Gear and Gear Unit Design: Theory and Practice
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Lecture - 37
Involute Spur Gear Tooth Correction: Tutorial (Contd.)

A module 7, we are still continuing with Introduction to Involute Gear Tooth Correction. And in this lecture involute spur gear tooth correction there will be another tutorial, this is workout example 2.

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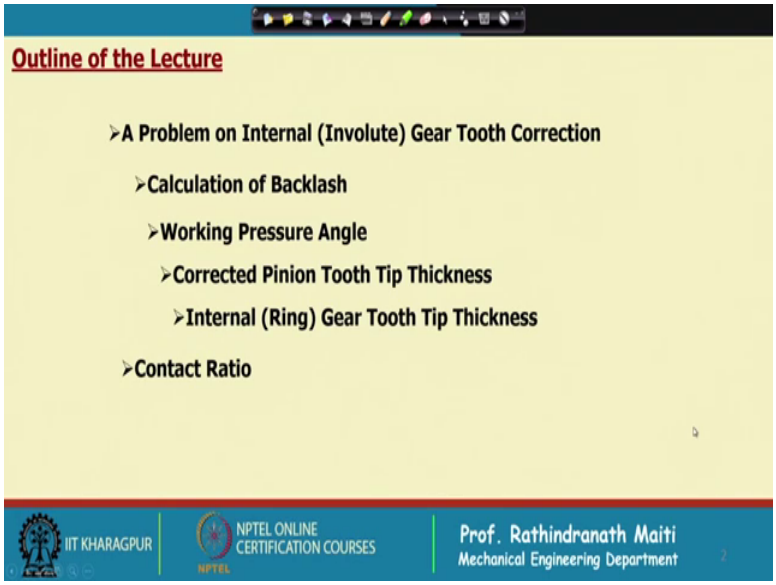

In this lecture I shall cover a problem on internal involute gear tooth correction. First of all I will show that after introducing correction what will be the backlash, working pressure angle, corrected pinion tooth tip thickness, internal ring gear tooth tip thickness and finally, when a gear correction is given usually the addendum is truncated or increased, some modification will be there. Therefore, it is essential to calculate the contact ratio also that we shall do for this problem.

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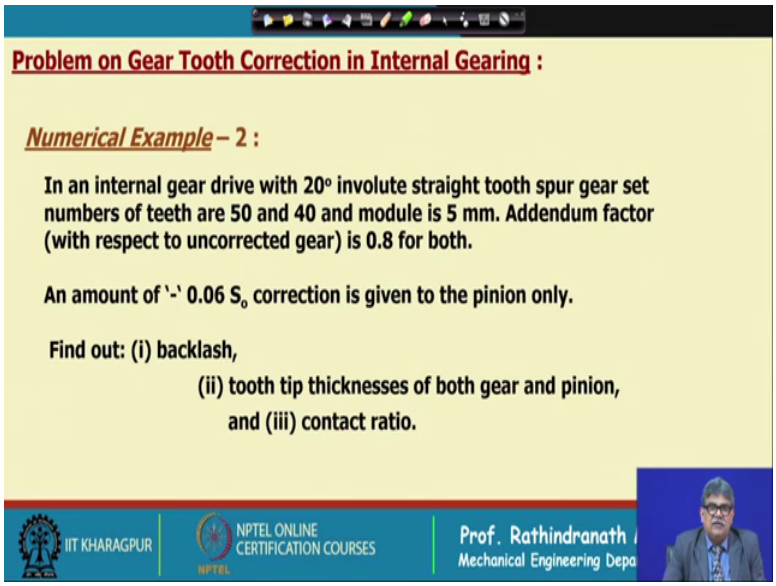

Now, the problem is a is that in an integral gear drive with 20 degree involute straight tooth spur gear set numbers of teeth are 50 and 40 and module is 5 millimeter. Addendum factor with respect to under uncorrected gear is 0.8 for both.

