Gear and Gear Unit Design: Theory and Practice
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Lecture – 39
Gear Train, Internal and Epicyclic Gearing

This is the last module on internal epicyclic and other special gearing. And the first lecture in module 8 is on gear train internal and epicyclic gearing.

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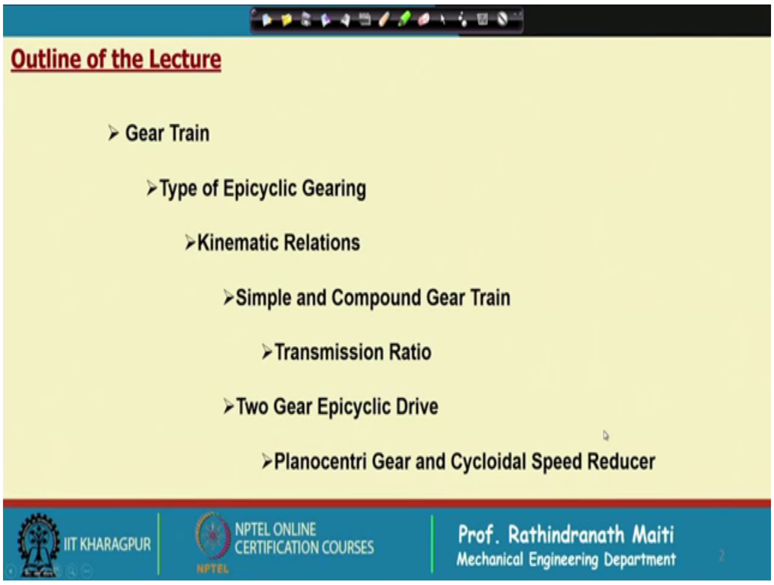

In this lecture, first of all I shall give you a brief idea what is gear train , type of epicyclic gearing, next kinematic relations, simple and compound gear train, transmission ratio, two gear epicyclic drive. Lastly, I will show you an example practical example of two gear epicyclic drive which is plain eccentric gear and also cycloid speed reducer.

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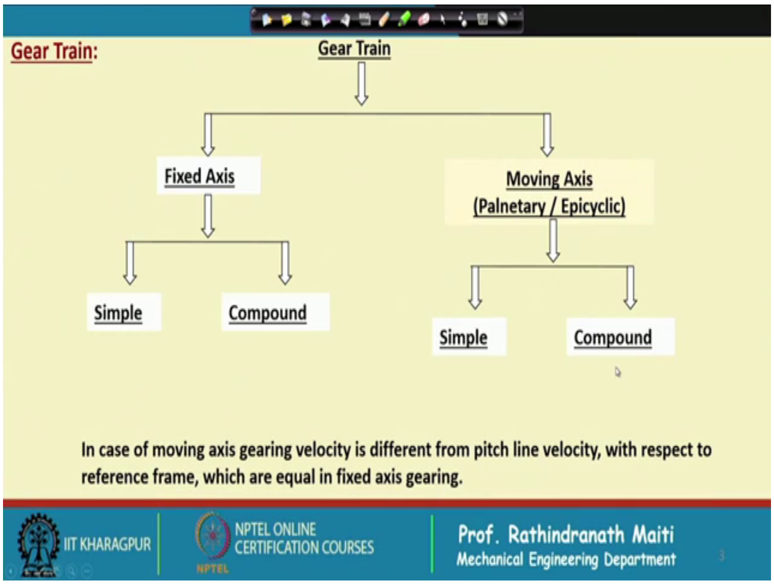

Now, considering the gear train first of all gear may be fixed axis, gear means gear unit, there will be pinion and gear may be in a single stage or stages may be two, three etcetera, etcetera. In fixed axis, first of all you will find the simple. Simple means in this case you may consider one pinion and one gear.

Even if we put an idle gear in between or idle gear between two gears in same line that means idle gear is meshing with pinion or drive one with the driven one gear, then also it is a simple train. And it might be compound train; compound train is that the gearbox unit which we have designed, the first stage, second stage and if it is a number of stages as more those are all compound gearing.

Now, it might be moving axis which is called planetary or epicyclic gearing. In that case again it might be simple gear train, but in this case there will be a sun which is usually driver and there will be a driven gear which may be internal or external; in between that there will be planet.

This is something like as in fixed axis simple gear, but in this case usually the ring gear will remain fixed. And from the planet carrier output will be taken. And in case of compound gears in the planet stage, there will be one gear meshing with the sun, and other gear on the same axis or integral width integral on the shaft or fixed on the shaft planet carrier shaft it is driving the ring gear or maybe it is the external gear.

So, in case of moving axis gearing, velocity is different from pitch line velocity, with respect to reference frame, which are equal in fixed axis gearing. This means that while we are designing the gears in planetary drives to find out the pitch line velocity of gears, we have to consider the actual motion gearing motion in between.

