Gear and Gear Unit Design: Theory and Practice
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Lecture - 38
Tooth Tip Interference, Avoidance and Contact Ratio in Involute Internal Gearing

Welcome to last lecture of module 7, that is week 7 which is on introduction to involute gear tooth correction, and this is the 6th and last lecture of this module. The title of this lecture is Tooth Tip Interference, Avoidance and Contact Ratio in Involute Internal Gearing.

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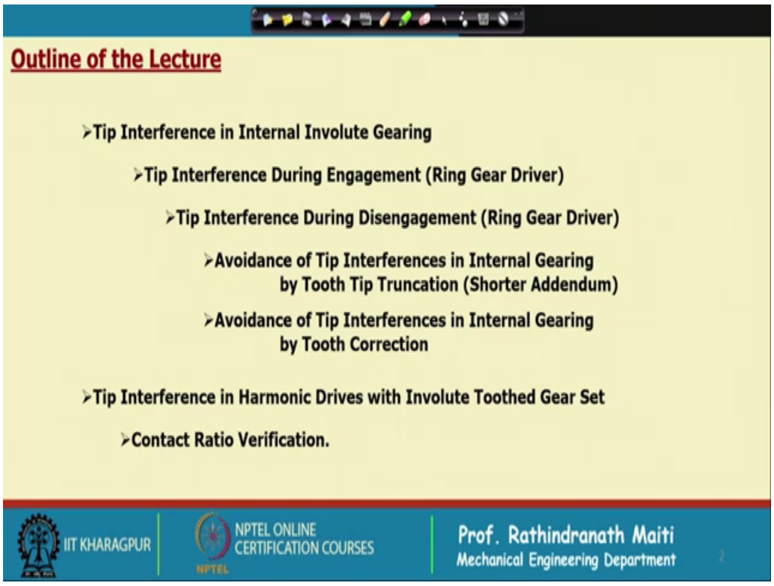

In this lecture first I shall give the definition of tip interference in internal involute gearing, then tip interference during engagement ring gear is driver, tip interference during disengagement, ring gear is driver; Avoidance of tip interference in internal gearing by tooth tip truncation which is sorter addendum. Avoidance of tip interference in internal gearing by tooth correction; Tip interference in harmonic drives with involute tooth gear set and finally the contact ratio verification.

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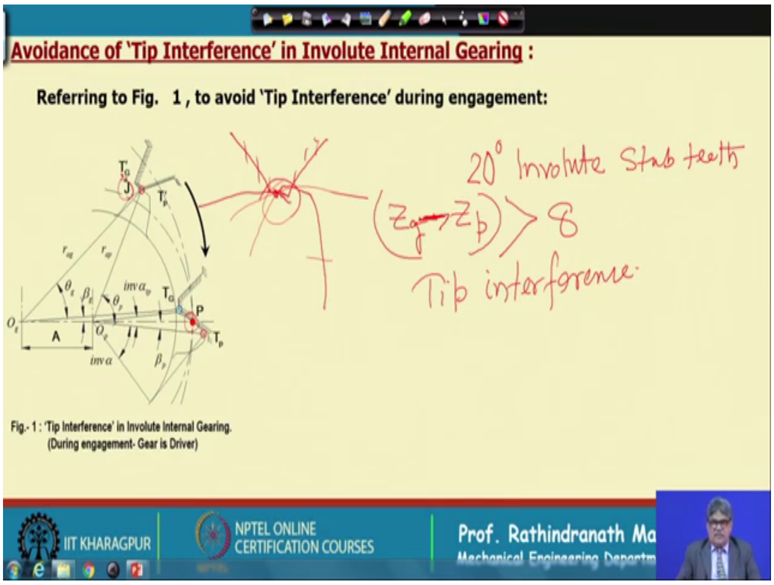

Now, first let me explain what is tooth tip interference. In internal gearing when it is used for epicyclic gearing normally there you will find son pinion and the internal tooth ring gear.

Now, if we think of the size if size of the ring gear is 100 then other two are less than 50 less than 50 size; that means, if the ring gear is of 100 teeth then the pinion son pinion and planets they will have teeth number less than 50 this is very common ok. In case of just external gear drive sorry internal gear drive with only a fixed ratio where the pinion is driving the internal gear. For an example inside an drum that is fixed in that case also there will be at least tooth difference more than 10 or even more because we want the ratio we want the transmission ratio there.

So, number of teeth in the external tooth gear that is the driving pinion you can say will be less than much less than the internal gear at least 10 tooth difference will be there or even more. But there are some other gear drive which is actually epicyclic they do not fixed access, but arrangement is different and it is very difficult to consider those as in a physically drive. One example is true we are drive where the transmission ratio will be more if the teeth difference is less. High transmission ratio is possible with teeth difference one.