Now, here 50 40 means definitely in the ring gear will have 50 teeth and the pinion which is driving it or vice versa will be 40 teeth and module is 5, 20 degree involute. Now, an amount of minus point naught 6 S 0 correction is given to the pinion only. Now, S 0 correction means there will be no change in center distance and minus correction means we will depend the cutter, this means that minion truth will be thinner. However, we are not going to change the addendum what was that a addendum factor was there we will give that one; that means, the blank opinion will not be changed only this amount of corrections very small correction will be given and has it is S 0 there will be no change in center distance.

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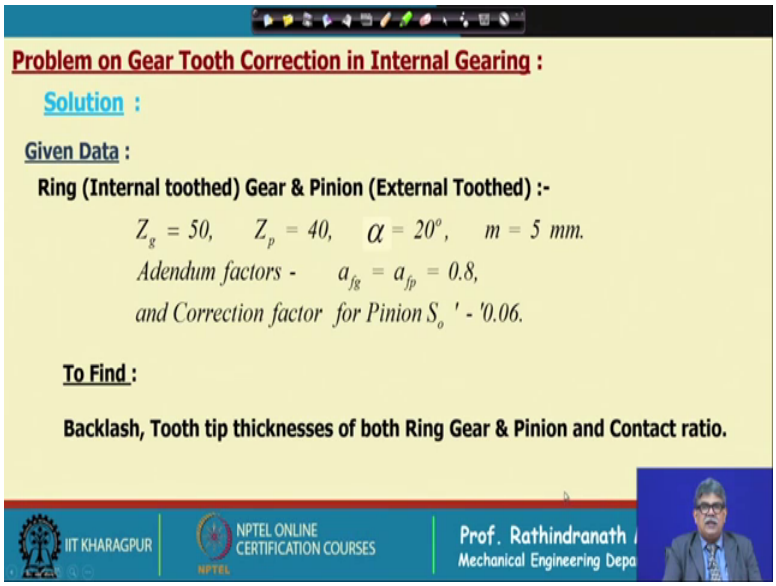

Now, we have to find out what is backlash. Tooth tip thickness of both gear and pinion and finally, the contact ratio. Now, the given data are Z g; that means, number of tooth of the ring gear is 50, Z p number of external tooth pinion is 40, alpha the pressure angle standard pressure angle is 20 degree and module is 5 millimeter. And addendum factors for both is that is a fg and a fp is 0.8, and correction factor for pinion is S 0 minus 0.06 and we have to find out backlash tooth tip thickness of both ring gear and pinion and contact ratio.

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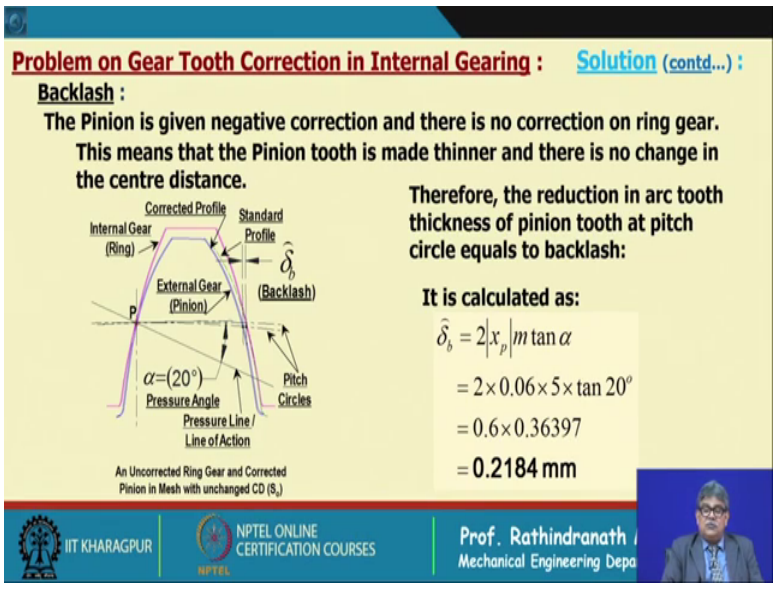

Now, first we will estimate the backlash. The pinion is given negative correction and there is no correction on ring gear. This means that the pinion tooth is made thinner and there is no change in the center distance, S 0 correction means there is no center distance change. Therefore, the what is the reduction of tooth thickness at the standard pitch circle to thickness of the pinion at the standard pitch circle that is the amount of correction.

