Gear and Gear Unit Design: Theory and Practice

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Lecture - 35
Involute Spur Gear Tooth Correction Part – III

We are continuing with introduction to involute gear tooth correction in module 7.

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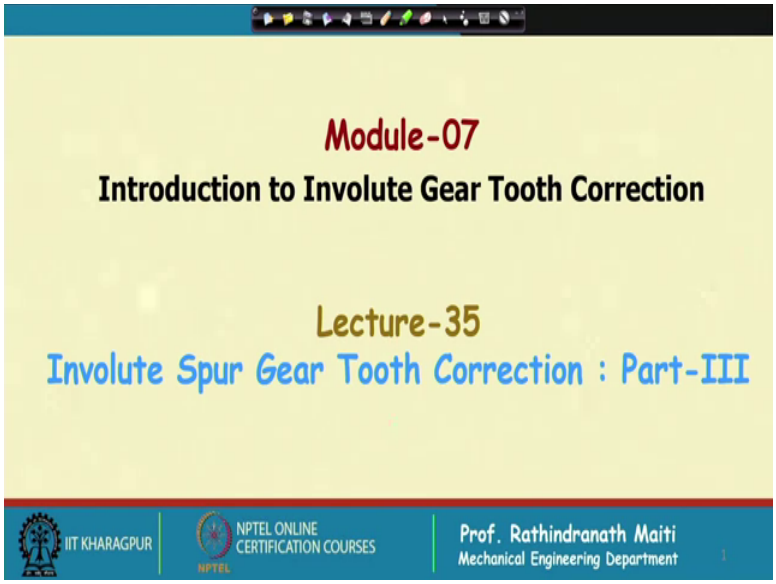

And this lecture is on involute spur gear tooth correction part 3.

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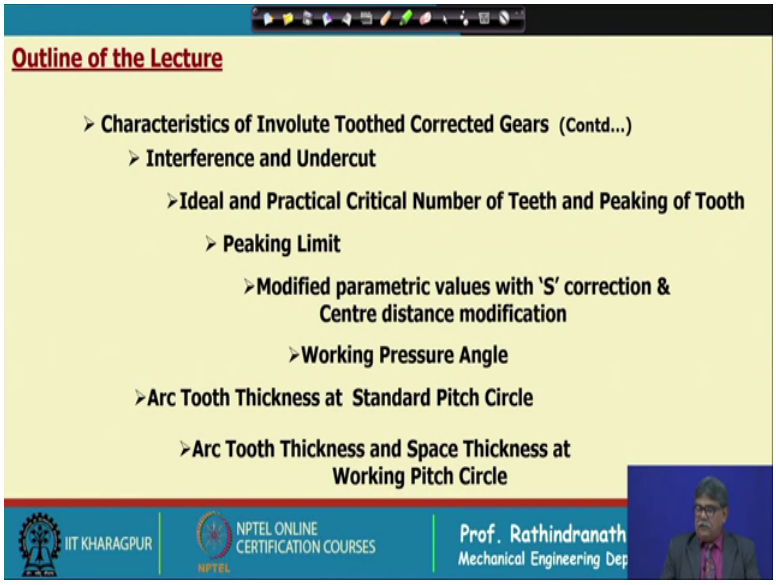

We have already learned a little bit about the characteristics of involute tooth corrected gears, and we are continuing with that. In this lecture, first I shall discuss about the interference and undercut, next ideal and practical critical number of teeth and picking of tooth, then peaking limit, then modified parametric values with 'S' correction; that means, center the distance correction or modification, and we will find out the working pressure angle, then we shall calculate arc tooth thickness at standard pitch circle and finally, we shall calculate arc tooth thickness and space thickness at working pitch circle and other dimension.

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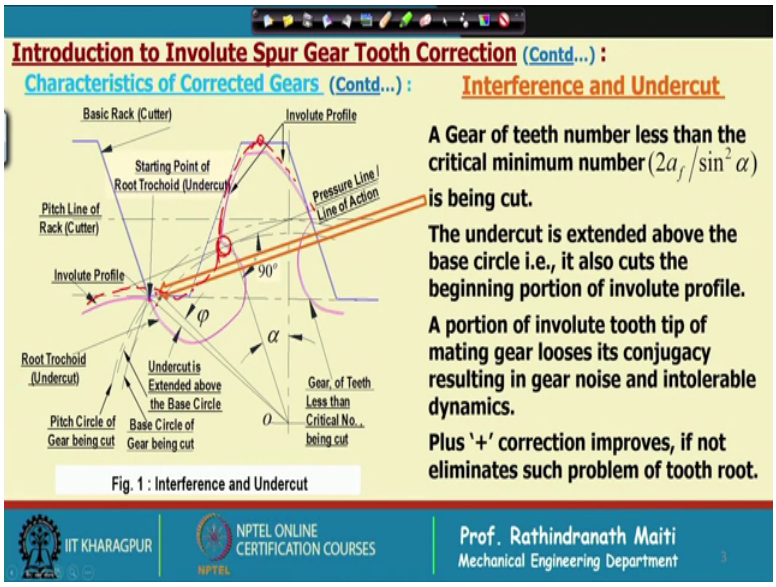

