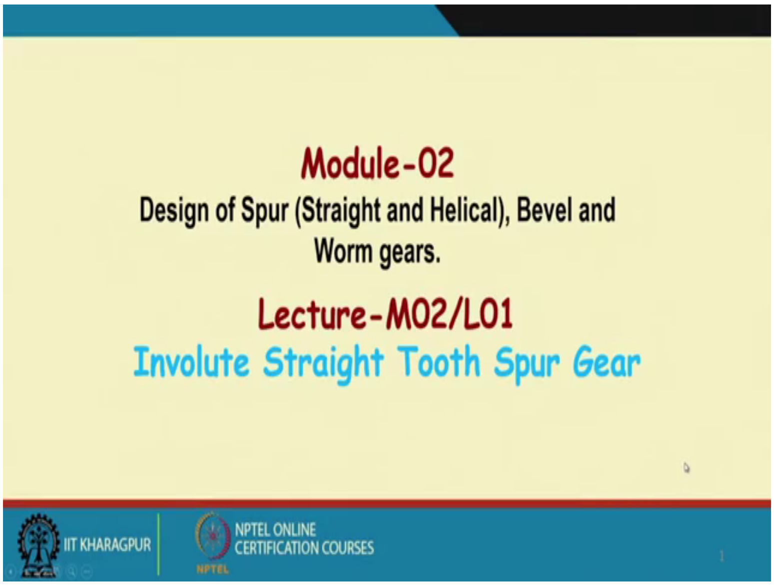

In module 2; we shall learn about design of Spur, straight and helical, bevel and one gear; worm gears. This is the fundamental technique of designing such gears with involute tooth. So, first lecture of this module will be on involute straight tooth Spur gear design.

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Now, in this lecture, I shall cover design of gear tooth based on beam strength; consideration of tooth accuracy that is, generation velocity factor and lubrication factor.

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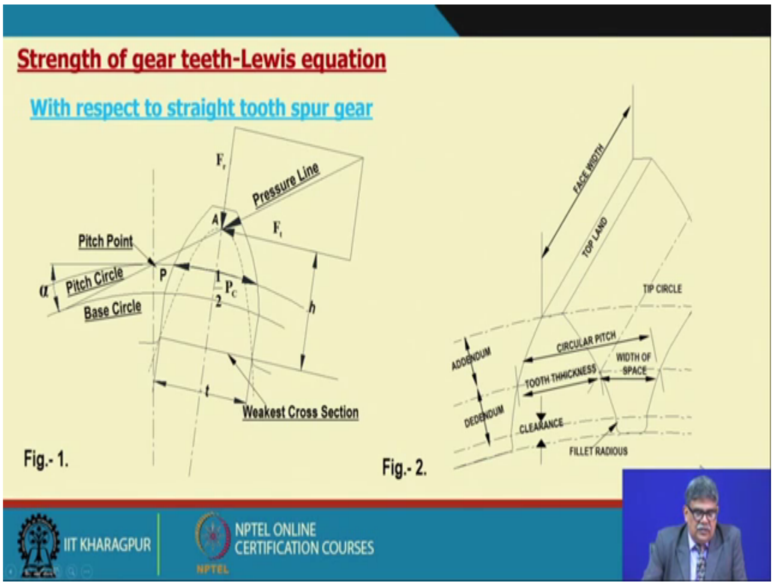

Now, if I consider a involute tooth, then while it is touching with another gear pair, then load is being transmitted through this line of action, this is the line of action, I have shown the different gear proportion and we have considering at this stage; the this is the standard gear. So, this two which circle are meeting at the pitch point through; which this line of action is drawn.

Now, this is the pitch circle and this is the base circle and this is the pressure line or line of action and this point is called pitch point. And here, we have consider this is the weakest cross section of this tooth profile, because the when the load is acting on the line of actions, then at first instant this will experience a bending stress at a root somewhere here and that stress will be maximum where the section is weak or the weakest section. So, we will consider that weakest section and this line can be shown as the can be considered as this profile is such that if I take any section at that section bending stresses will be equal.

Now, this pressure angle is alpha this is the point where the load is acting and this is the normal force is acting in this directions, but we can consider the tangential force, the component of this one is acting at the perpendicular direction of the line joining the center and this is the radial component of this load. Now this is the height; tooth height where the from the load point a, the top point of the loading point to the distance of the weakest cross section.