We shall learn in the next section about those, but it is not possible to make the teeth difference one in case of involute internal gearing ok. So, if we consider 20 degree stab teeth say let us consider 20 degree involute stab teeth. Standard stab teeth means as I told earlier the eternal point will be 0.8 standard tooth eternal factor is 1, standard stab teeth is pointed internal gearing usually standard stab there used if not corrected. With 20 degree involves stab tooth then if we consider the teeth number of gear is Z g and teeth number of pinion say minus Z p that usually should be greater than 8 otherwise there will be a tip interference.

What it is? That if we consider this figure. So, let us consider the ring gear is driver ok. So, it is driving the pinion which is the most cases in case of such tool gear efficiently drive, in that case if we draw the geometry and if we consider the pitch point here this is the pitch point p then let us consider the intersection of two which is called tip circle diameter or addendum circle diameter of pinion and gear is given by j. Intersection of the tip circles of the gear and pinion is designated by j ok.

And now if we consider that T p is the tooth tip corner here and T g is the truth corner here. Now, when they are coming in being engaged then T p dash during the time of engagement or you can say this is edg it is called engaged they are they are engaging mode. So, that T p dash should cross the T dash g or or in other words T dash p should cross g before T dash g would come there. I repeat again.

Let us consider the internal gear drive where the gear is driving the pinion and direction is shown then tooth tip corner is of the gear is designated by T g and it is of the pinion is designated by T p. Now, they are coming and they I will reach at pitch point. Then with reference to this pitch point if we consider when they are coming in they the T dash p should cross the point of intersection of addendum gears which is j earlier than T dash g, otherwise what will happen you may find that if this is the teeth of external gear and tip this the pinion.

Suppose here is the j this is the tip circle radius and this is the tip circle radius here was the j, but they have come together at that point this has not crossed this pinion has not crossed earlier that point then they will interfere which is called tip interference which is called tip interference. This occur in internal gearing in involute internal gearing if the design is not proper, and there is for the standard start tip or even with full lift this number this difference should be minimum as I have specified here it is 8. If the pressure angle increases this number will reduce if the pressure angle decreases this number will increase.

So, 8 is the value it actually it is 7 point something for which this win this T dash p will be reached at j at the same time T dash g also which reach at that point. Suppose it is 7.8 then if you make it 8 then in that case T does T dash p will cross that point just a little before T dash g and there will be no interference, ok.

So, we need to verify such interference by theoretically, how to do it that we will learn here.

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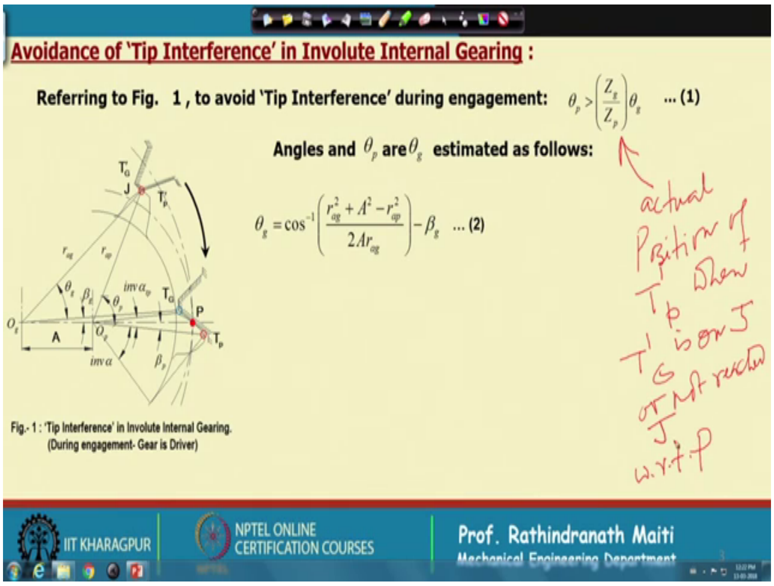

So, the condition is that if you look into this geometry that with reference to the pitch point then theta p. What is theta p? Theta p is the point of intersection the point of j from the pitch point the spread of the j and T Z with respect to the center of the pinion; that means, you can clearly see what is theta p. That theta p must be greater than Z g by Z p into theta g.

What is theta g? Theta g again the on angle with respect to the gears and j to p up to pitch point j to you know sorry this is just the angle of tip rotation respectively rotations. So, this is a little confusing to visualize this, but think about that if theta p is greater than then theta g Z p by theta g this means that actual position the left hand side is designating the actual position of the T dash p, actual position of T dash p this is with respect to p.

So, that you have to visualize that you have to understand clearly this angular conditions ok.

Now, the we can calculate this theta g this is geometric clearly this geometric point of g which is if we consider this triangle that O g j into b dash sorry op then r ag square plus A square minus rf square divided by twice A rg minus beta g; that means, whole angle theta g plus beta g is this much minus beta g must be equal to theta g, ok.