In epicyclic gear, there is a coupling motion and as well as there is gearing motion. So, we have to separate our gearing motion to calculate the pitch line velocity. For example, if we had to calculate the cv factors, in that case we have to find out gear velocity exactly. However, it is beyond the scope of this course that limited time is there, so we will learn only kinematics and few arrangements.

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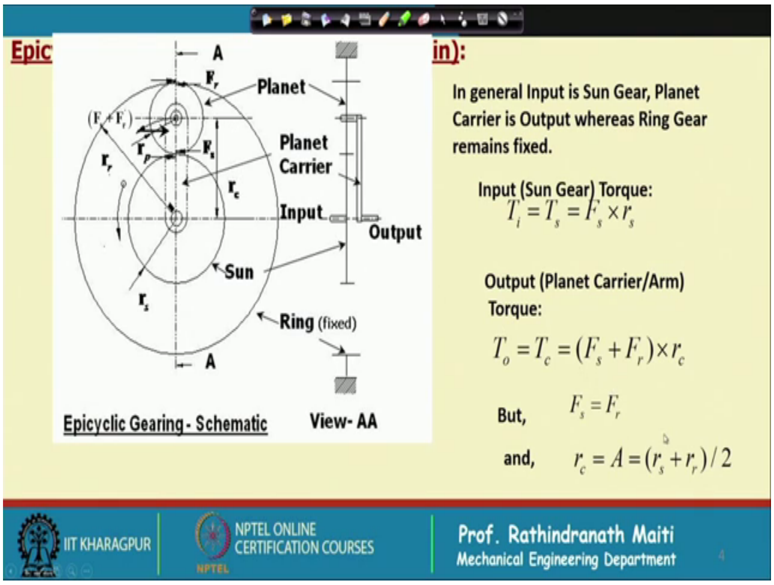

Now, so this is an epicyclic gear it is shown that there is this is the ring gear we have obviously in taken the internal gear and this is the sun gear and this is the planet gear, and this is the simple train.

Simple train means, the sun gear is meshing with this planet gear, and then it is meshing with the ring gear in a in the same line. Now, to understand epicyclic motion, I will show you the motions using two coins.

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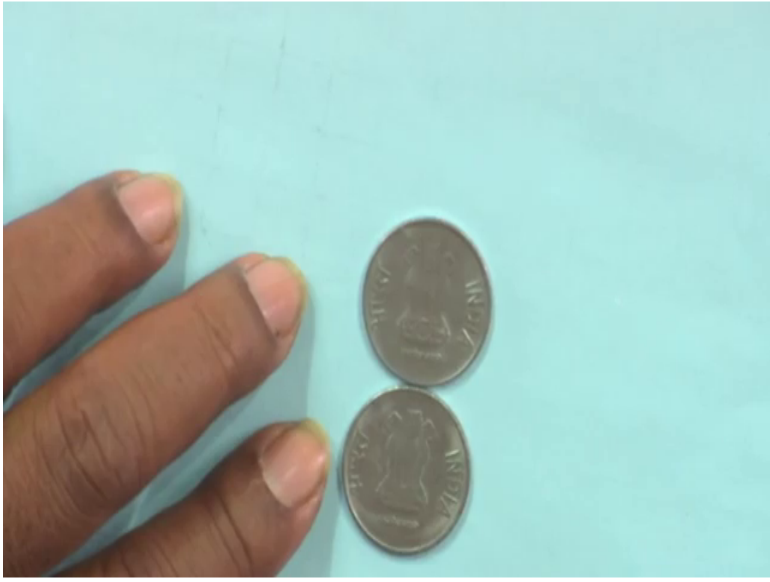

Now, in this case, perhaps you can see that these two coins, I have taken, this is 2 rupees coin and both are of equal periphery. And we have placed one above the other keeping the symbol of Ashok Chakra in the upward direction. Now, as they had periphery equal if there is a the upper one is rotated with slipless, there is no slip in that case apparently we will feel after a full rotation again this will be in the vertical direction. If we go up to half rotations, the bottom one will be in the upward direction, this will be if we ask anyone or in the first instant we will feel like this.

But let us see now I am rotating this coin very slowly. You can see, but the it is not slipping only we have rotated half the circle, but if you see that the this has become in the same directions that is the upward. If we rotate another half, then again it is vertical that means, if we give the motion to the upper one rotating about its own axis slipless with the other one and revolving around the central axis, then it will have two rotations we are gaining plus one, apparently it would be only one motion.

If we rotate at a fixed axis, you will find this will be 1 is to 1. This means that by allowing the touching gear, the allowing the touching gear to rotate about says own axis and revolve around the other axis of the fixed gear, we will gain some motion that is the epicyclic motion that is the planetary gear motion. So, in this case also, if we give the outer one fixed, and if we rotate the sun gear, then if we take out the motion out of this arm, we will have some additional transmission ratio and that is the advantage of epicyclic gearing.