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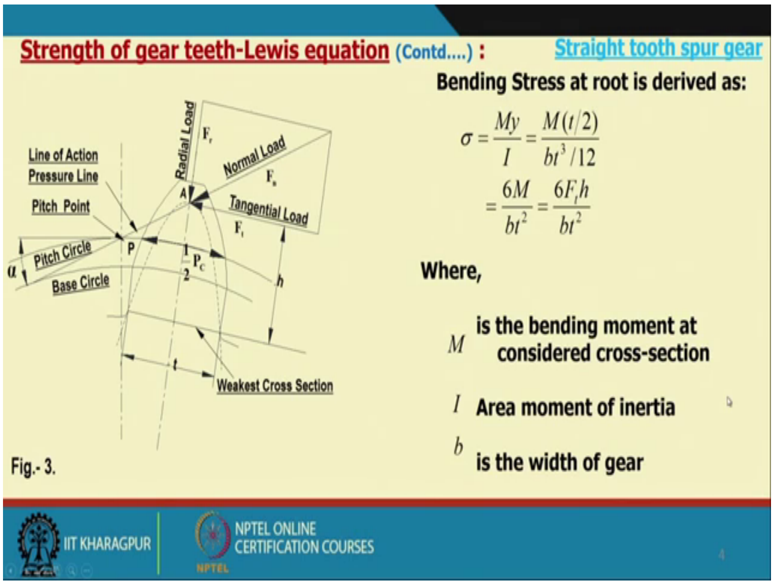

Now, this is as I have shown already this is the tangential load. This is the normal load and this is the radial load acting at this point. Now, if we would like to find out the stress at this point, we can write down this equation which is sigma is equal to M y by I whereas, y is the distance from the neutral section to the point extreme point where the stress will be maximum and I is the area moment of inertia of this cross section, say we have taken this front view of the teeth and along this direction the width is there. So, this section is this tooth thickness as a width and length of the rectangle is the width of the gear. So, which is b?

Now, this y must be equal to t by 2, whereas, this I they area moment of inertia of this area if we consider this area, then this is the neutral axis about which we can write this is b t cube by 12. Now, this becomes 6 m my by b t square and again if we consider this moment which is equal to nothing, but this tangential load into the h is this beam length then it becomes 6 f t into h by b t square.

So, this already I have given this m is the bending moment the considered cross section I is the area moment of the inertia and b is the width of the gear.

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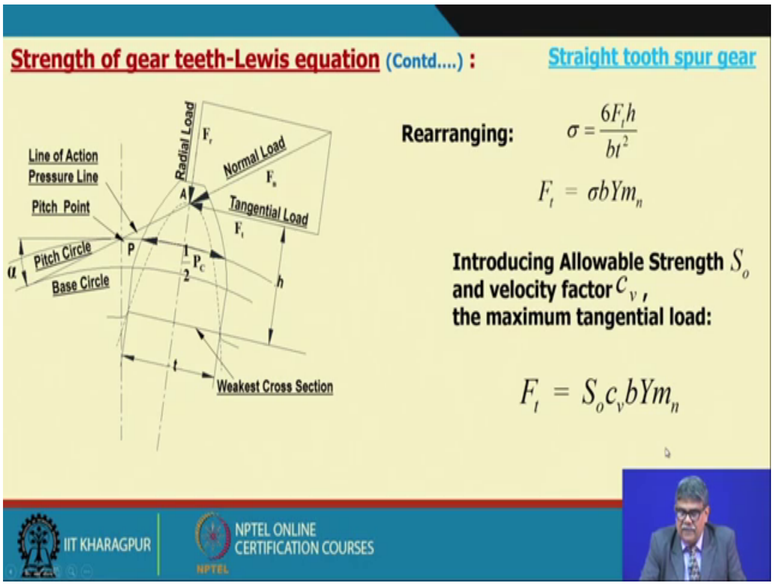

Now, t square by 6 h that can be related with the I m n means that is the pitch. Pitch is equal to pi into m n. M n is module, we will come to the module concept, we will we will we will show that this each is pi n m n and this can be expressed as y into pi m n because the t with a selected module, this t will have definite distance.