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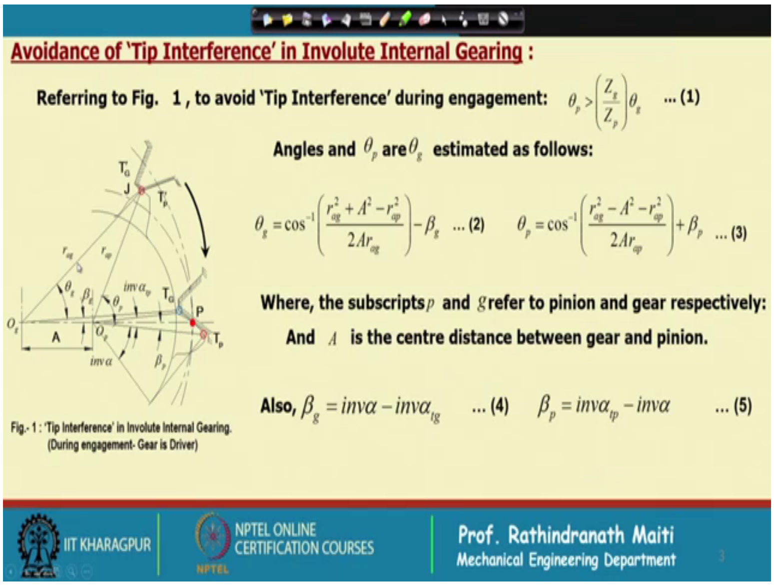

And similarly we can calculate the theta p also you can just compare it with this geometry and this value will become like this, where the subscript p and g refer to the pinion and gears respectably and A is the center distance between gear and pinion.

Now, beta g say this angle this angle here it is given what is beta g that can be found out by involute alpha minus involute alpha T g here is not shown alpha T g, and similarly beta p will be involute alpha T p involute alpha. This involumetry we learn at the beginning also during the gear tooth correction we have learned a little more. So, you have to be careful about calculating this ok.

So, now, that means, from the geometry of the gears we can easily calculate this to calculate r ag we should consider the standard pitch circle and then addendum we have to add two addendum with the channel pitch circle diameter. Now, addendum it might be 0.8 in commonly within case of internal gear. If you truncate it it will be less or even it can be more that you have to accurately calculate you have to put it there and center distance if it is not corrected gear then easily you can calculate what is the center distance there.

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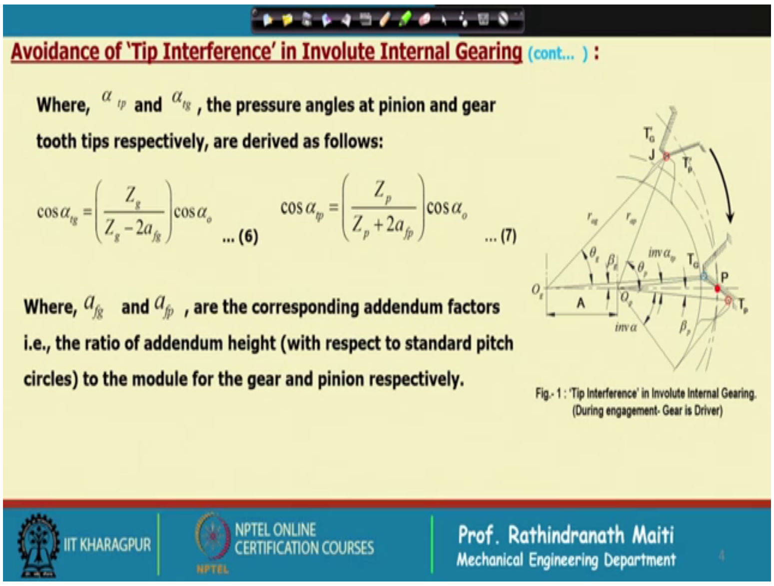

So, other values can then be calculated with no difficulties, where alpha T p and alpha T g the pressure angle at pinion and gear tooth tips respectively and these are derived as say Z g minus 2 a fg in the denominator 2 a fg, a fg is the addendum factor and alpha 0 alpha 0 or alpha o subscript o is the standard pressure angle.

Say if we have considered the 20 degree involute gears then we have to take this is two entity whether it is corrected or not corrected. For this calculation we have to consider the water is the standard angle we have considered and what we have considered the addendum factors with respect to the standard pitch circle, ok.

And a fg and a fp are the addendum factors for the ratio of addendum heights. This means that addendum height can be given by addendum factors into module. So, it will be a fg into module or a fb into module, in case of gear and pinion respectively.