Now, let us consider a gear of teeth number less than the critical minimum number which is 2 into addendum factor divided by sin square of pressure angle. Now we this is being cut; so what will happen? There will be undercut. It is called interference this interference means cutter is interfering at the root of the tooth it is cutting some portion of the involute and it is like that the cutter while it is generating this trochoid; trochoid means this root, then it is cutting some extra portion or the trochoid is being continued above the base circle and it is cutting some portion of involute tooth. In fact, if the tooth number should be more than the critical number, these perhaps the tooth could be perhaps something like this; that means, involute portion could be continued up to the base circle.

Now, if we look into this figure more carefully; we will find that the intersection point of the trochoid and the involute is here it is a little bit about below the pitch circle it not necessarily it will be below the pitch circle it might go above the pitch circle tooth number is much less in the in this case, it is shown that it is above the base circle this is the base circle, but it is below the pitch circle at there is an angle is formed, which can be calculated on the process of if we the gear cutting is calculated. However, as this portion is non involute, it is not involute we have cut this portion. So, it is not involved.

So, definitely there will be if it is allowed to mate with another gear. So, there will be violation of gearing law and extra dynamics will be there which is definitely cannot bear out. Secondly, if we look into the route, which is taking the bending strength, that is become much thinner and this will fail. So, by no means this gear is allowed for power transmission, but in many power transmission we need this number less than the critical number this is to optimize the size of the gears.

So, what is to be done the gear correction is given in this case it is profile shift we shall go for the plus positive correction plus corrections of the gears and with plus correction we expect that this profile will becomes this profile will become something like this and we will get the use useful involute profile above the base circle. Now this is shown that it is being a portion of involute tooth teeth mating gear losses it is losses it is conjugacy resulting in gear noise and intolerable dynamics which I have already discussed.

Plus correction improves, if not eliminate such problem of tooth root; this man this means that definitely with some plus corrections we will get involute tooth, but with that

there is another problem may arise what it is; say suppose with this improvement of teeth we have made the teeth like this see this is the correction, but with this correction we have to increase the addendum as well otherwise that we will not get the enough tooth height and enough contact. So, at one point we will find this has become pointed which is called peaking.

So, keeping that peaking limit it may not be possible that we give the full correction; that means, full envelop involute profile, but this portion will be improved this point will come down very close to the base circle. So, that is the target we shall keep in mind.

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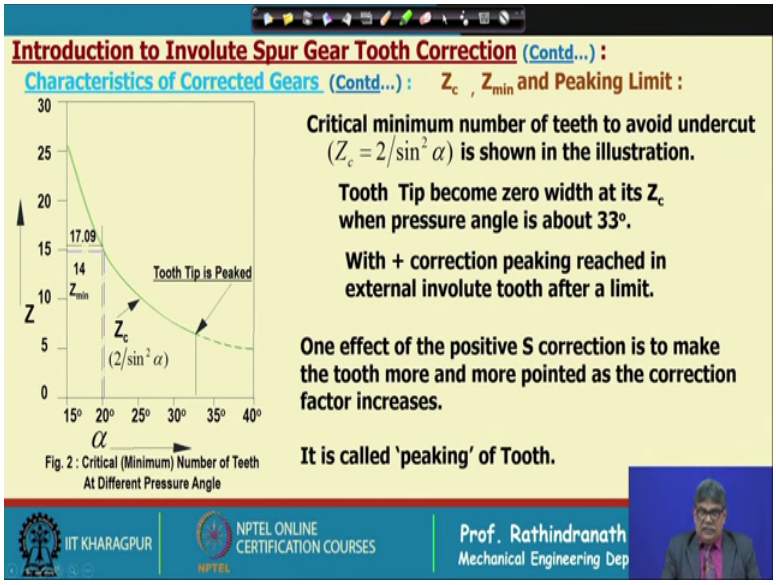

If we plot the critical minimum number of teeth to avoid undercut, then the plot will look something like this what is given in the in this figure 2. Now in this figure we have drawn a line which is actually showing the critical number of teeth cal calculated by (Refer Time: 08:43) twice in to addendum factor into sin square of pressure angle, but in this case or we can we should say that as the standard in 20 degree involute is normally one module addendum is one module. So, we have taken 2 by sin square alpha and we have plotted that curve. In that from this curve or from the calculation also what we find that for 20 degree the actual number of teeth is 17.09 here, if this point is here 17.09.

