Gear and Gear Unit Design: Theory and Practice Prof. Rathindranath Maiti Department of Mechanical Engineering Indian Institute of Technology, Kharagpur

Lecture - 36 Involute Spur Gear Tooth Correction : Tutorial (Workout Example)

We are continuing with the introduction to Involute Gear Tooth Correction module 7 and this lecture is an workout example on involute spur gear tooth correction; this is you could may consider as a tutorial.

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Outline of the Lecture			
> Workout Example of Gear Tooth Correction (Profile Shift)			
Problem- Leading to Non Standard Centre Distance			
>Selection of Teeth Numbers, Working Centre Distance etc.			
>Calculation for Working Pressure Angle			
>Some Useful Formulae			
Estimation of Profile Shifts for Gear-1 and Gear 2			
>Calculation for Topping.			
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Now first of all, the example is on profile shift correction. First we pose the problem, it will lead to non standard center distance, selection of teeth numbers, working center distance, calculation for working pressure angle, then some useful formula, estimation of profile shifts for gear 1 and gear 2 and finally, calculation of topping.

Now, in this lecture we solve a problem where we could have go for all the rigorous calculation which we have present in last 3 lectures, but instead of that we will use some standard formula which is which can be directly we can arrived into those parametric values. For example, this topping value and etcetera. This formula has been suggested; it is from the analytical solutions as well as some practical empirical relations we will we can calculate those. But these through this problem will learn how the gear correction can be made from the practical purposes.

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Involute Spur Gear Tooth Correction :		
Modified parametric values with 'S' correction & centre distance modification:		
<u>Numerical Example</u> – 1 :		
A straight tooth spur gear set of $(lpha=)20^{\circ}$ involute teeth has a gear ratio of		
i = 3: 2 = 1.5, centre distance $A = 210 mm$ and module $m = 10 mm$.		
Select the teeth number and necessary correction keeping i and A exact.		
<u>Solution</u> : $A = (Z_1 + Z_2)m/2$ Now, $i = \frac{Z_2}{Z_2} = \frac{3}{2}$		
or, $(Z_1 + Z_2) = 210 \times 2/10 = 42$		
Solution of teeth numbers, $~Z_1 \ \& \ Z_2~$ expressed by above two equations does not lead to any whole numbers.		
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Now, the problem, in this problem we have considered a straight tooth spur gear set of alpha is equal to 20 degree involute teeth and it has a gear ratio of i is equal to 3 is 2 1.5. Center distance is 210 millimeter and module m is equal to 10 millimeter. And what we have to do? Select the teeth number and necessary correction keeping the ratio i and the center distance A exact, this means that this is for a specific purpose.

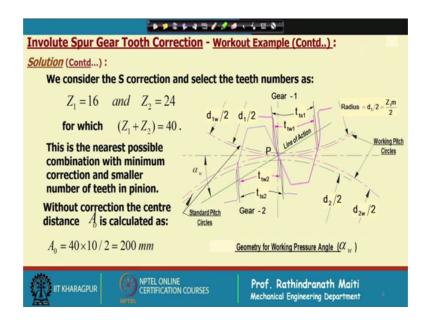
So, it has been specified that we have to stick to the transmission ratio fixed gearing transmission ratio is 3 is to 2 that is 1.5 and center distance 210 millimeter and module should be 10 all are specified and pressure angle is 20 degree. Now, here first question one may ask that if we the 3 by 2 ratio 1.5; that means, every after 6 revaluation will be same tooth peered in come in contact. So, there will be tooth hunting and how we are allowing that. Now, answer is that this is a specific purpose gear and in such case we have to make it the exact ratio and it should be usually precision gears, we cannot take the rough cut here we should make the gear set as much as precision possible, ok.

Now, let us see what we can do. In solution first of all we will calculate the center distance and considering the center distance and module we can calculate the summation of teeth number. In this case A which is specified as a 210 millimeter and module is 10 millimeter. So, from there directly we get from the first lesson A is equal to Z 1 plus Z 2 into module divided by 2 from there we find the Z 1 plus Z 2 is equal to 42. Now, again with the combination of 42 if we would like to make Z 2 by Z 1 is equal to 3 by 2 it is

not possible, say if we take 25 and 17 it will not be 3 by 2. If we take 16 it is close to 3 by 2, but not 3 by 2.