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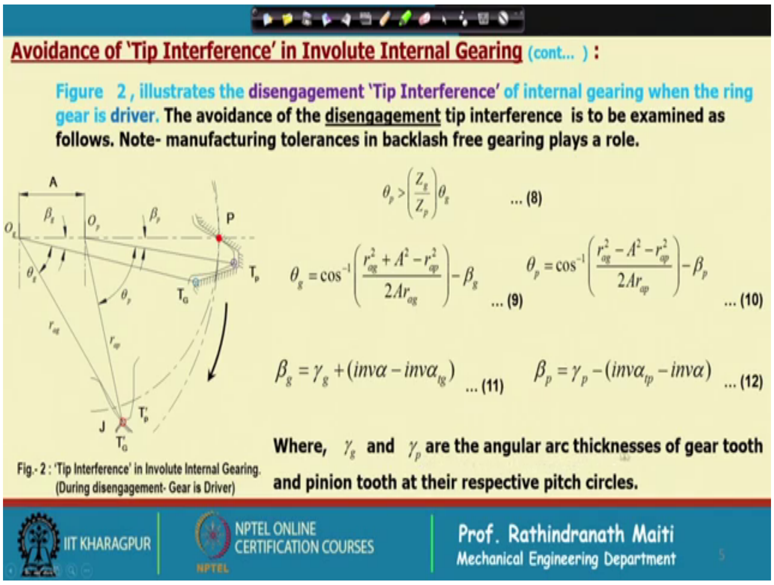

Now, then question is that what will happen during suppose there is no interference during engagement the ring gear is driver ok. We will be there any interference during disengagement say suppose it has entered inside the tooth pockets and you trying to go out will be their interference.

The simple answer is that usually it will not happen because if there is a little bit a backlash then you will find it will never happen it will never happen ok. But still it can be verified with the same geometry. Here I have presented this one. So, this is procedure is same you can calculate all these angles and then finally, we this is the condition, condition at 8 that should be satisfied where here this another angle are gamma and rp are the angular arc thickness of gear tooth pinion tooth at their respective pitch circle. This are gamma g and gamma p at pitch circle for standard gears this would be equal, ok.

Ideally this would be equal, but usually gamma g will be slightly less than gamma p because to avoid the say jamming there will be backlash that r g minus r p can be considered as the backlash there ok, are viewed less. So, that due to this backlash automatically this T tip interference will be avoided provided it has not tip interference during the engagement it is automatic. But still we can consider everything and we can calculate in this way.

Now, next question is that if the pinion is driver then what will happen. If the pinions become driver then the case is opposite during the engagement. So, this will be the engagement you can say and the arrow of rotation which is showing the direction of rotation it will be just opposite. So, it will be in that case it will be more critical during the disengagement, disengagement of pinion from the gear it will be more critical.

So, for example, we have used a very big pinion to drive a ring gear due to some reason it might be we do not need fixed axes; in that case we need to verify the disengagement interference considering the pinion as the driver that we should remember, ok.

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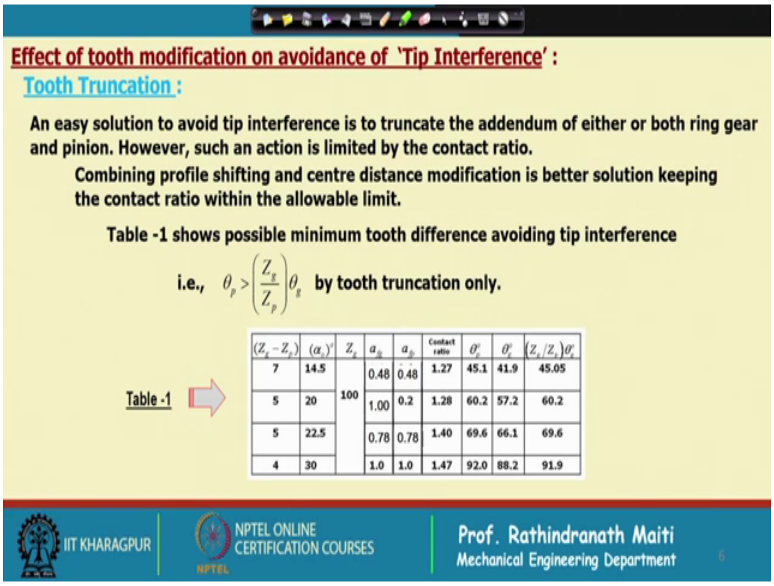

Now, how do avoid I the two tooth tip interference? As I told if they are interfering simply trim that that is the first way that is the easy way, but while you are trimming this teeth then you may find the contact ratio may not be sufficient although in case of internal gears the contact ratio will be higher than of same teeth number in external gear.

For an example, if you take 100 teeth gear and 80 teeth pinion say for external drive external tooth gear drive what will be the contract ratio for the same amount of addendum and addendum? It will be higher in case of internal gear if you take 80 teeth pinion and 80 teeth ring a 100 teeth ring gear with same proportion of tooth height that you should remember.