So, we can easily take 17 teeth for all practical purposes, but earlier we have seen if it is very high speed gears in that case we should go for uncorrected gears we should go at least for 18, but as well what we have seen that; if we if the speed is low then in that case

we can go up to 13 without any corrections; that means, this curve will be somewhere here and we are getting another curve through this point or another point at least the 14 which have shown this means that what will happen will not be there any undercut yes there will be a little bit undercut, but that is not recognizable in open eyes and neither it will affect the dynamics of the gears, because this contact usually will begin a little bit above in that case then the ideal point of contact, but in the line of action it will not be affected much; so there will be not much increase in dynamics also it can be allowed.

However, it is always better if you can go for corrections, but the correction means we have to not only calculate the amount of corrections, but all also the machines should be such that we can adjust such amount of shift of the profiling tooth tip becomes 0 width at it is Zc when the pressure angle is about 33 degree. Now it is shown in this curve that; if we gradually increase the pressure angle, then with the increase in pressure angle what will happen; the tooth will become something like this say if it is a tooth for 20 degree involute, then for 30 degree, it will become something like this.

And in case of 35 degree slightly more you will find that this will be pointed whereas, the addendum height is slightly less than that actual addendum; that means, one the factor will become less than one addendum factor. In fact, at 33 degree this will become a tipped.

So, we should consider about this peaking point which is called peaking and about these limitations, because why are we giving the positive correction to the gear having teeth number less than the critical number; we cannot go for any amount of correction, because at one point the peaking we will start; do we have to increase the addendum as we are shifting the cutter in the in the last direction; that means, we are not allowing the cutter to cut the depth as it is in the standard.

So, we need to increase the addendum also, but we have to consider that peaking limit now with plus correction peaking reached in external involute tooth after the limit one effect of the positive ‘S’ correction; that means, center decision modification is to make the tooth more and more pointed as the correction factor increases it is called peaking which I have already described.

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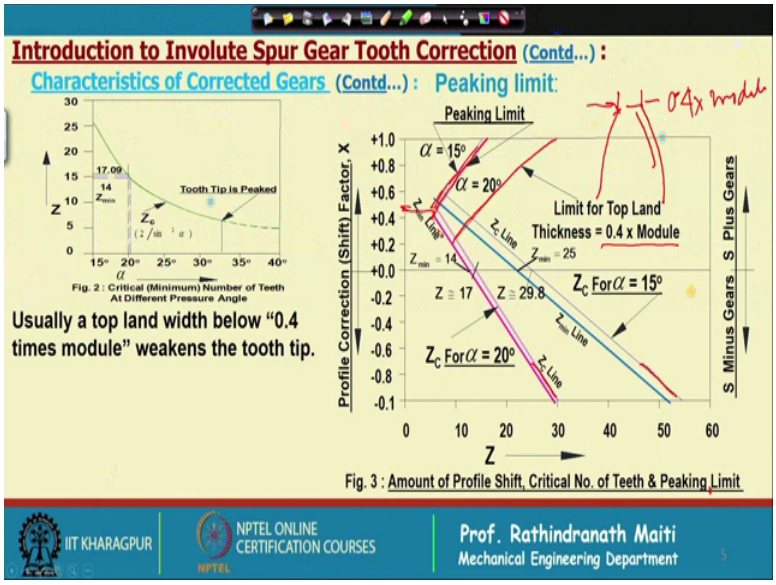

Now, let us look into the peaking and their limits. Now, in this in the graph right side what we have done; first of all we have considered a line this is for 15 degree, this line is for 15 degree pressure angle and this line is not the critical number of teeth what we have considered; no; this is we have this line we have considered for critical number of teeth; that means Zc; this is in this case 2 by sin square alpha; that means, pressure angle and this line is here.