So, we shall measure that amount of reduction and thickness on the pitch line of the ring gear and that amount is the backlash this is the answer is simple here. As we are not changing the center distance then the pitch circle, the working pitch circle will not change the pressure angle will not change. But as shown in the figure as shown in the figure there will be reduction in thickness pitch line; this amount which is given by delta b and with cap. This means that this is the arc thickness on rack cutter pitch line, this is the straight thickness but here it will be the arc thickness, arc distance, length along the arc you can say. So, that is that amount we need to calculate.

Therefore, the reduction in arc tooth thickness of pinion tooth at pitch circle equals to the backlash. It is calculated as delta b cap 2 into mod of x p correction factor because we are calculating the amount. So, x p is expressed with negative sign as it is negative correction, but here we will take the magnitude only.

So, we have used x p mode and m is the module then 10 alpha that is the standard formula and that becomes 2 into 0.06 into 5 into tan 20 degree which comes to 0.2184 millimeter, say it is 0.22 millimeter. So, that amount will be the backlash; very small backlash, but that is good enough for running the gear in good environmental condition.

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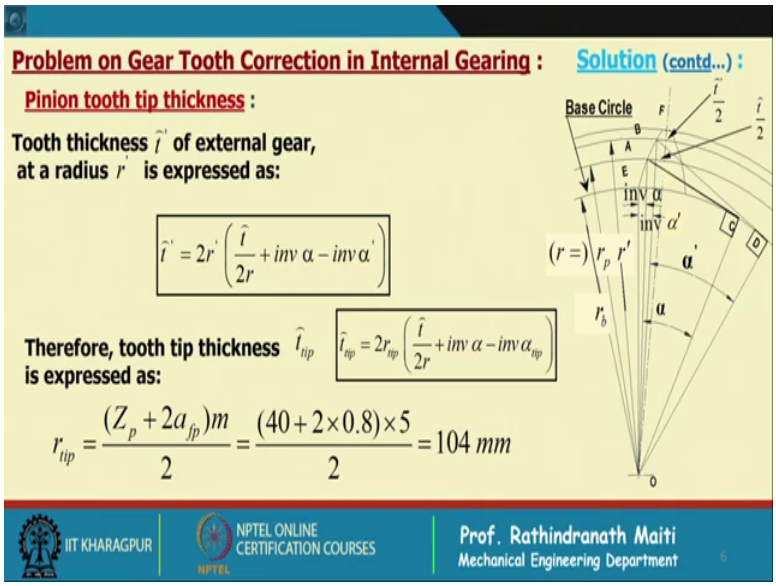

Now, we need to calculate the pinion tooth tip thickness. Now, this tooth thickness we have designated as t dash with cap that is we will take the length; we will measure this length also along the arc of external gear. At a radius r dash which can be expressed as t dash is equal to twice r dash whole into t divided by twice r plus involute alpha minus involute alpha dash.

Now, involute alpha means the involute of the standard pressure angle and alpha dash is the pressure angle at radius r 0, r dash, t is the arc to thickness of the pinion external gear at the pitch circle radius and r is the pitch circle radius. Therefore, tooth tip thickness t teeth will be equal to twice r tip because r dash is replaced by r tip whole into the t by 2 r plus involute alpha minus involute alpha tip; this r here these shown in the figure r is equal to r p that is the pitch circle radius. Now, r tip that is the radius of the pinion at the tip is equal to number of teeth of the pinion plus twice into addendum factor whole into multiplied by module divide by 2. The teeth number is 40 and addendum factor is 0.8 and module is 5; therefore, that value becomes 104 millimeter.