Now, input let us consider in the form of torque. Then inputs Sun Gear is having torque input T i that must be equal to T s; T s means torque in sun that is equal to a force acting here F s and into the radius of the sun which is r s. Now, this torque it is transmitting torque mean F s is the force one force is acting on the planet and another sorry force is acting on the sun, and the same force is acting on the this planet pinion. Now, this is again touching this is meshing with the ring gear here. So, on that definitely it will have the equal force.

And what will be the directions, it is also pushing in that suppose it is rotating in this directions, this is rotating in this directions anticlockwise, so this is rotating the planet in this direction; that means, force acting on this planet edge in this directions. But while it is trying to push this ring gear, they wrote the load direction will be same on this planet. That means, if we consider the force over here that will be equal to twice F s.

So, we can write that output planet carrier arm torque; that means, this two force is again acting at that point which is nothing but that F s plus F r that is into the radius of the arm or the arm length that is the r c. So, therefore, now as I told that F s is equal to F r, therefore we can write r c that means, torque here is equal to A plus oh no sorry r c is nothing but the center distance between the sun and the planet which is r s plus r r radius of this one divided by 2.

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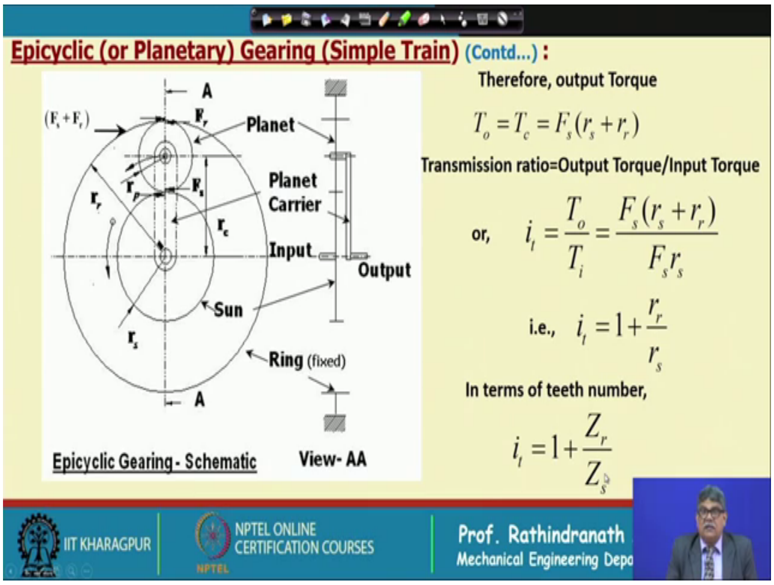

Now, therefore, the force F s is equal to r s plus r r. And therefore, the transmission ratio the T output divided by T input which is F s by r s by r r divided by r s the radius of sun, the radius of ring plus radius of sun divided by radius of sun that means, it will become 1 plus r r by r s.

Now, in normal case, if we drive this sun as input and the ring is output then we will get the transmission ratio is r r by r s. Here we are getting extra one which is which I have shown by rotating the coin. Now, in terms of teeth number, this can be written as 1 plus Z r, Z r is the teeth number of this ring and divided by the teeth number of sun.

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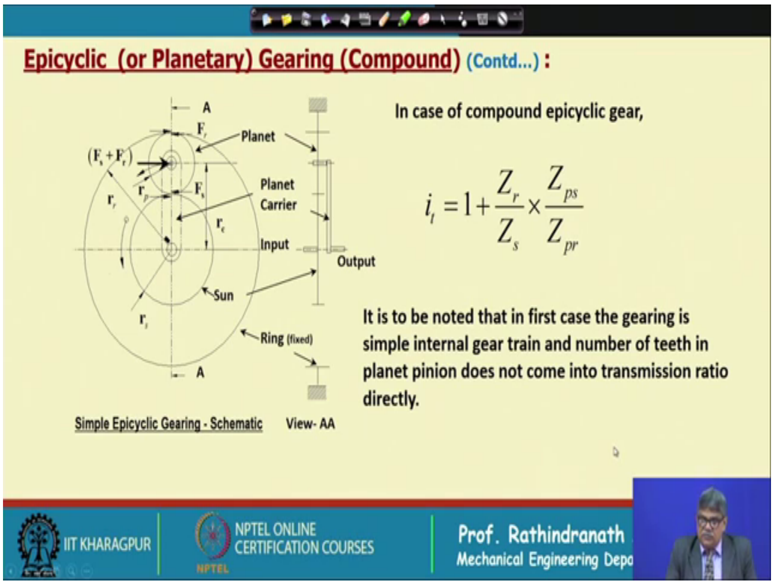

In case of compound gear, which is not shown here, this is the this is not the compound one, this is the simple one. In case of compound one this will be Z r into Z s into Z ps by Z pr that means, this is the here now there will be two gears we will show see into the next slide.