But still while we are truncating definitely each and every case, suppose after we give small truncation and we will check whether it is tip interference is avoided or not ok, then each and every case we need to verify where the contact ratio is satisfactory. It should be at least 1.4 as case of power drive.

Now, what is done? One exercise is done that we have truncated the teeth say 14 and half we are definitely the number of teeth will be more, ok. So, first let us verify the case of 20 degree teeth 20 degree involute. In 20 degree stop teeth; that means, with 0.8 addendum we found that two the difference should be 8, but what we have done? We have truncated and the tooth say and we have shifted the center like that mostly truncated this is this example is for truncated the teeth then it is very peculiar although peculiar and not may be may not be feasible that we have kept the addendum factor of gears is 1.

Whereas pinion only 0.2, somehow we managed to have the contact ratio is 1.28 tooth difference is 5 ok. Then this tip interference is just avoided just appetite, but these are not feasible design. But we have shown if we agree with this 1.28 very slow speed it might be possible if you keep the contract with say 1.28 and addendum factor opinion key 0.2 no correction it is clear then we can have Z g is equal to 100 and tooth difference is 5; that means, 95 it is possible.

If we go to 14 and half in standard case much higher difference is much higher it is I think 10 or 11 tooth difference would be there, but still we can come down to 7 keeping both addendum factor of gears and addendum factor opinion 0.48 where contract issue is somehow you can get only 1.27 and these right side angles are given to how they are crossing this as you can see this. One is 45.1 and this is 41.9 and this ratio is 45.05, just it is less, ok.

Now, we are increasing the pressure angle with 22 and a half which is used for internal gearing in precision type in aircraft internal gearing sometimes this angle pressure angle is used it is not standard for common industrial application there we have kept the addendum factor 0.78, 0.78. So, contract issue has improved to 1.4 and we find that it is possible it is possible, so perhaps 22 and half of this design is acceptable ok.

And to with 30 degree obviously, with tooth difference only 4 we can make even the addendum factor 1, above 0.8 above standard stab teeth and contact issue is 1.47 perhaps with stab teeth it will come 1.4 or even a little higher and in that case this is avoided. This means that if we can have such a drive with say 30 degree it is better with true difference only for, but by no means we can reduce the teeth number less than that. So, at least it should 4 at least it should 4 with only by truncation. But if the correction is introduced probably the situation will improve.

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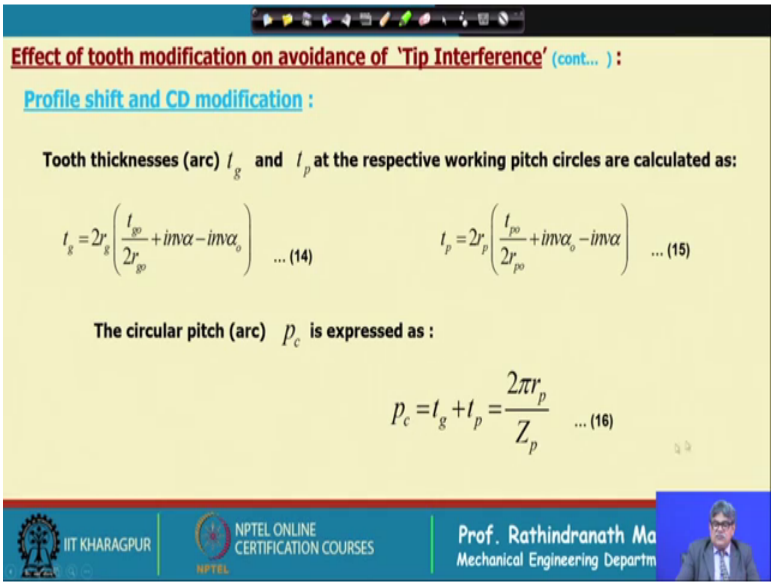

So, let us see that now we have modified center distance and in that case this is the repetition, how to calculate that tooth thickness there and that circular piece will be expressed by this one now.