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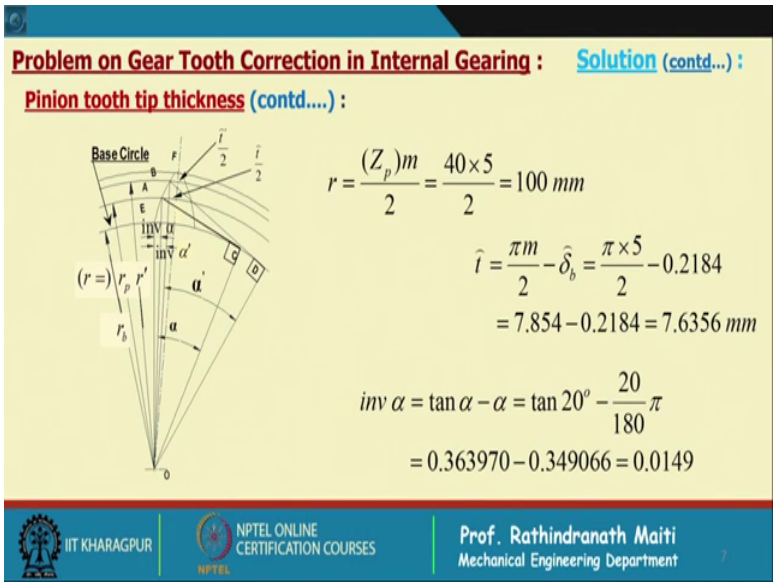

So, we have calculated it is 104 millimeter tip radius and the p circle radius is we calculated as number of teeth into module divided by 2 because this is the straight tooth spur gear. So, this is 100 millimeter.

Now, tooth thickness at the pitch circle is now, the standard tooth thickness at pitch circle that is circular pitch divided by 2 pi m by 2 minus the amount of correction or in this case we have considered that is the backlash that is del b. So, straightaway we calculate these tooth thickness the present tooth thickness which becomes the original tooth thickness was 7.854 minus the backlash 0.2184. So, this becomes 7.6356 millimeter.

Now, involute alpha; alpha 20 degree already it is shown earlier. So, that is calculated as 0.0149 again 0 and as instructed earlier, I again mentioned that we should take at least 5 decimal values for such calculations because the differences are very small, an involute of the angle is also very small amount. So, we have considered 5 decimal or 6 decimal places.

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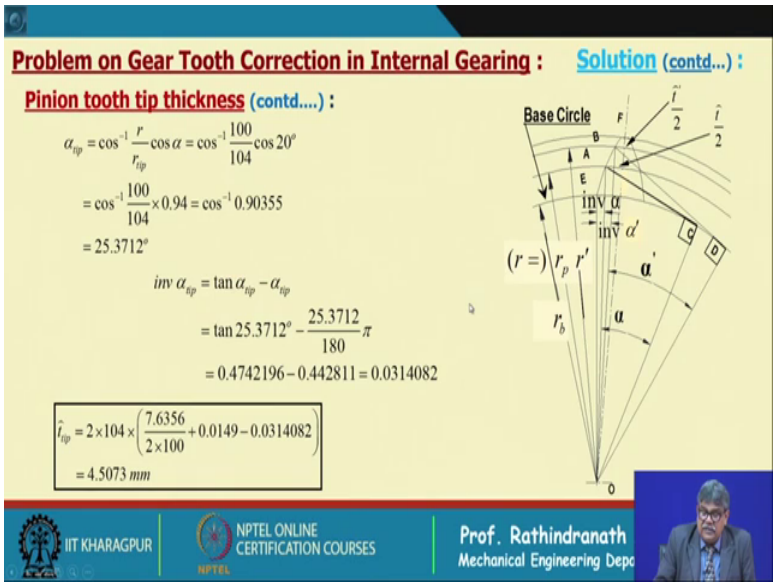

Next we calculate alpha tip; that is given by if pressure angle is known at any radius, in this case we have considered the standard pitch circle and the standard pressure angle.