And that means, we are getting again additional transmission ratio here. This is something like the gearbox which we have designed the Z r by Z p r into Z p s by Z s. This Z ps is the pinion mating with the sun and Z pr is the pinion mating with the ring gear. Next slide we will show it. It is to be noted that in first case the gearing is simple internal gear train and the number of teeth in planet pinion does not come into transmission ratio directly.

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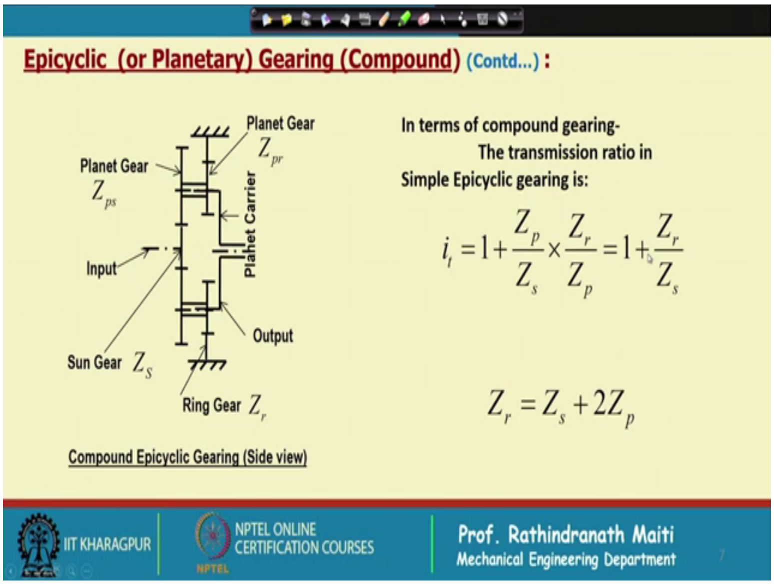

So, this is the compound gear, it is shown like it is the side view is shown here. This is the sun gear and it is mating with the planet gear teeth number of which is given by Z ps. And then on the same shaft, there is another pinion which is of smaller diameter and than this one and the teeth number is given by Z pr and this is the ring gear.

So, in case of compound gear we are getting this ratio; whereas, if this is a simple gear then definitely Z sorry in terms of compound gearing the transmission ratio in simple gearing is Z pr, Z p ps Z r by Z p. So, this will become 1 plus Z r by Z s. This if we make this teeth number same, and also we can see Z r is equal to Z s plus twice Z p number of teeth in the planet in simple gearing.

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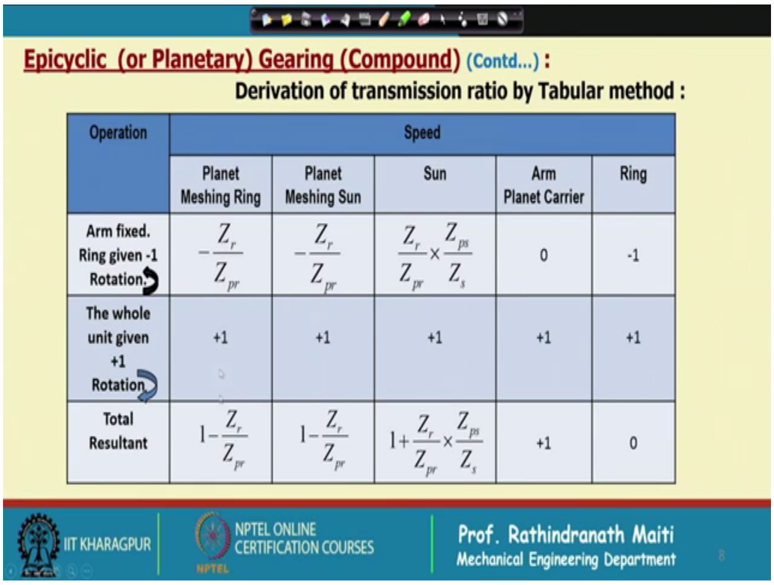

Operation	Speed				
	Planet Meshing Ring	Planet Meshing Sun	Sun	Arm Planet Carrier	Ring
Arm fixed. Ring given -1 Rotation	$-\frac{Z_r}{Z_{pr}}$	$-\frac{Z_r}{Z_{pr}}$	$\frac{Z_r}{Z_{pr}}\times\frac{Z_{ps}}{Z_s}$	0	-1
The whole unit given +1 Rotation	+1	+1	+1	+1	+1
Total Resultant	$1-\frac{Z_r}{Z_{pr}}$	$1-\frac{Z_r}{Z_{pr}}$	$1+\frac{Z_r}{Z_{pr}}\times\frac{Z_{ps}}{Z_s}$	+1	0

Now, the how to find out the transmission ratio in case of epicyclic gearing; There are first of all I have shown the we can consider the force, and from there we can equate the what will be the calculate what will be the torque in input and output as well as in the ring gear. And from there we can find out output torque divided by the input torque will give us the ratio, but there are several methods, other methods if you go for the motion analysis if you find out that each pump point of contact velocity from that point of view we can find out also what will be the transmission ratio; Another popular method which is used in practice that is the tabular method.