Now, this can be further modified to y into m and capital Y into m n. Now this small y is called the Lewis form factor and whereas, capital Y which is pi into small y is called modified Lewis form factor Lewis is an engineer who found out all such dimensions and he proposed this factor. So, that is why this factor that is why it is called Lewis form factor.

Therefore, we will we will be able to express the stress in terms of this form factor and this form factor can be written in this form a table is also available because Lewis he developed the table where the y is for different size of the teeth. It was given, it will not depend on the module it will depend on the teeth number. So, if the teeth number is more this form factor more if tip number is less, this phone factor will be less; however, this can be expressed by this formula; where z dash can be given by z into cos cube beta where j is the number of teeth of the gear and z dash this is called formative number of teeth and where beta is the helix angle of the gear.

Now, if this helix angle is 0, then it is a straight tooth Spur gear and then z dash is equal to z. So, we should remember that why we are calculating y, we should consider this as a formative number of teeth which is expressed by the actual number of teeth and the helix angle of the gear.

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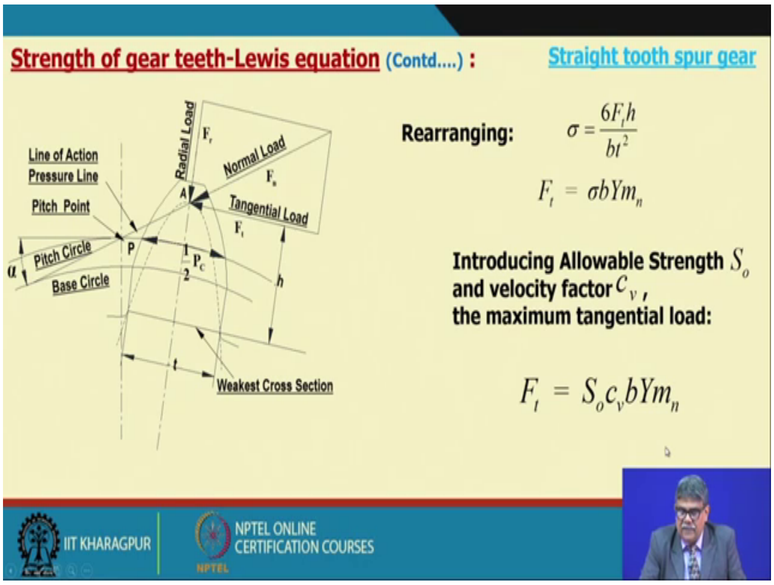

Now, this helical gear, I have not yet discussed we are we shall come later, but we should remember this formula and when we I shall discuss about the helical gear there it will be further explained.

Now, if we rearrange this formula that is the stress now can be expressed as 6 into tangential load into this teeth height where up to the weakest cross section and then b divided by b is the width of the gear into t square t is the thickness of the t teeth at the weakest section. Now edge this is b t square by 6 is y and Y m n. So, we can express that f t is equal to the tangential load is equal to stress developed there into width of the gear in to form factor into module introducing a allowable strength now we will have to design the gears. So, we shall calculate the stress later, but initially we have to design the gears depending on the material strength we have to find out what is the size of the gears; basically we have to calculate; what is the module of the gears?

So, this stress we shall replace by the allowable strength which is defined by S 0 S o S subscript o and the velocity factor we shall also introduce velocity factor this velocity factor is to be introduce because depending on the velocity the allowable strength to be considered; that means, if it is of high velocity and not very accurate then this allowable strength will be less if it is very accurate gears as well as if the velocity is less, we can go for higher strength the maximum tangential load.

Now can be expressed is f t is equal to S o into c v the velocity vector into b Y m n, but please note that this is not the replacement of the stress this we have introduced instead of expressing the stress we when we calculate after the considering the design gear, but when we are trying to design the gears, we will consider the arrival load allowable strength and the velocity factor.