If we take 24 divided by 8 divided by 18 again it is not 3 by 2. If we take 26 by 16 again it is not 3 by 2. So, no solution is possible we cannot take the exact number of teeth which will give ratio 3 by 2 and the summation of teeth number is 42.

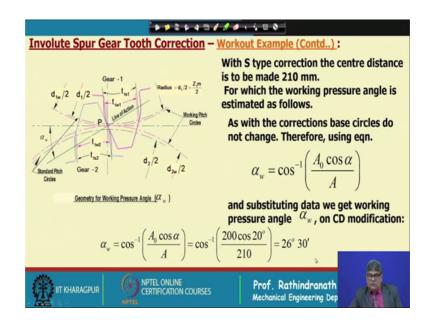
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Then the solution closed solution is possible with teeth number Z 1 is equal to 16 and Z 2 is equal to 24 for which the ratio is 3 by 2, but that summation of teeth number is coming 40. Now, with 40 teeth if we do not give any correction then center distance will become 200 millimeter because forty into 10 40 into 10 the module divided by 2 is equal to 200 millimeter.

So, what is the way? Way is that we need to provide some correction, it might be positive correction to both; we have to go for positive corrections positive correction for both so that we have into the center distance is equal to 210 millimeter. Now, one solution is that we give equal amount of corrections that is 10 millimeter increasing center distance from there we can find out that x 1 plus x into m is equal to 10 millimeter. So, x 1 easily and x 1 is equal to x 2 and we can give the same amount of corrections and we can arrived into the solutions. But looking into this strength point of view and which is almost depending on the number of teeth that pinion and gear some formula is suggested from where we can find a better amount of corrections and that procedure is followed.

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So, this is definitely S type of corrections. This center distance is being made 210 millimeter for which the working pressure angle is estimated as follows, as which the corrections base circles do not change. Therefore, using the equations the working pressure angle is equal to cos inverse the standard center distance into cos of the standard pressure angle divided by the standard center distance, sorry A is equal to the working centered distance; in this case A is equal to 210 and A 0 or A o is equal to 200. And substituting the data we get working pressure angle on CD modification. And the calculation which shown below for which we get the working pressure angle will be 26 degree 30 minute, ok.

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Involute Spur Gear Tooth Correction :				
Modifi	Modified parametric values with 'S' correction & centre distance modification:			
Some Useful Formulae in summery form :				
Table-1				
	Standard centre distance	$A_{o} = m \frac{Z_{1} + Z_{2}}{2}$		
	Actual centre distance (after pushing)	$A_{o} = m \frac{Z_{1} + Z_{2}}{2}$ $A = A_{o} \frac{\cos \alpha}{\cos \alpha_{w}} = m \frac{Z_{1} + Z_{2}}{2} \frac{\cos \alpha}{\cos \alpha_{w}}$		
	Working pressure angle	$inv \alpha_w = 2 \frac{x_1 + x_2}{z_1 + z_2} tan\alpha + inv \alpha$		
	Sum of profile correction factors	$\mathbf{x}_1 + \mathbf{x}_2 = (\mathbf{Z}_1 + \mathbf{Z}_2) \frac{\operatorname{inv} \alpha_{w} - \operatorname{inv} \alpha}{2 \tan \alpha}$		
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Next here we suggest that we can again I can mention that we can now calculate the amount of corrections. Say for example, we have taken 16 24, 16 teeth if as it is a high speed we can directly we can take we will corrections such that it will come to close to 18 teeth that we can do. So, plus correction we can calculate from there and rest of the correction we can assign to the 24 teeth gears, but which this help of this formula, some suggestion is there we can calculate what will be the amount of corrections, correction distribution of amount of corrections there.

Now, this formula are first of all this standard center distance that we can calculate like this, this is the basic formula already we have done it and the new center distance is equal to standard center distance into cos of standard pressure angle divided by cos of working pressure angle and which is given by this formula and then working pressure angle which can be written in this form. This can be equated as well as we can that can be written in this form.