So, alpha tip can be written as cos inverse r by r tip into cos of alpha and r is equal to 100, r tip is equal to 104, alpha is equal to 20 degree and from there we calculate that the at alpha tip the pressure angle will be 25.3712 degree that means, alpha teeth will be this amount. So, next we calculate involute alpha tip, the same way; mind it that in the formula 10 of the angle involute means 10 of the angle minus this angle this angle on the angle that should be expressed in radian. So, we have expressed in radian here 25.3712 divided by 180 into pi and finally, we have calculated that involute alpha tip is coming 0.0314082. We have considered here 6 decimal places.

Therefore, now, we have all the values therefore, we can calculate the tooth thickness of the corrected pinion at the tip, which becomes 2 into r tip is equal to 104, 2 into 104 into the tooth thickness at corrected to thickness at the standard pitch circle which is 7.6356 divided by 2 into 100 because 100 is the pitch circle radius plus involute of standard angle 0149 minus the involute of the pressure angle at the tip which is 0.0314082 and finally, we calculate the tooth thickness is 4.5073 millimeter, 4.5073 millimeter. So, we can consider that tooth thickness is about tooth tip thickness is about 4.5. For a uncorrected gear at the tip it will more or less same; it is 4.5 millimeter; that means,

addendum if he the addendum is taken 1, that is just for your information. Anyway, this our concern is that for this gear we have calculated the tooth tip thickness 4.5073.

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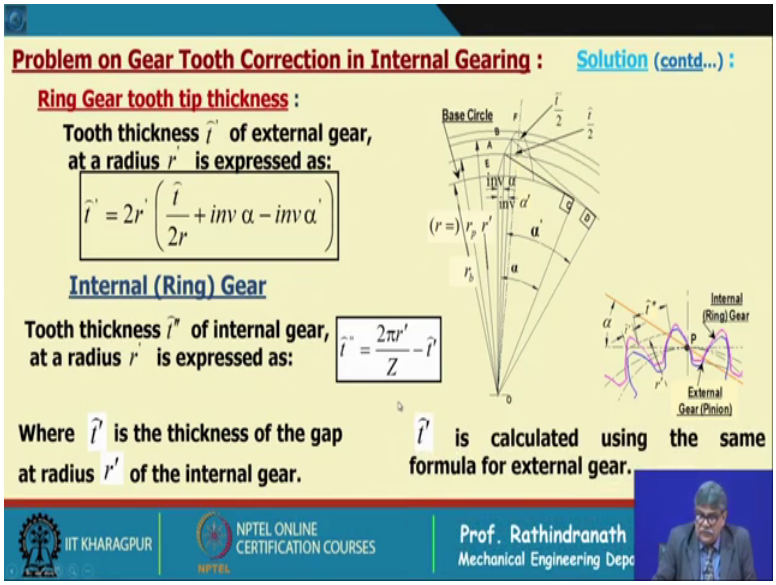

Next we move to the ring gear tooth tip thickness. Now, tooth tip thickness of external gear at a radius r dash is expressed as again the same formula t dash is equal to twice r t by twice sorry twice r dash into t by twice r plus involute alpha minus involve alpha dash. I remind it here that to calculate the tooth thickness of the ring gear what we are doing? We are first we are calculating the gap thickness because gap thickness means of this ring gear; that means, the external gear; external tooth gear that we will calculate at a particular r. And then from the pitch there we will subtract the amount of tooth thickness there to get the tooth thickness of internal gear ok.

Now, tooth thickness t double dash of internal gear at a radius r dash is expressed as t double dash is equal to twice pi r dash because at by Z the number of teeth of the ring gear that will give the pitch at that radius minus t dash which is the thickness of the gear; that means, external tooth thickness there.