And as well we have also considered the 20 degree pressure angle and then we have got this line; the right hand line of this purple colors, what we find we have also plotted a curve here, this is the limiting line at which the peaking is being appeared that mean peaking will be there at this point what does it mean actually say for example, to integrate pressure angle and we have considered the Zc line in that case the peaking will be at about 0.45 plus correction at 0.45 plus correction there will be the peaking with this line ok.

Now, if we consider the another line parallel to this which is for Z mean; that means, what Z means this critical number what we can go for practical purposes in this case what we have found that it is at no correction this is the number of teeth 14. So, in that if you plot that line again we will find this correction amount will be slightly less for that. So, we can go up to 0.41 or 42 some things like this as we see from this graph. Similarly, for 15 degree you will find this line this is the peaking limit; and you can calculate we can consider that in case of 15 degree possibly we can go for 0.5 a slightly more correction among.

And as we go on increasing the pressure angle that that amount of allowed positive correction will be less and less from the peaking point of view. Anyway; this curve if it is plotted properly then we can use this curve to find out roughly estimate; what should be the amount of corrections and taking that is a basic we can start we can calculate further that what exact amount of correction we can give.

Now, here for 20 degree; I have shown another line, which is with the top will be; that means, teeth sorry this one is 0.4 or into module. And usually this is desired; normally in the one problem what we have calculated; this is out 0.5 of the module is the tooth tip with a standard tooth and if we go on correction it will positive correction it will be reduced; however, we should keep it preferably we should keep it 0.4 into module.

So, we can we will have another limiting line below these lines and, which we can give the correction up to that amount. So, that is for to select the; what should be the allowable limit or minimum corrections we should give what should be the maximum correction and what we can give.

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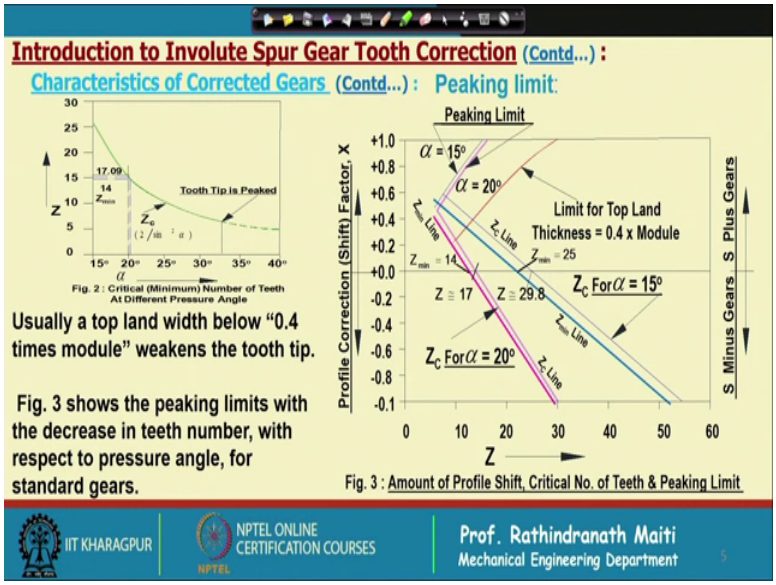

Now, in this figure what we have already shown and we have discussed.

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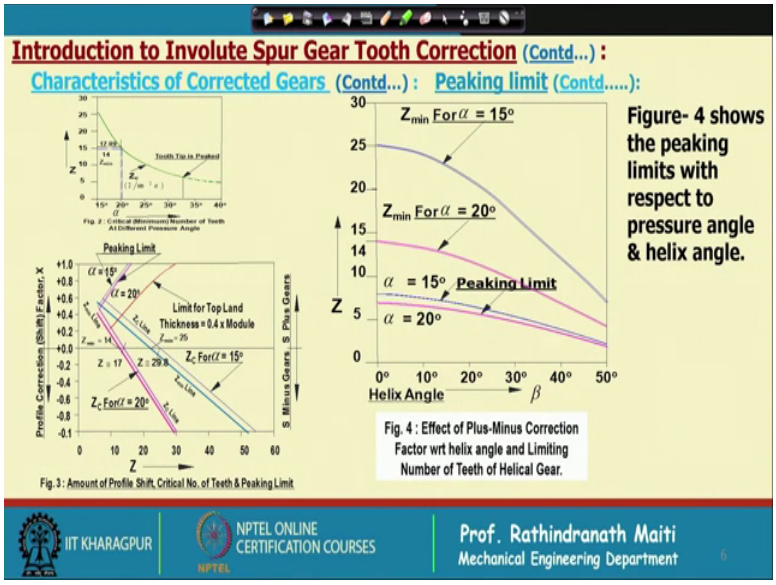

Now, from these 3 graphs; if we keep together these graphs then these two graphs and then another graph we have also plotted for which we have considered the helix angle; that means, we are going for helical gear. In that helical gear when peaking limits those are plotted against different pressure angle 15 and 20 two different pressure angles and also we have plotted graph for minimum number of teeth; that means, this is a practical limit what we have considered.