Look at this is 20 into the amount of the correction factor of the gear 1 plus correction factor of gear 2 divided by teeth number of the gear 1 plus teeth number of gear 2 into 10 of standard pressure angle plus involute of the standard pressure angle. Some of profile correction therefore, can be written in this form, ok. So, this is one set of formula.

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Involute Spur Gear Tooth Correction : <u>Modified parametric values with 'S' correction & centre distance modification</u> :			
Some Useful Formulae in summery form : Table-2 : Dimensions for S- gearing			
Description	Pinion	Gear	
Number of teeth	Z1	Z2	
Pitch circle diameter	$d_1 = Z_1 m$	$d_2 = Z_2 m$	
circle diameter (with topping)	$d_{a1} = 2(A + m - x_2m) - d_2$	$d_{a2} = 2(A + m - x_1m) - d_1$	
Root circle diameter	$d_{f1} = d_1 - 2(1.25 - x_1)m$	d _{f2} = d ₂ -2(1.25- x ₂)m	
10oth thickness on pitch circle	$\mathbf{t_1} = (\pi n/2) + 2 \mathbf{x_1} \mathbf{m} \tan \alpha$	$\mathbf{t_2} = (\pi m/2) + 2 \mathbf{x_2} \mathbf{m} \tan \alpha$	
Topping	$ym = A_0 + (x_{1+}x_{2})m - A_{2}$		
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Another set of formula is that the if the pinion teeth number is Z 1 and Z 2, the diameter can standard diameter we can calculate and the addendum circle diameter sorry this has it is not this is addendum circle diameter or you can say simply tip circle diameter with topping can be calculated as d a 1 is the tip circle diameter of gear 1 is equal to 2 into modified center distance of A plus module minus correction amount of correction or correction factor of gear 2 into module whole minus the pitch circle standard pitch circle diameter of gear 2.

Similarly, the tip circle diameter of the gear can be calculated which is shown in the second column. Now, root circle diameter also can be calculated knowing the factors and the tooth thickness on pitch circle, here this is also tooth thickness of on pitch circle that also calculated by t 1 is equal to standard tooth thickness plus 2 into x 1 into m of into sorry module into 10 of pressure angle. Similarly t 2 can be calculated also the same way.

Now, the topping value which is y into m y is the factor width factor for the topping that can be given by standard center distance plus x 1 plus x 2 into m minus A.

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	Involute Spur Gear Tooth Correction – Workout Example (Contd):			
1	Using involutometry: $inv\alpha_w = \frac{2}{(Z_1 + Z_2)} \tan \alpha (x_1 + x_2) + inv\alpha$			
	Substituting the values :			
	$inv\alpha_w = \tan \alpha_w - \alpha_w = \tan(26^\circ 30') - \frac{26^\circ 30'}{180^\circ} \pi = 0.4985816081 - 0.4625122518$			
	$\therefore inv\alpha_w = 0.03606935627$			
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Now, we shall use involutometry and involute alpha w can be given by twice by total number of teeth that is summation of the teeth number of pinion and gear into 10 of pressure angle multiplied by summation of the correction factors plus involute of standard pressure angle that formula already we have derived earlier. And in this case substituting the values we get we have already calculated the working pressure angle. So, 10 of that minus the alpha w. Keep in mind this alpha w in radian that we should keep in mind we should not make a mistake. So, this can be calculated by this into pi, ok.

So, we get this value and involute alpha w in this case 0.03606935627 we should take the value up to at least 5 decimal correctly. So, this calculation for this calculation we should follow the accuracy. If it is in computer no problem usually 8 decimal places will be taken there. In case of calculator we should be careful to consider up to 1 2 3 4 5; that means, in this case better to take up to this value. In case of maybe there we should take up to this value so that it comes close to this value ok. (Refer Slide Time: 16:09)

Involute Spur Gear Tooth Correction – Workout Example (Contd):		
Using involuto	metry: $inv\alpha_w = -$	$\frac{2}{(Z_1+Z_2)}\tan\alpha(x_1+x_2)+inv\alpha$
Substituting the va	alues :	
$inv\alpha_w = \tan \alpha_w - \alpha_w = \tan(26^\circ 30') - \frac{26^\circ 30'}{180^\circ}\pi = 0.4985816081 - 0.4625122518$ $\therefore inv\alpha_w = 0.03606935627$		
Similarly,	$inv\alpha = 0.01490438387$	
[Note: In calculating involute of any angle at least five decimal places of all components in equations are to be considered a		
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So, we have calculated involute alpha; similarly, involute of the standard pressure angle in this case 20 degree that becomes 0.01490433, but usually this is taken up to this value. So, one can remember this value for 20 degree pressure angle involute alpha will be 0.01490. So, in calculating that is already I have discussed that we would take the 5 decimal places.