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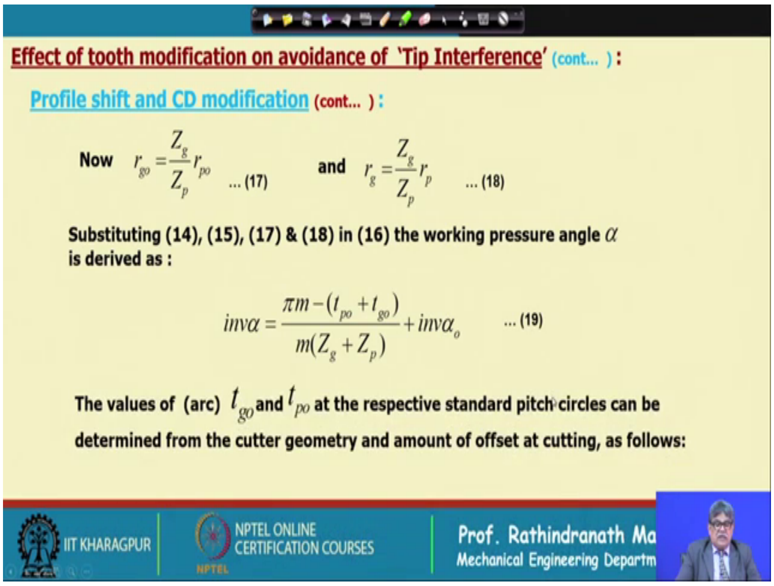

And then which CD modifications with tooth corrections then these relations this is shown earlier also, this can be introduced there. And we can you can step by step this is the procedure of gear tooth correction interval and gear tooth correction which is shown earlier that is repeated here. And we can calculate finally, the center distance ac between the pinion cutter and the ring gear being cut for an offset x g can be expressed as because this we need to find out the two thickness after the cut, ok.

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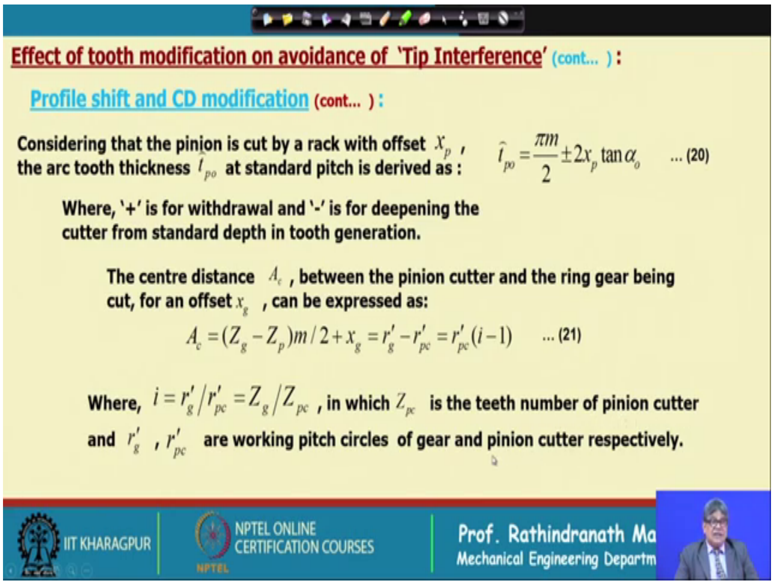

And these are the relations for new I mean the cutter which circle radius working pitch circle radius r dash species and working with certain radius of ring gear is r dash g which is given by r Z Z g by Z pc, in which Z pc is the teeth number of pinion cutter, ok.

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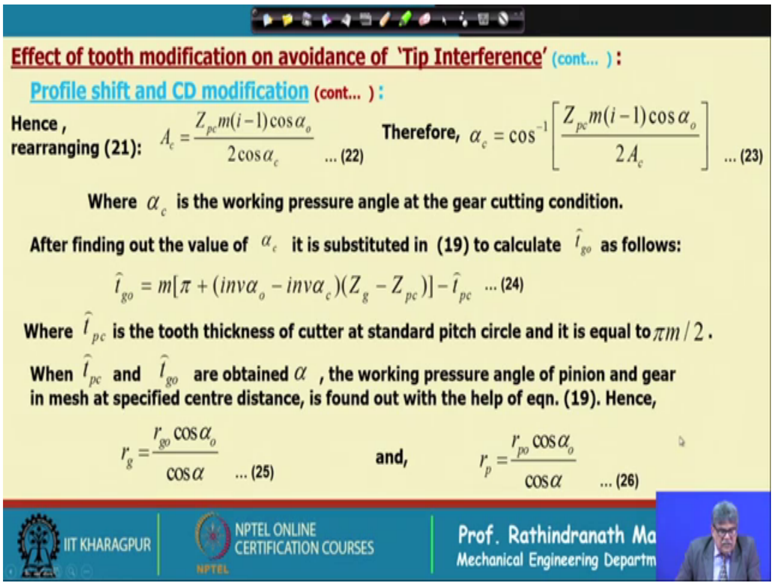

And then finally, rearranging 21 we find that center distance of cutter A c will be this and we if the A c we fixed that is the center distance which can be fixed depending on the which can be found from the depending on how much correction we have given amount of corrections from there we find.

And then we find the alpha c, alpha c is the c is the working pressure angle at the gear cutting condition and from there we find out again the thickness at the pitch circle. And again we calculate the r g and r p with that respect.