In case of tabular method, what we did do the first of all we consider the arm is fixed and ring is given minus 1 rotation, and minus 1 means anti clockwise given rotations. Then definitely for compound gearing the planet meshing ring we will have the motion Z r by Z p r. If we just remember the figure then Z r is the teeth number of ring gear, it is more than the planet meshing with the gear. So, this means that if we give this is one rotation then this will rotate. Say suppose this teeth number is one-third of this one then in that case it will rotate three times, but the direction will be same because the internal gear and the extra meshing is external gear they will rotate in the same directions.

Now, the planet meshing the sun as this is on the same shaft which Z pr that we will have the same motion (Refer Time: 19:24) in these motions. Now, the sun we will have motion Z r by Z p r into Z ps by Z s that is the same we do it two stage gearbox, but in this case rotation of this sun we will be opposite to the planet. And that means, in this case it is clockwise which is which is plus means clockwise; this is ring is rotating anticlockwise, this will be clockwise. Now, the we have kept the planet carrier fixed. So, this motion is zero and ring is rotated rotating at minus 1.

Now, the whole unit take the whole unit and rotate it in clockwise direction plus 1. So, simply for that each one we will have extra one motion as I have shown by rotating the coin. So, this is plus 1, plus 1, plus 1, plus 1, plus 1 ok. Now, the total motion if we add these two motions then this is become 1 minus Z r by Z pr, this is also becoming the same, and this becomes 1 plus Z r by Z p r Z p s by Z s this is plus 1 and this is 0. Now, this if this is the motion of the sun then arm is rotating only one rpm which is the output, this is the input, this is the output when we keep the ring fixed.

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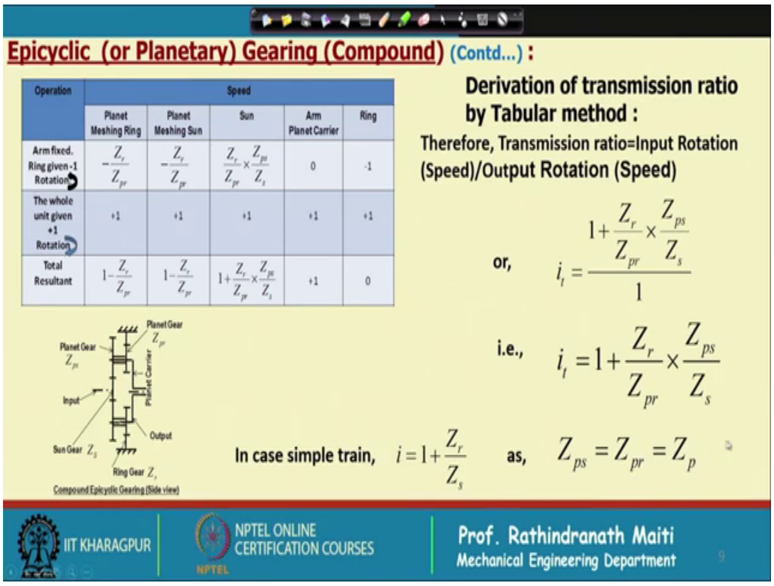

Then from this table, we find out that the transmission ratio is the input rotational speed divided by output rotational speed in terms of torque, these are otherwise output torque divided by the input torque. But in this table, we have found out the motion. So, according to that this output say input motion input speed divided by output speed that means rotation not speed because we have not considered the time part.

So, it is the rotation, and in that way we get the transmission ratio is equal to 1 plus Z r by Z pr into Z ps by Z s which I which we have shown earlier which we have derived from the torque ratio. In simple train, i is equal to 1 plus Z r by Z s because Z ps is equal to Z pr is equal to Z p which already we have discussed.