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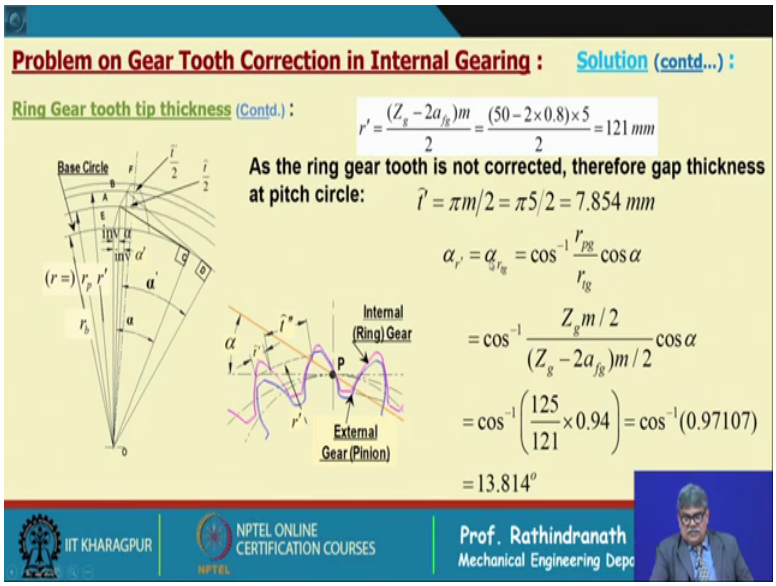

Now, the procedure will be same to calculate t dash. Here it is shown that we are calculating tooth thickness at the radius we are calculating tooth thickness at the radius we consider r ok. This is r dash and then we are considering this pitch, pitch minus t dash will give you give us t double dash that thickness.

Now, here the tooth tip thickness of the ring gear is number of teeth minus twice of addendum factor of the gears into module divided by 2. Now, tooth number is 50, so 50 into 2 into 0.8 divided by 2 into module will give us the tip circle radius of the gear is 121 millimeter, it is 121 millimeter. As the ring gear tooth is not corrected, therefore gap thickness at pitch circle is equal to t dash is equal to pi m by 2 standard pitch divided by 2 circular pitch that is pi into 5, 5 is the module divided by 2. So, 7.854 millimeter and alpha r dash that is at tooth tip thickness that is alpha r t g is equal to cos minus alpha into standard pitch circle radius of the gear divided by r radius of tip of the gear into cos of standard pressure angle.

We substitute the values and we get this angle is 13.814 degree. Mind it in case of what we observe in case pinion when we went to the tip, it was more than the standard pressure angle. The reason is that this was far away from the base circle. In this case this is close to the base circle; tip of the internal gear has to be slightly above the base circle in all cases, if necessary the addendum is truncated to move above the base circle. Usually teeth number are more than the critical teeth number; say for example, in case of

20 degree pressure angle critical tooth number for external gear is 17 and they are very rare that internal gear will be of 17 teeth it is usually 50, 40 more 30.

So, in that case the tooth tip which is if we take given one addendum below the pitch circle that will be above the base circle. In this case; obviously, it is above the base circle. So, at that point that is when it has been generated by the time it has opened by 13.814 degree. So, this is the pressure angle there which we have calculated.

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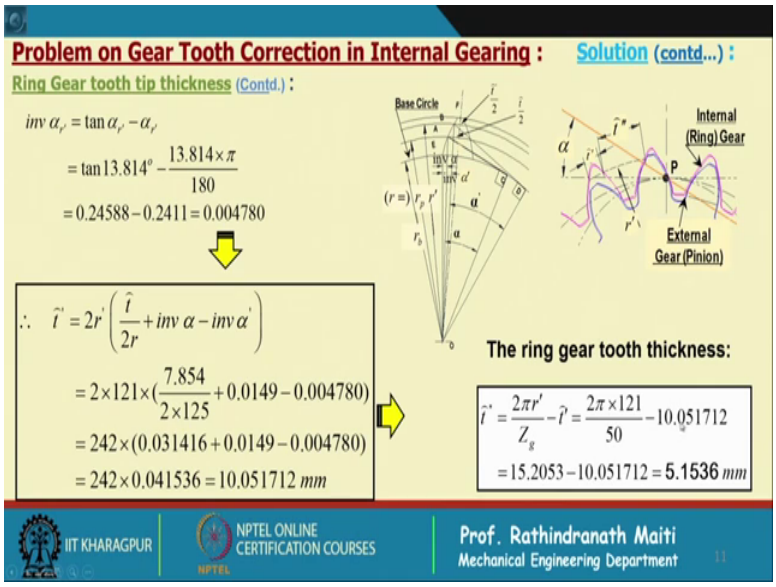