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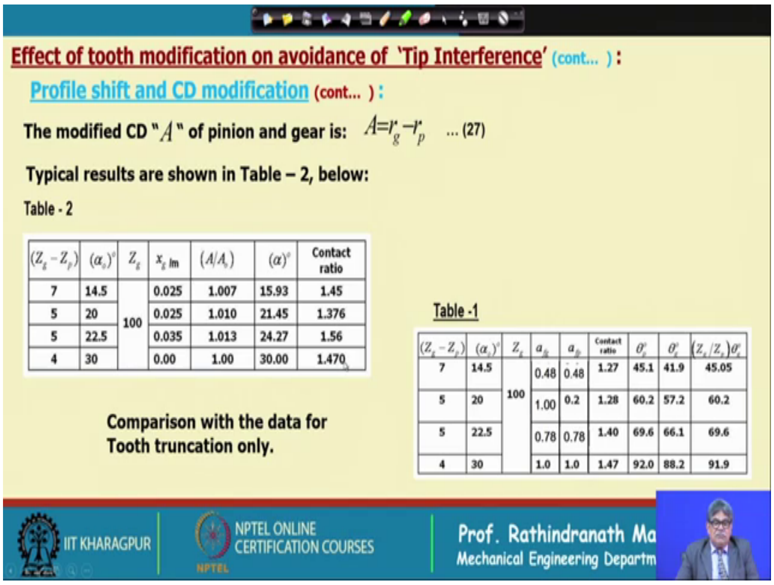

And then with such corrections if we introduce the corrections then what we find for 14 and half tooth difference we have kept 7 the same A by A 0 this correction amount of that that is showing that how much correction has been introduced 1.007 and we are getting contact ratio 1.45 avoiding the tooth interference, avoiding the tooth interference.

So, what is there that with such corrections, with corrections means if we change the CD there is no meaning keeping the teeth same. In this case the it is being reduced or if you go for minus correction it will be it can be also increase in the distance, but here new center distance is A 0 whole center distance was A; no, sorry.

New center distance is A 0, whole center distance is A; that means, A 0 is less than A and amount of correction x z by m by module it is 0.025. In case of 20 degrees are also 0.025. Here the in case of 20 degree it is improved to 1.376 for 22.5 it is 1.56 and from

30 it is 1.470. So, in case of 30 we need not change the center distance regard it is not required as shown in table one which we discussed earlier. So, you can see that how much improvement can be done with intuition gear tooth corrections.

So, in case we have to go for less number of teeth with pressure angle 14 not normally used for internal gearing say 20 and 20 and half then we can introduce the center distance correction to the modifications and we can have improved result, in case of 30 degree which is not required.

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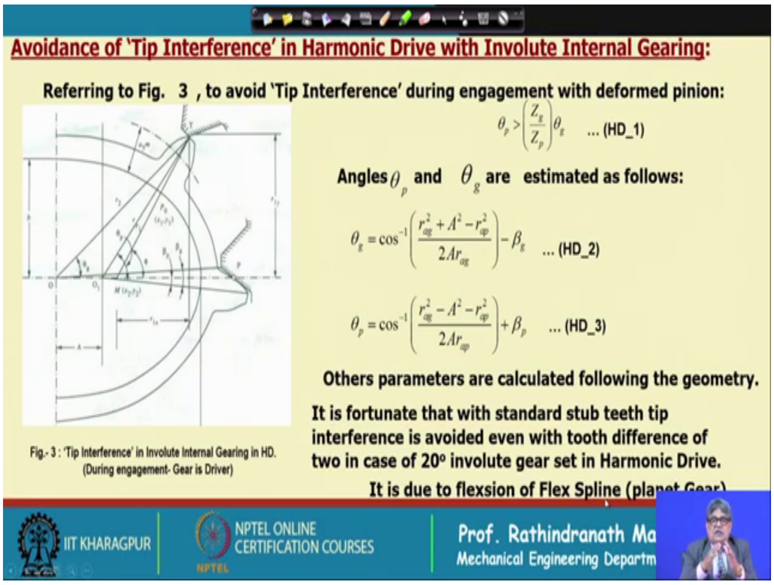

Now, another interesting matter is that in the next lecture the module 8 we will learn about the harmonic drives ok. In that harmonic drives the pinion is on thin rim and that is deformed elliptically and rotates inside the ring gear ok. Now, there the tooth difference is kept two; obviously, correction is introduced if we go for involute gear there other other profiles are also used, but in case of involute profile still two teeth difference is made with 30 degree ok.

Now, if we would like to verify the tip interference there then we have to consider the different conditions of the pinion also which is shown in this figure. But the procedure will remain same only geometry will become a little more complicated, but that also can be calculated. There are few references which you can study and weak you can find out ok. So, this is simply just by the equation normal equations the conditions are shown ok.

And other parameters are calculated following the geometry and then it is fortunate that the standard stab tooth tip interference is avoided even with tooth difference of two and 20 degree involute gear in case of harmonic drive ok.