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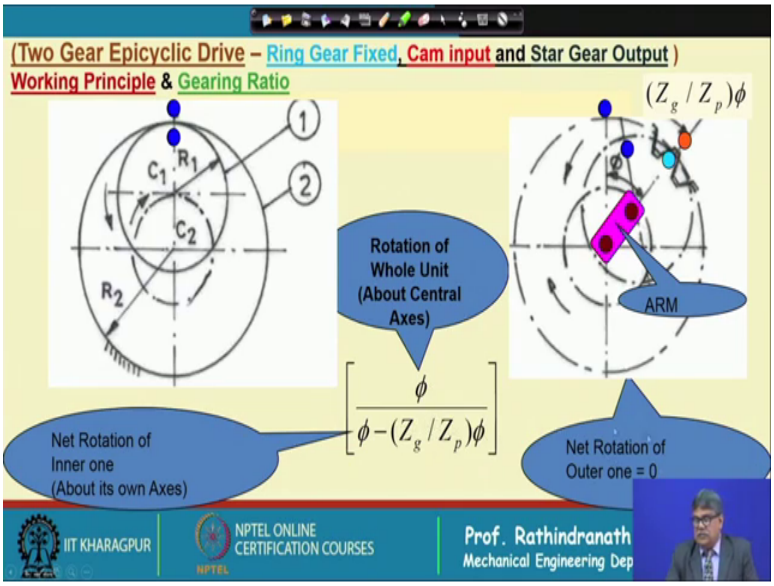

Next we shall consider a little complicated may be cyclic drive. In that case, we have only two gear. Gear 1 is the pinion external teeth in this case we have shown external teeth. And gear 2 is having the internal teeth internal gear. Now, let us consider a point fixed on the ring gear, and this is the position vertical vertically meshing. The planet is or the gear inside pinion is meshing with the gear vertically and so we take another point on the pinion.

Now, what we do along with this arm there is a arm here I will show you this arm here, first we rotate the whole unit by an angle phi. So, arm is like that, so we have rotated everything every unit in this way. Now, then definitely this point the point here the point on the pinion has arrived here and as well as point on the ring gear also which on the same line, it has arrived here. Now, what we do we keep this arm fixed, and along with this arm this was locked because we have rotated everything.

So, we will open the lock and then we rotate this ring gear in the opposite direction, so that this point this orange color point again come back to its original position. But by the time the pinion we will rotate the gear has no motion. Now, we can say, but the pinion we will come back you we will come to this point where you can consider the arc from the light blue point to dark blue point on the pinion must be equal to the arc length on the gear from orange to deep blue point on the gear.

So, rotation of whole unit about the central axis is phi, whereas the net rotation of inner one is this much, because this first is say it has rotated in the direction by phi angle. But now it has rotated in the opposite directions by an amount number of teeth of the gear divided by number of teeth of the planet into phi. It has rotated a more amount. So, therefore, this is the total this first rotation input rotation is the arm this one, and output rotation is this much. So, we will find that net the ratio is like that.

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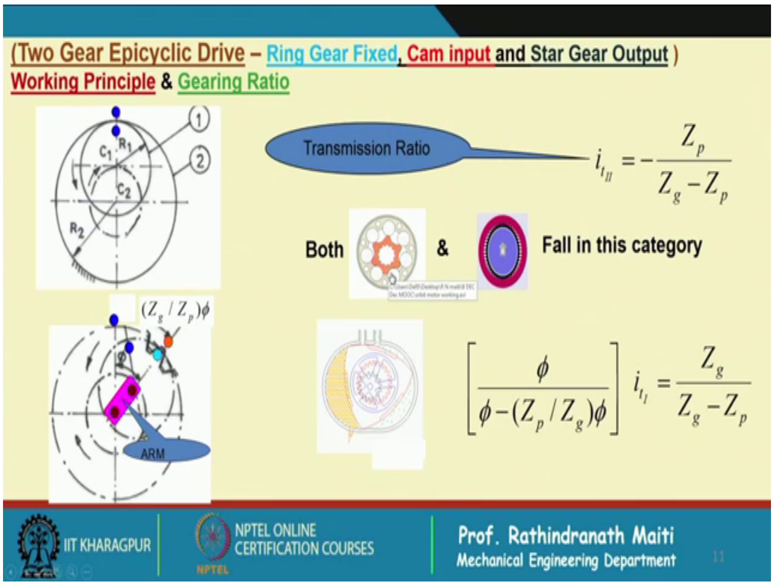

And from there we will find the transmission ratio , this is i 2 we have designated at 2 because we are keeping this ring gear fixed and we get minus Z p by Z g minus Z p. If we equate the earlier one, if we equate this one, we will arrived into here. Now, this the two gear epicyclic later we will show that can be put into where this is a another special gear the same principle it is working on the same principle.

And what we find look at this is quite interesting that the teeth number of pinion divided by this difference. If we make this difference very low, then transmission ratio will be high. And minus sign has become that the arm is rotating clockwise and the output is through output speed is the speed of the inner pinion and it is in the opposite direction. So, this minus sign has become automatically.