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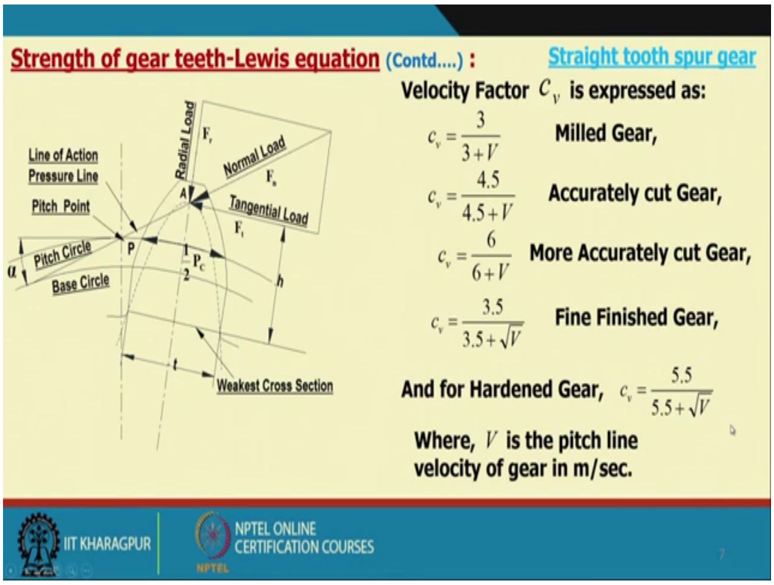

Now this velocity factor will vary depending on the accuracy of gear and how it is cut now this velocity factor can be consider 3 by 3 plus V; V is the pitch line velocity in meter per second b is the pitch line velocity; that means, if this is the pitch point the velocity at this point which is constant for both the gears and that is in meter per second.

Now, what is milled gear this is usually the not the generated by the cutter, but a milling cutter is used by which the gear tooth are generated; obviously, these are not for very high velocity. So, somewhat this is a crude design the milled gear and for which usually you will find this factor is taken in such a way it will protect the safety of the gears as strength wise.

Now, next comes accurately cut gear like if the gear is cut very accurately then this value will become like this 4.5 by 4.5 plus V it is empirical, but this gives us the you can say that optimum gear optimum model of the gear while we are using the formula which are which we have proposed.

Now, this will become 6 by 6 plus V more accurately cut clear please if you look into this, suppose the velocity is here, let us consider, this is 4.5 in that case this is 0.5 if it is 4.5 here then definitely this value will be less than this and if it is 4.5 here, this value will be somewhat different from this 2 and if when this will be considered this gear you will find the automatically the allowable strength increasing or decreasing in the formula.

Now, very fine finished gears, we can use this formula and if it is hardened gear then we can use c v as this one harden and this is also ground gears ok. So, c v factor we can directly take it from here.

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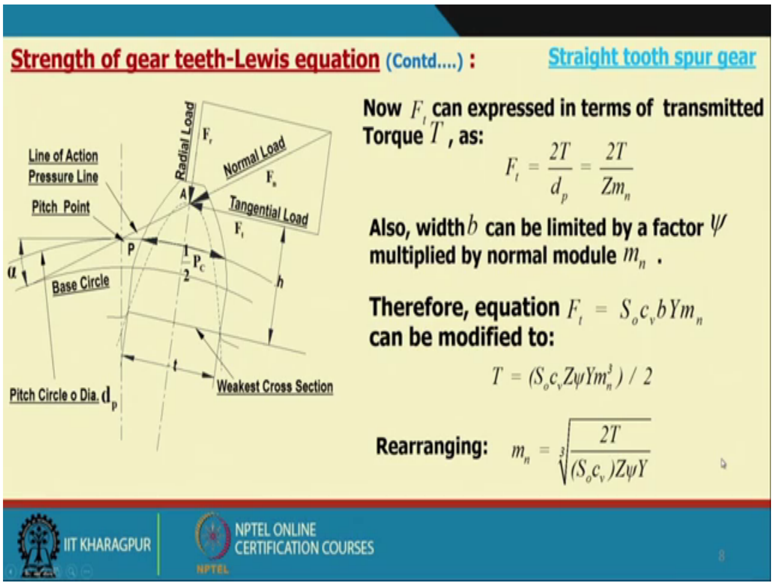

Now, F t can be expressed in terms of transmitted torque T, suppose this is transmitting at of T. So, remember that torque will be different from 2 meeting gears, if they are equal, then the torque is same otherwise one of that if it is input other one will be more or less depending on the proportion of the gear that will come later that how this torque is calculated.