We are fortunate that this is there it is essential that keep tooth difference should be at least 2, then of course, it can be kept 4, but most of beneficial will be if we use the tooth tip difference and which is very common applications. It is due to the flexsion of flex spline as the flex spline is being fixed. So, it is you can say forced deflected and it helps in disengagement as well as in engagement, ok.

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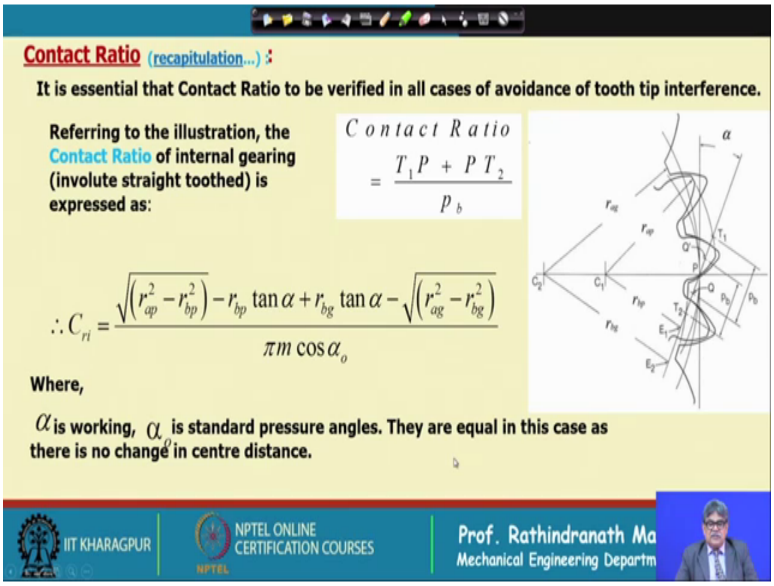

But as I told each and every case when we are truncating or we are introducing the gear tooth correction center distance modification we should verify the contact ratio. We have already discussed this earlier with reference to the figure of the internal gears.

The contact ratio can be given by T 1 P by P T 2 summation of that; that means, if we consider this distance from here to here plus here to here one is engagement during engagement and other is disengagement, that divided by best pitch which is fixed whenever the pressure angle is chosen it will not change pb will not change even if we go for corrections. But this T 1 P, P T 2 will change so that we need to calculate carefully.

And the formula what we use here what we, you can see this formula is written in form of if we know the addendum circle that is the blank on which this gear on pinion has cut r amplifier, r ag that can be easily calculated; r bg and r pb as the best circle once the tooth is a pressure angle is finalized then we can easily calculate and then alpha tan alpha and tan tan alpha component is coming over there.

This alpha is the working pressure angle that we need to calculate on the basis of center distance where they have made say calculate; corrections is one part to thickness etcetera one part. But if you know the center distance finally, if you know these dimensions easily you can calculate the what will be the contact ratio. So that need to calculate each and every time while we are trying to verify the interference.

So, alpha is working and no alpha degree the standard pressure angle and they are equal in this case as there is no change in center distance. This I have I have mentioned, if if there is no center distance then they will be same. If there is a same, even if only corrected gear if the centre distance is not changed then working pressure angle has to be the same as the standard pressure angle that you should remember. Say for example, good example is the plus minus correction tip thickness are varying, but this angle is not varying. If we increase the angle or decrease the angle only in that case the working pressure angle will vary that to we calculate very carefully.

So, thank you for listening this part. This is the end if end of week 6 lectures where I have given a fundamental idea of gear tooth corrections only on state tooth spur gear not helical spur gear. With helical if the teeth are made helical this will be more complicated, in case of bevel here it is further complicated when they are corrected. In case of we will gear normally specified by the manufacturer. But in case of we will helical gear it is although it is difficult still it can be calculated it is very tedious to calculate that the amount of corrections to thickness itself etcetera etcetera. However, it is possible.

Now, another I would like to tell you at this stage up to the week 5 lectures that is mostly on the general purpose gear and a gear box designed that should not be difficult for any student whether it is a third year bachelor student or maybe first year first graduate student it should not be difficult. But a gear tooth corrections and the next which is a special gearing that might be a little difficult for the students, but here one thing I i can tell you that in this learning process to give the horrible idea of the gear design I have introduced to this correction part and the new gears also, special gears in the next week.

And the assignment, in that assignment we will learn how to correction in the correction tooth tip interference etcetera, etcetera.

But if we think of the final examinations definitely no difficult questions will be there from week 7 and week 8 lectures, no difficult questions ok. That you B.tech students, whether you are B.tech or M.tech students you need not worry about this complications and tedious calculations it will be very very simple you should only understand what are these, ok.

Thank you very much. Next we will go for week 8 lectures.