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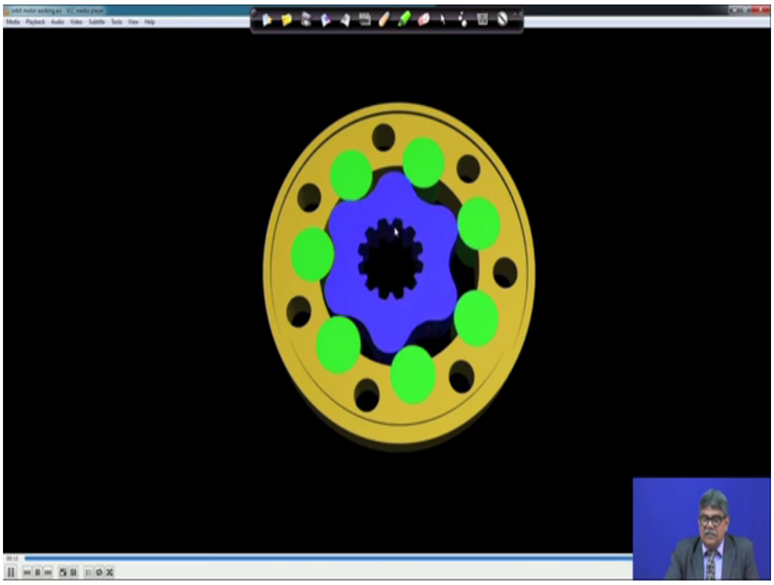

Now, have a look into this motion this is of course, not involute gear, this is the cycloidal or epitrochoidal gear. But as you see if we consider if you consider a point here then this is coming. While this is rotating by one full revolution, we can see that this is rotating one for full revolution about the main axis, whereas this point is coming only this much. In that case, you will find the transmission ratio is nothing but six. Now, we can see also this one. This is the gear is moving like that here what we find this is a flex gear.

And if we consider this point, just one revolution of this is giving the motion two only one teeth actually two teeth motion will be there that we will learn later. So, this means that by this such special gearing we get huge amount of transmission ratio in single stage. And these gears are used it is called as harmonic drive, and this gear is also in getting it is used as well as this is of course, say figure of the view of the motor.

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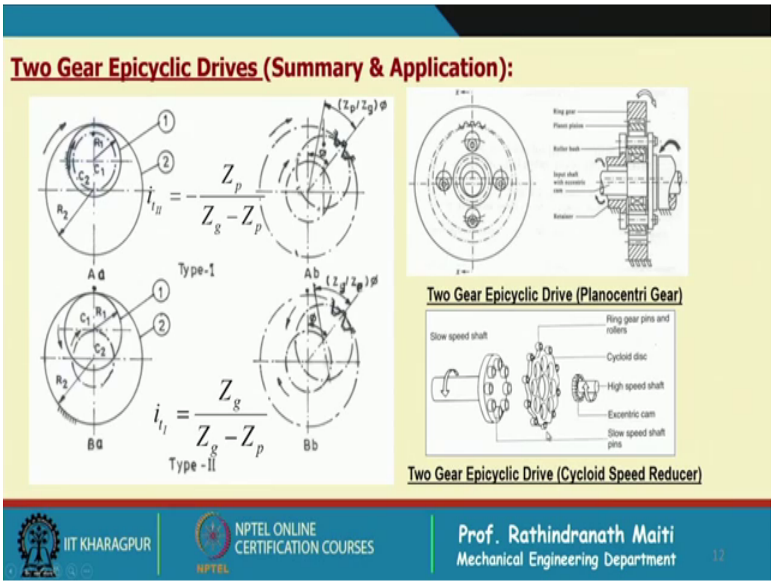

And this is the Wankle engine and this one is the Wankel engine and for these in all such cases transmission ratio is opposite Z g by Z g the minus Z p. And this is motion one. This is not this motion this is other one in that case motion of this inner one it kept fixed in this case in case of Wankle engine.

So, basically what we find that we have to this first of all we have described this one type one in that case after rotating the whole unit we have allowed the gear to come back its own position so that gear remain fixed and the output is the pinion. It can be done in opposite way that this we constrained the motion of the pinion. So, that it is this axis on the pinion always remain vertical that means, the point here we bringing up to this point.

In that case, this ratio of rotation of the outer gear will be Z p by Z g into phi; in this case Z g by Z p into phi and then type on transmission ratio will be different from this one as you find here. Sorry, it is this one will come here, this is for two, this is the two motion and this is motion one. So, this is the transmission ratio of one, they want stand for type, one two stands for type two.

So, we have misplaced this one anyway with this type of gearing arrangement with involute like teeth not exactly involute with very small difference of teeth because it will be an advantageous only if we give this difference it is very small. So, we have a gear gearing arrangement it is which is called planocentric drive, where this planet carrier is somewhat not the normal it is allowing this gear to rotate like this with a hole this planet carrier is put this hole is having the center distance equal to this one. So, that we get the desired motion. This is called planocentric drive. And usually you will find the two planets or three planets at equispaced so that dynamic balance is maintained.