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Involut	Involute Spur Gear Tooth Correction – Workout Example (Contd):		
	Substituting the values in $inv\alpha_w = \frac{2}{(Z_1 + Z_2)}$		
	we get: $(x_1 + x_2) = 1.163$		
	For such a higher value and as the centre dist correction both the gear and pinion must be g		
	Also, considering equal root stress we use eq and we put x_1 in pinion and x_2 in gear res		
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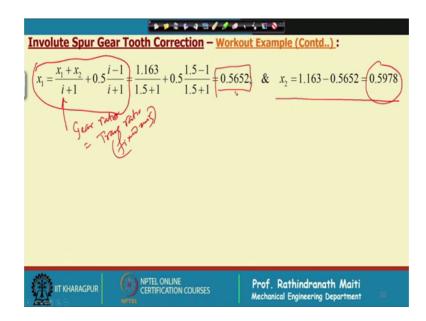
Now, substituting the value in involute alpha w, we can for formula for involute alpha w we can find 2 by Z 1 plus Z 2 into 10 alpha into x 1 plus x 2 plus involute alpha. We use

this formula and we get the x 1 plus x 2 is equal to 1.163, 1.163; for such a higher value and as the center distance is being increased in tooth correction both the gear and pinion must be given S plus correction.

Say, suppose if this value could be less then x 1 could be more and even x 2 could be given minus value, but here as x 1 plus x 2 is 1.163 and from the peaking point limit of value perhaps for these 16 teeth we cannot go for more than 0.4 for sure. It is better to give plus plus correction to both.

Now, also considering equal root stress we use equation for distribution of corrections and we put x 1 in pinion in x 2 in gear respectively. Now, we follow the formula what we have suggested that is more or less keeping in mind that what will be the increase in root so that the tooth strength become more or less equal. But here I would like to mention we do not know yet what is the material. So, from material point of view consideration will be different. But usually what the materials may gear materials is usually taken as slightly weaker, but not very weak. So, possibly that formula can be safely used and we will arrived in to the tooth thickness has that they will have more or less equal strength.

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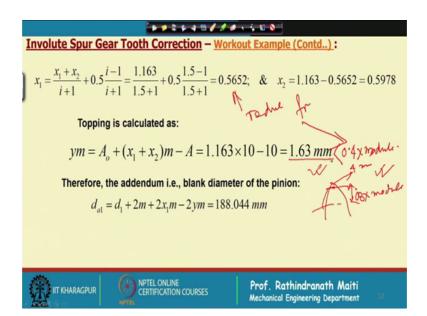
Now, we have in that formula we have written this formula of course, it is not shown in the table, but we need to follow this formula for here that we can use for the distribution $x \ 1$ is equal to $x \ 1$ plus $x \ 2$ divided by i plus 1 plus 0.5, i minus 1 was i by i plus 1. And

this i is nothing, but the gear ratio and in case of is equal to transmission ratio in case of fixed axis.

So, using this formula and we have calculated already x 1 plus x 2 is equal to 1.163 and this is transmission ratio is 1.5. So, we get the value x 1 is equal to 0.5652. Now, for 20 degree pressure angle and with 16 teeth, 16 teeth probably we can go for 0.5652 because for 14 what we found it is around 0.45, perhaps it is possible, but we need to accept this value of x 1 after calculating the tooth teeth thickness what it is coming; however, with x 1 value like this x 2 becomes this much. So, x 1 is this much x 2 is this much; this is more or less same for both gear and pinion, but this is recommended for the pinion.

We can go least for this we can go slightly higher for this, but first of all we will take what would be the tooth thickness at the tip.