So, practical limit in case of 15 degree it is 25 teeth and in case of 20 degree we have considered 14 whereas, actual number in case of 20 degree it is 17, and actual number of teeth in case of 15 degree it is 29.8 anyway; if these two graphs we use and along with that peaking limit we can use, then from there we can find out what should be the amount of correction for against the teeth number four different helix angular ok.

So, roughly using these three graph; we can we can consider both straight tooth spur gear and the helical gear and we can find out what will be the amount of correction that we can afford for a particular number of teeth. Now we have to match our design with that amount of corrections, and then what we wish whether this is given to have the more strength of the pinion as well as to avoid the undercut.

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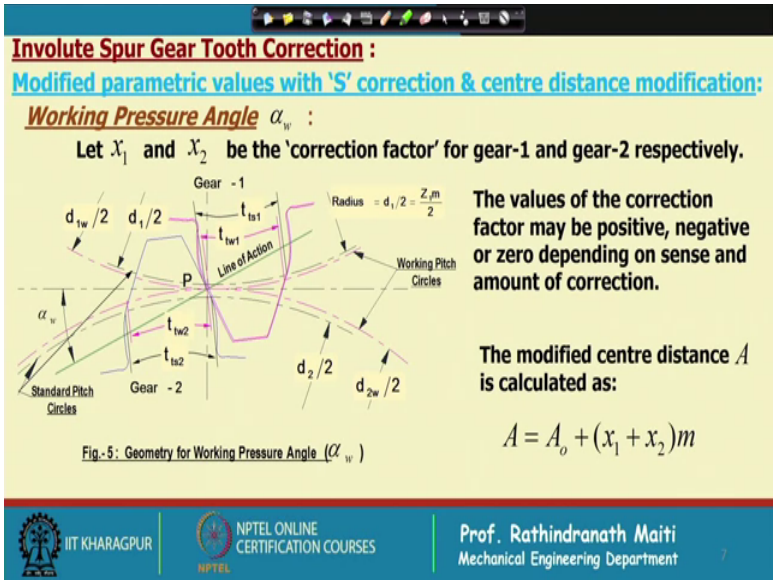

Now, we will we need to consider the modified parametric values with 'S' correction and center distance modifications. Now as already discussed the 'S' correction means there will be change in center distance, and 'S' zero correction means there will be no center distance corrections; that means, center distance will be kept same. In that case this is not a difficult task to find out the correction for other gears say for example, we have taken a 10 teeth spur gear straight tooth spur gear and say 31 teeth gear pinion is of 10 teeth.

Now, from the procedure; what I already discussed we can calculate what will be the amount of correction for that 10 teeth pinion; say it comes 0.25 positive. And, in that case if you would like to go for a 'S' 0 correction; that means, no center distance change we will simply give negative correction to the gear same amount; that means, 0.25 negative corrections minus corrections. In that case, definitely in case of 20 degree involute 31 teeth will not go for undercut the route will be acceptable only the route will become thinner, and in case of the 10 tooth pinion the root will increase.

Now, if it is found they are strength wise it is not the weak any of that we weaker than the much weaker than the other in that case we can accept that surplus corrections, but very often we need to go for 'S' correction; that means, there will be change in center distance. The change in center distance means from the same gear 'S' 0 is sorry 'S' factor for the first one, that is that the pinion is plus 0.25 and the second one we can keep as it is only center distance will increase by some amount or we can give for minus corrections a little bit, but not to the extent. So, that the center distance remain same or else, we can go for plus collection of gear also center distance will increase by a little

more amount. Now, that is from another point of view that we are making the center distance of square value.

So, all this to be calculated tent we should come into a decision that, what we would like to do? Now, considering that is correction; that means, center distance is being increased; what is what will be the shape of the gear? In this case as it is shown, that the top gear is gear one it might be pinion and bottom one is gear two; and this was originally the gear two was having this is the pitch circle this was the pitch circle and for gear one standard pitch circle was this one ok. Now, what we have done; we have given plus correction to both pinion and gear and center distance is increased by some amount and we have got a new pitch circle pitch circles, which is purple in color for working. Now as a result what has happened definitely this line of action is more inclined and this angle is greater than the standard pressure angle.