Also, we will have the two gear epicyclic drive which is sorry with the cycloidal teeth which is called cycloid speed reducer which I have shown. So, this is the cycloid speed reducer. In that case these are pins are one gear, and this wavy cam is another gear. And this is rotated by usually a cam; the input will be cam like this.

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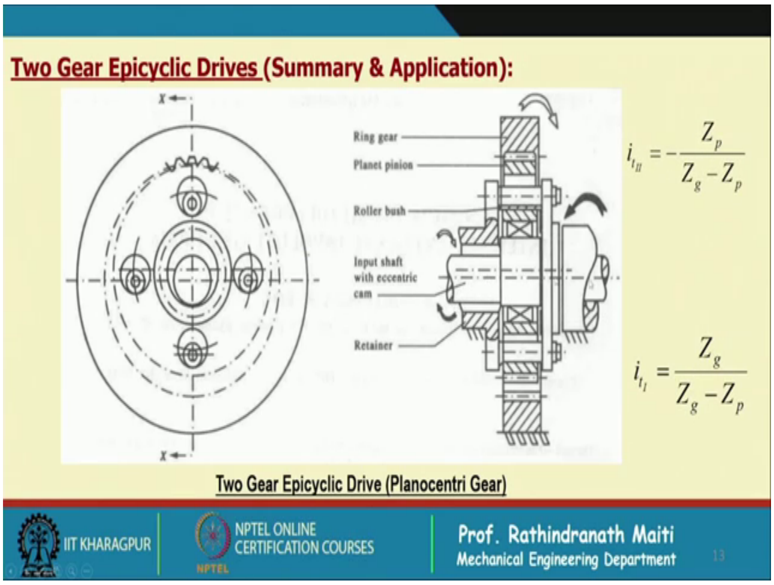

So, we have shown again, and this is the view it is if we can get type one Z g one divided by Z minus Z p. In that case the output is gear; whereas, in this case output is the pinion and it will rotate in the opposite directions; and in this case it will rotate in the same directions.

Now, interestingly with involute teeth although we cannot make this difference one, but cycloidal class of teeth that is epitrocal teeth we can make this difference is equal to 1. That means, the model which I have shown rotating model in that case Z g the number of low we in outer was 7 and this difference is 1. So, we get reduction in 7.

In case of this such planocentric drive this difference usually 3 and 4. Whereas, the tip number of pinion may be 100 or 99. So, in that case we will get transmission ratio is equal to 33.

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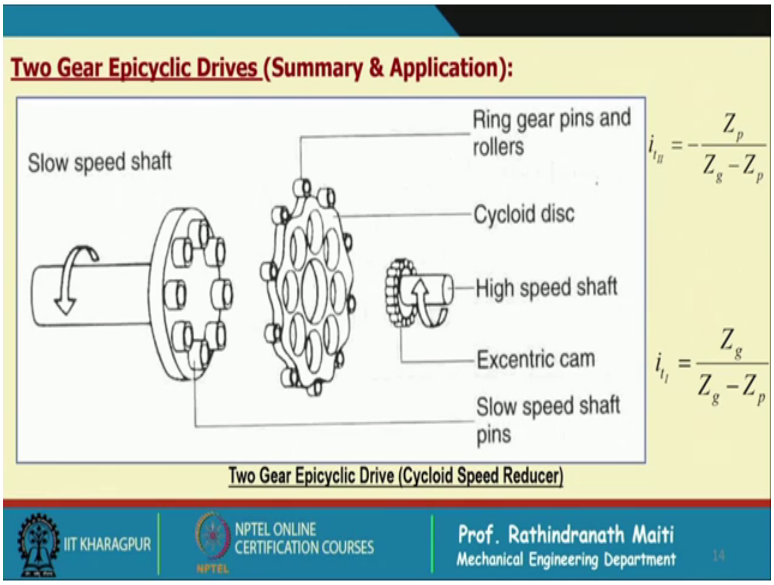

So, in this case, it is possible we can make this transmission ratio simply the teeth number by one of that either this or this circular one the teeth of the outer one and these lobes are the teeth of inner one the difference is one So, we can have the transmission ratio either the number of the lobe on the inner one divided by the difference one or number of lobe by the outer one divided by the difference one. The, it will be type one or type two depending on which one is fixed and through which we are taking the output.

In these case, we should remember the input is always the arm which is in which is a cam, cam sort of things which is put inside here. This cam means it is on the shaft this is and another eccentric is mounted and here is the center distance center distance is nothing but in this case one to the difference into module divided by the 2.

So, if we can involute teeth, suppose it is a 5 module and tooth number of gear is 100 and pinion is 99, then center distance will be 1 into 5 divided by 2, 2.5 millimeter. So, eccentricity will be of that amount in this case, also it is very small.

So, thank you.