Now, we calculate the involute of that angle 13.814 and we get this value involute of alpha r dash in we have considered the in we are considering the internal gear which is 0.004780, 0.004780. And then we substitute all the values in the formula standard formula and r dash is equal to 121, tooth thickness had the standard pitch circle 7.854 and pitch circle radius of this ring gear is 125, involute alpha is 0.0149 and involute alpha dash 0.004780.

Now, you look into this as this value is very close 0.005. So, if you do not go for 5 decimal places definitely there will be error and what we find that tooth tip thickness not to tip thickness, the thickness at the radius of internal gears tooth tip that is at 125 millimeters from the center of the gear. The thickness of the gap is 10.051712, 10.051712; that means, it will be here, this thickness here this thickness.

So, we can consider that the thickness here this gap thickness, this is the gap at the tip of the ring gear is 10.051712. Therefore, the tooth thickness there; that means, this much a small amount that can be calculated as calculating the pitch there. That means, this point to this point which is 2 pi r dash; this is the r dash here in this case tooth tip divided by Z g minus t dash here 2 pi into 121, we have calculated the tooth tip radius is 121 divided by the number of teeth 50. So, that minus 10.051712 and we get the tooth thickness tooth tip thickness of the internal gear is 5.1536 millimeter, 5.1536 millimeter which is slightly higher than the tooth thickness we have calculated for the corrected pinion which is 4.5.

So, this gear would work and this will work and this is the way how we can calculate. This is a very simple problem I have shown and we can calculate what is the tooth thickness.

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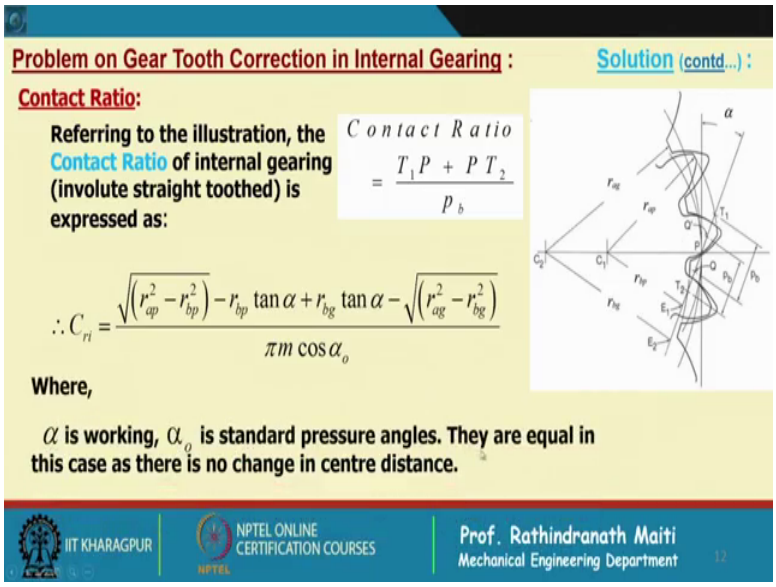

Next as I told once the correction is given usually tooth tip is truncated thicknesses are changing. So, we need to calculate the what will be the contact ratio? In this case we shall also calculate the contact ratio. If we consider this line of contact then this contact will be from starting from this point to this point, it will end to this point; suppose it is moving in this directions so contact will end at this T 1 and it will start at T 2. Referring to this figure, we can write the contact ratio is equal to T 1 P, P is the pitch point plus P into T 2 divided by P b means base pitch we have we shall compare the total length of contact with the base pitch because base pitch is the distance between two teeth.