So, we need to calculate, what the working pressure angle with that value is; we should need to calculate the center distance and other parameters. Now here one clue again I will tell you that once we select a standard cutter and tip number then one thing will not change in the gear that is the base circle this base circle will remain same that base circle diameter will always be Z into module into cos of this standard pressure angle; this is the straight spur gear.

First of all, we will calculate the working pressure angle alpha w let x 1 and x 2 be the correction factor for gear 1 and gear 2 respectively; x 1 we have selected let us consider x 1 we have selected from the point of view how much correction it is needed as the teeth number is less than the critical teeth number, but x 2 we have finalized depending on, what will be the center distance and what amount of correction will give to the other gear; so that the center distance become a square value it is multiple of 5 or 10; as well as the tooth strength will be some it will be it will improve of pinion and although it will reduce for the gear, but it will be at a level that both of more or less equal strength let us consider like that and from there we would like to select that what we will x 1 a sorry x 2.

Now, the values of the correction factor may be positive negative or 0 depending on the sense and amount of correction sense means plus minus and the amount of correction that is for, how much we would like to give from center distance square center distance,

ecetera, etcetera. The modified center distance A is calculated as A is equal to A 0 or A o subscript o original plus x 1 plus x 2 into m; directly the amount of profiles shift can be added to the standard center distance that is A o to get the new center distance in case of straight tooth spur gear.

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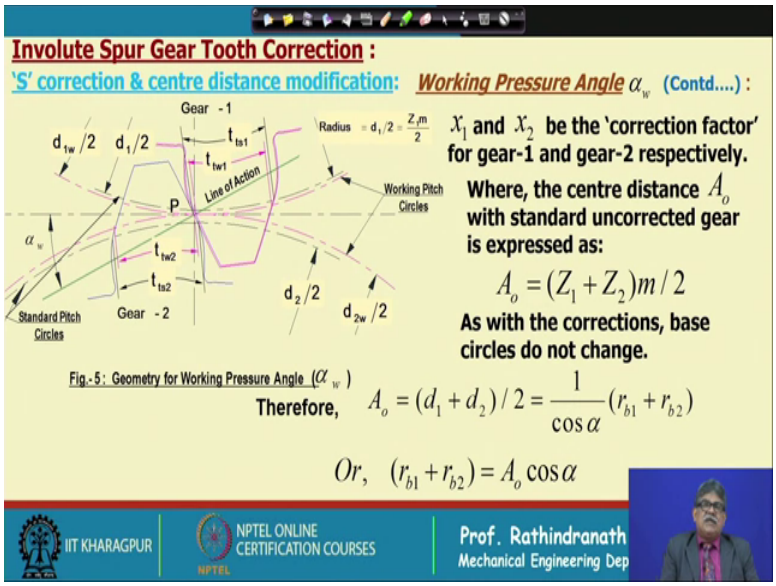

X 1 x 2 the correction factor where the center distance A o with standard uncollected gear is expressed as A o or Z 0 is equal to Z 1 plus Z 2 into m module the total tooth number give pinion plus gears divided by into module divided by 2. As with the correction base circle do not change then A o d 1 plus d 2 by 2 is equal to 1 by cos alpha in into rb 1 plus rb 2 whereas, rb 1 is the base circle radius of gear 1 and rb 2 is the base circle radius of gear 2 and that then we find the summation of this base circle is equal to base circle radius is equal to A 0 into cos alpha.

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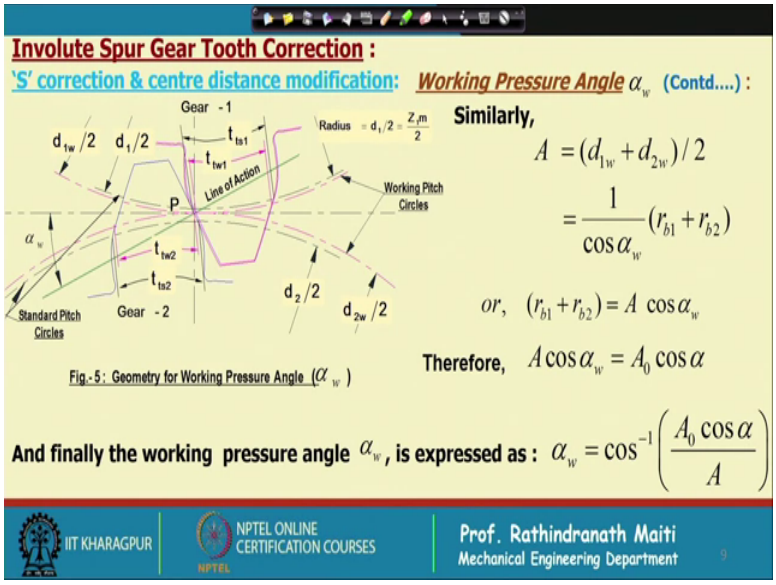