Anyway if we know the torque on this gears then this f t can be calculated divided by 2T by d p whereas, d p is the pitch circle diameter of the gears. Now this d p for straight tooth Spur gear can be expressed as number of teeth into normal model.

Now, also width b can be limited by a factor psi multiplied by normal module; that means, the width what we have considered b that cannot be taken of any dimensions this should have there is a limit this is gear should not be very thin as well as gear should not be very wide for there are several factors, but most important factor is that if you take very thin gears that may buckle if we take very wide gear, then this contact for gears may not be uniform everywhere and one side it will be more other side it will be less and it is found in practice that width beyond certain limit of no use.

So, considering these we can limit this b in terms of a factor multiplied they consider module, therefore, equation F t there you get equation we can write into S zero into c v into b the width Y m n where this b is replaced by Y m n and sorry b is replaced by psi into m n and this torque equation as you have put it here. So, we are getting here m n cube and this 2 is coming over here.

So, torque; now can be expressed in terms of the allowable strength velocity factor number of teeth width factor modified form factor Lewis form factor and into module. Now if we rearrange then this module normal module, in case of straight tooth's per year can be expressed as cube root of twice into torque divided by allowable strength into velocity factor into number of teeth into width factor into form factor.

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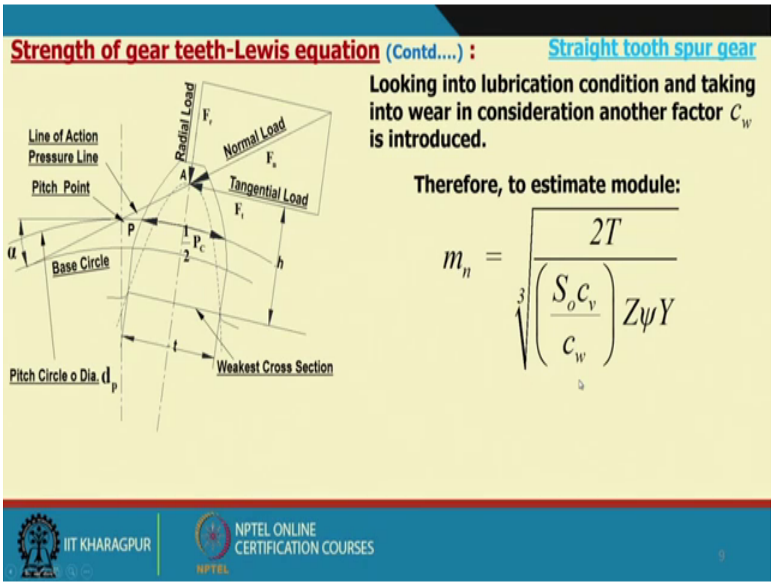

So, looking into the lubrication condition now while we are designing the gears it is not that the gears are being operated at the same conditions gears tooth profiles are of same accurate. So, we have to consider all such things in case of lubrication the gear may be usually these are oil lubricated and it is splash lubrication, if we consider the gearbox the gearbox some certain portion of the gear will be in inside the oil and while gear rotates that is placed over each other. So, this oil into the contact thus the surface is lubricated and wear is do friction and wear is reduced, but depending on that lubrication condition we have to consider some factor in design.

Now, this lubrication as I told it is a splatter lubrications for very high velocity or precision gears accurate gears this might be the force lubrication in that case the oil is it is not injected it is pumped, it is through the nozzle at the contact point in that case lubrication condition is better and for that we can go for higher allowable strength because these are well maintained.

So, we have to we need to consider and a lubrication factor or wear load factor in this formula and in this case we have put this lubrication factor into denominator and definitely it will be more than one and if we look into this.