Now, this formula the formula for contact ratio can be written as in terms of the tip circle radius and the base circle radius of both gear and pinion. It is written as r ap square that is r into addendum circle of pinion square minus rb p square; that is addendum circle of the pinion square minus r bp means this circle sorry this is the base circle of the pinion into 10 of the working pressure angle. In this case working pressure angle alpha is same as the standard pressure angle because we have not changed the center distance ok. Then base circle radius of gear into 10 alpha that is working pressure angle 10 of working pressure angles minus under root addendum circle radius square of the pinion gear minus the base circle radius of the gear divided by pi m into cos of standard pressure angle that is the base pitch. After correction when the working pressure angle, if the pressure working pressure angle changes still we need to compare with the base pitch because base pitch base circle is not changing.

So, we calculate the length of contact along the line of actions which is passing through the pitch point that pitch point again the intersection point of two working pitch circles, but we compare with the base pitch this is understood. And here it is written alpha is the working pressure angle and alpha 0 is the standard pressure angle.

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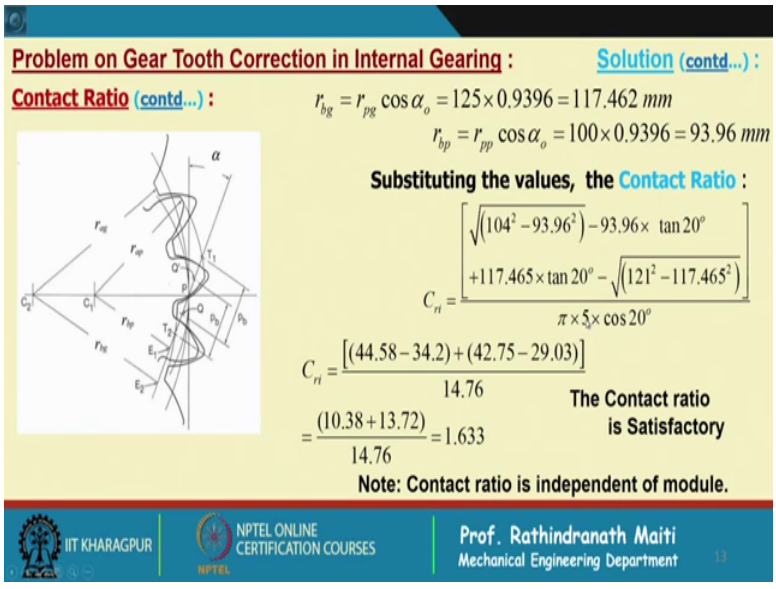

Now, we calculate all these values. This is the base circle radius of the gear which is nothing but the pitch circle radius of the standard pitch circle radius of the gear into cos of standard pressure angle which comes to 117.462 and base circle radius of the pinion

becomes 93.96 we have calculate in the same way and we substitute these values all these values to find out the contract ratio because the tip circle radius already we have calculated earlier. So, we have substituted these values. And finally, we get that the contact ratio is 1.633.

Now, this is satisfactory; the reason is that usually ideally, contact ratio can be 1, if there is no vibration if there is no deviation from the accurate dimension but we want win one pair is leaving its contact, there would have at least one pair and in that way it is found for practical purposes, for industrial gears we can have the contact ratio is equal to 1.4 at least 1.4. And in this case we have got it is 1.633.

For your information I would like to tell you with the same number of teeth if it would be external gear; that means, pinion is 40 and gear is external tooth 50 of same modules, same proportion everything, this contact ratio will be slightly less you can calculate this you will find this will be less but still that will be also more than 1.4 and that will be satisfactory and edge the two thick addendum height we have taken 0.8 we should call in that case these are the stub teeth standard stub teeth gears.

Now, one thing can be given, the contact ratio is independent of module that means here all these calculations module is included, but if you divide by this module; that means, all term can be divided by 5 then still this formula would work, ok. So, contact ratio does not depend on the module it depends on the tooth proportion and the number of teeth pressure angle, etcetera.

Thank you.