Again if we think in terms of now new working pressure angle, then this must be equal to A into cos alpha w alpha w is the working pressure angle.

Now, if we equate A cos w cos alpha w must be equal to A o cos alpha, because both are the summation of two base circles radiates. And finally, we can find out the working pressure angle alpha w will be cos inverse A o into cos alpha divided by A; that means, if we know the new center distance and standard center distance and standard pressure angle we can easily calculate, what will be the working pressure angle for the corrected gears?

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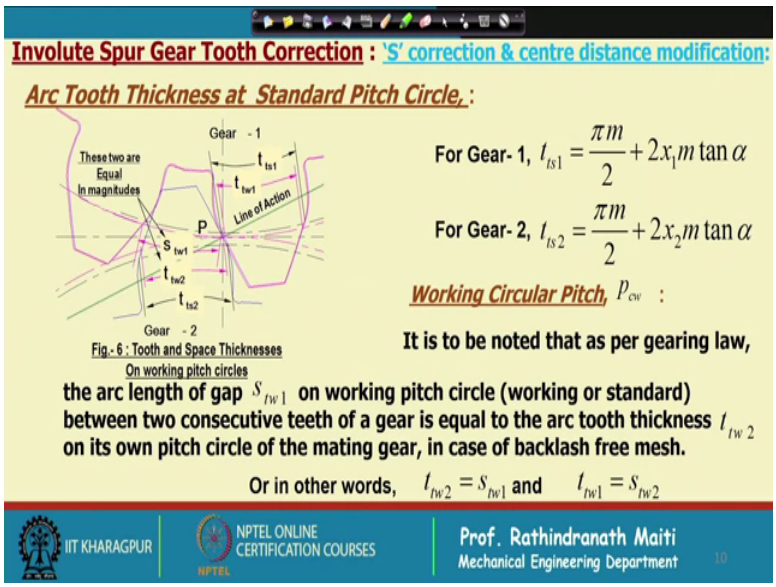

Once we find out this angle then we can find out what will be the tooth thickness no tooth thickness of that amount of from the amount of correction itself we can find out the two thickness at standard pitch circle. So, t subscript t s 1 means tooth thickness standard of gear 1 is equal to standard thickness; that means, pi m by 2 standard circular piece divided by 2 plus 2 into x 1, that is the correction factor into module into 10 of alpha that is 10 of standard pressure angle that already we have shown that how this is coming.

Now, here x 1 itself may be plus or maybe minus; that means, if we give minus correction automatically in place of x 1 we will put that minus value; so tooth thickness will reduce in case of plus correction it will increase. So, this one can be calculated similarly for gear 2 also we can calculate the tooth thickness; and then working circular pitch; that means, circular pitch at working pitch circle can be calculated this is summation of this two.

Now another interesting part is that from the gearing law the arc length of gap is t 1; that means, this what is the corrected gear for that the gap on the working pitch circle will be equal to the tooth thickness on the working pitch circle of the; sorry, this one of this one sorry not this one this one this no this is the space thickness of the corrected gear on the working pitch circle and this is the tooth thickness tooth thickness of the of the other gear and they must be equal to they will be same they will be same these two will be same.

So, we should give in this mind this means that after the correction of the gear if we consider they are working pitch circle at this that pitch circle the gap thickness of one gear will be the tooth thickness of the other gear. So, from that point of view we can write t tooth thickness working of gear 2 is space thickness of working one and vice versa tooth thickness of working gear 1 is equal to space thickness of the working 2 on the new pitch circle.