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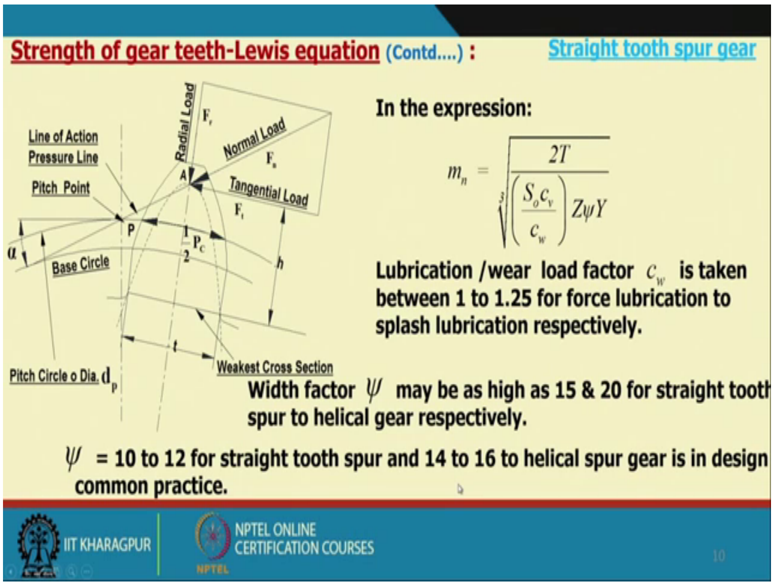

This factor may be considered between 1 to 1.25 for force lubrication to splash lubrication respectively splash lubrication means; this is teeth is put inside the oil and oil is being carried out with the rotation of the gears and one means there is a pump and directly through a nozzle oil is put into the contact.

So, this is designer can choice these factors as well as these are recommended by the gear manufacturers or gear designers. Now apart from that it might be also of grease lubrication or very poor maintenance in that case definitely this factor should be hired. Now, width factor can be taken as 15 to 20 for straight tooth Spur to helical Spur gear respectively.

This is this means that if it is a higher side is that 15 for straight tooth Spur gear and higher side for a helical Spur gear it is 20 helical is having better load carrying capacity and we can go for better contact also and we can go for higher width.

However this is just a recommendation. In fact, it will be more and it should not be very thin; that means, we should not take 2-3 times the module at least it should be 10 times the module ah the width of the gear. So, in general this is followed that ten to 12 for state tooth's Spur gear and 14 to 16 to helical Spur gear in design of common practice this is the recommended arrange and these are used in practice.

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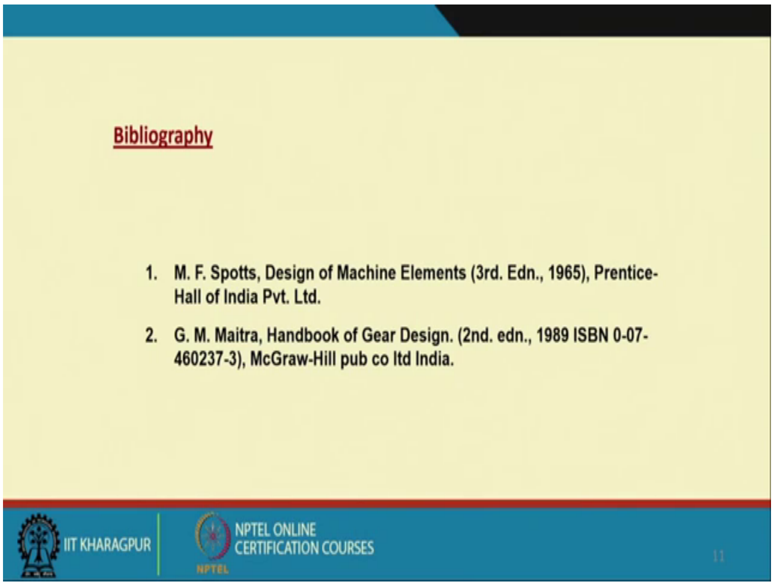

So, in this lecture what we have learned that how to design the gears and what should be the factors to be considered for considering these bending strength calculating the bending strain or calculating the module considering the bending strain and we should remember that the velocity factor that depends on that how the gears are manufactured and at what condition this gear will be used.

So, we can choose say for example, if it is a generated gear still there are 2 theme different formula proposed for the velocity factor because if it is hardened and ground, then definitely, we can go for higher strength of the gears if it is just simple hobbed cut or the cut the cutter, then we have to take a higher strength for the material whereas, if it is a milled gear or cast gear or very rough cut gear in that case definitely strains should be taken less.