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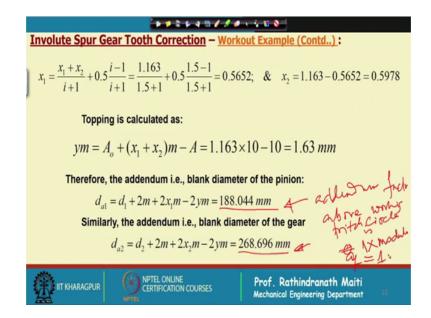


Then topping is calculated as ym the y is the width factor at the top m is the module A 0 is the standard center distance plus x 1 plus x 2 to m minus A. So, what we find? This is 1.163 into 10 minus 10 is equal to 1.63 millimeter because this A 0 minus A is equal to 10 millimeters. So, thick this is this is correct. So, therefore, the addendum that is blank diameter of the pinion that is tip circle diameter it can be considered d 1 plus twice module into twice x 1 into m minus twice y m is equal to 188.044. And let us check this one say this has become, but this is less than 0.4 into module. We wanted 4 millimeter there, but 1.63 in this case it has been accepted.

And we calculate. So, for that to keep this much definitely we would reduce this one this to be reduce for; so there is no harm, one can go for such calculation. So, that we can maintain this one and we can reduce this one, we can increase this one. But increasing the this correction factor in gear it will happen that the gear will become much more stronger than the pinion. We can go for much weaker material of course, so that decision we have to take, but one thing should be verified after this calculation that what will be that contact ratio? Will it improve you should reduce this we increase this one or if we keep this one whether the contract issue is satisfactory?

In that case we can truncate this one say instead of making it like this we can make the teeth up to this one say in that case this height after above the working pitch circle maybe 0.8 into module that is also acceptable say close to stub teeth and for that possibly it will come like this and we can verify the center distance. However, at instant we can accept this one ok.

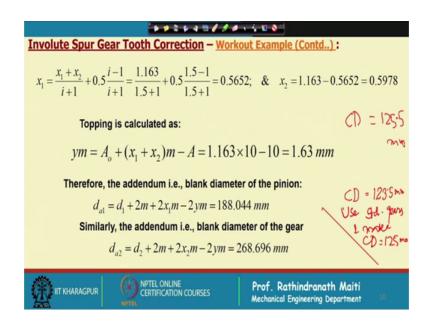
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And similarly the addendum blank diameter of the gear we can calculate as this one and both of these is having addendum factor above working pitch circle is 0 sorry 1 into module that is this means that a f is equal to 1. So, this will be acceptable.

Now, this is the procedural standard procedure to calculate the what should be the amount of corrections. Here we have taken the problem in such a way that due to maintain the contact ratio sorry gear ratio and the center distance we had to provide the corrections. But in many cases we need not go for correcting the center distance, we need not go for correcting the profile teeth, correcting the teeth or profile shift, but still it is done for several reason as I told to make the center distance square to provide a backlash or other factors are there.

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See for example, in a case suppose if it is like that CD has become center distance has become 123.5 millimeter. In that case if you use then suggestion can be given huge standard gears and make CD is equal to 125 millimeter; this is this can be allowed because due to this there will be backlash. If this backlash amount is not very high one can easily calculate what will be the backlash for that knowing also geometry and then it can be recommended. So, this gear is 125 millimeter center distance which such and such gears standard gears and the pressure angle sorry the backlash is this much; this is also allowed.

Suppose this CD becomes 125.5 millimeter, in that case there are two possibilities that the minus correction is given to either of that or both of that so that center distance can be made 125 plus the correction is such that there will be certain amount of backlash or make it 130 millimeter with some corrections; positive corrections mainly to the pinion and maybe a little bit to the gears also. So, that there with the center distance in which working pressure angle also will increase, but as well as there will be some backlash. Backlash calculation is usually why we are considering the backlash that is introduced

with such calculations. So, that these factors can be further modified for a backlash but that amount is very less, ok.

So, in this lecture what we have learned how to calculate the amount of corrections knowing the center distance, new center distance. This can be also in the modified form that if we give some corrections without considering the center distance, at the beginning we can arrived into a new center distance from there we can calculate the other parameters, ok.

Thank you.