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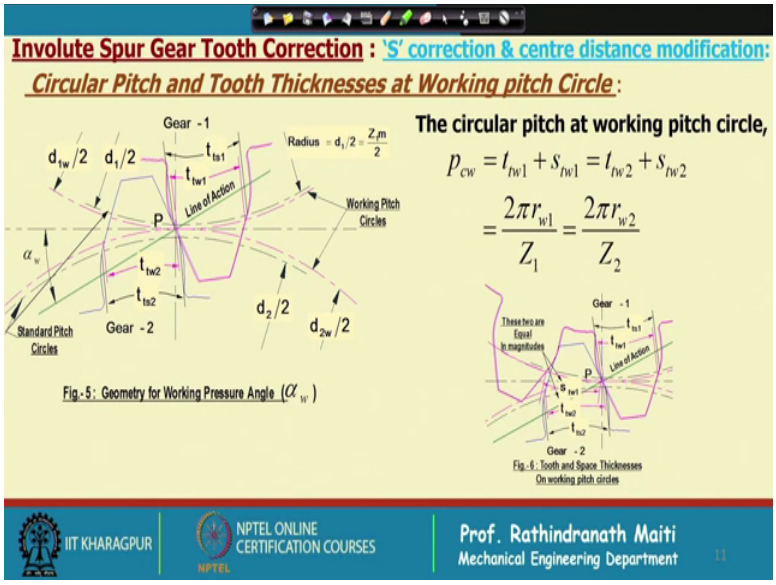

And then this circular pitch circle at working pitch circle can be given from this equation that is P subscripts c w; that is circular pitch at working it must be equal to tooth thickness that working pitch circle of one plus base thickness of working of one that must be equal to again tooth thickness on working pass pitch circle of gear 2 plus space thickness on the working pitch circle or gear 2; and that must be equal to twice into working pitch circle radius divided by Z 1 is equal to twice pi into working pitch circle of the gear 2 divided by the teeth number of gear 2.

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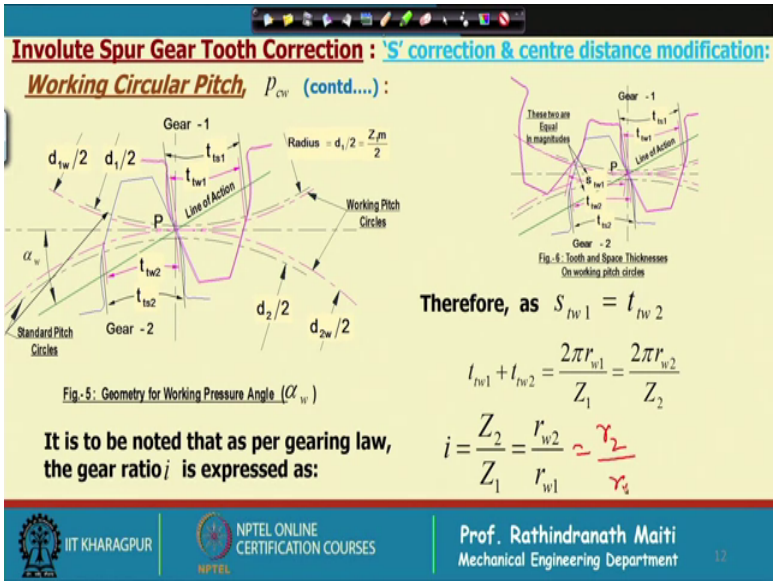

Now as S tw 1 must be equal to t tw 2, then t t w 1 plus t t w 2 is equal to twice pi r by wn 1 by Z 1 and is equal to twice pi r w 2 by Z 2, this means that once we find the

working pressure angle and the new centre distance from there we know what will be the radii of the working radii of the two circles and we can calculate the tooth thickness of one and from there we can calculate the two thickness of the other one from these relations and from that two thickness we can calculate how much correction it requires ok, because we know the ratio I the gear ratio in this case or transmission ratio for fixed axis gearing must be equal to Z 2 by Z 1 is equal to r w 2 by r w 1 which is standard case also r 2 by r 1 this ratio will always remain same.

So; that means, whatever may be the amount of corrections this will become r 1. So, thank you for listening. So, in this section what we have learn; knowing the number of teeth if it is smaller than the critical teeth number the pinion usually pinion smaller one, then we first of all we would we can estimate what would be the amount of positive correction we need to give to avoid undercut or interference to the cutting interference and then we can also select what will be the amount of corrections of the making gear, because we would like to also give correct that one by changing the center distance.

That means, we have not only corrected the pinion we have also selected the center distance cannot be altered due to some reason may be to make it square or something else. And from there we can calculate knowing the center distance, what should be the amount of correction of other gear; and finally, we introduce those corrections to these pinion and gears and from there; we can calculate what will be the other parameters.