Gear and Gear Unit Design: Theory and Practice

Prof. Rathindranath Maiti
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 03
Laws of Gearing, Kinematics and Geometry Part – II

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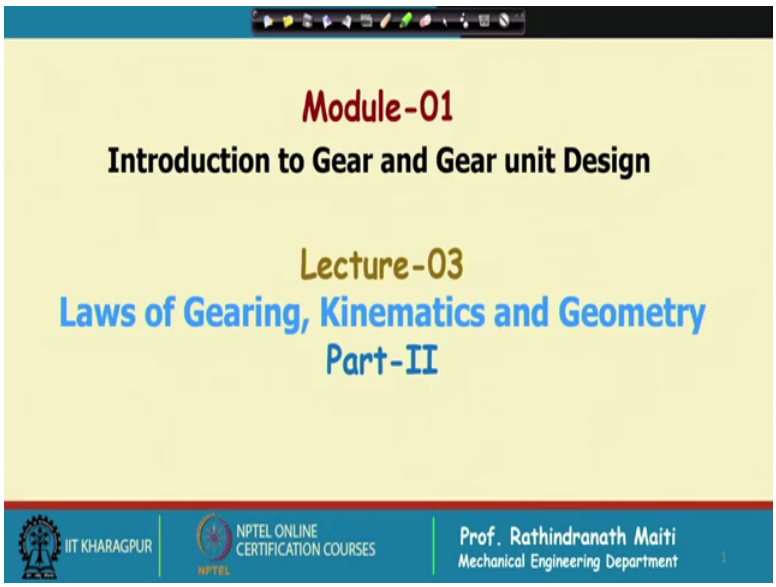

Introduction to gear and gear unit design and this lecture is on laws of gearing kinetics and geometry part 2, I have already discussed some bearing laws kinematics and geometry in part one and in this lecture.

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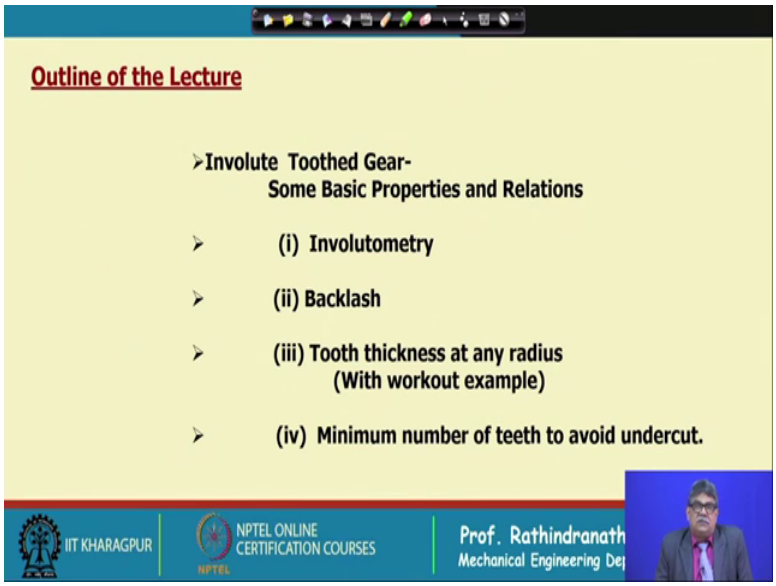

We shall continue with involute tooth gear; some basic properties and relations involutometry, it is an very important property of involute gear tooth, it is required to calculate different proportions of gear tooth corrections, etcetera. So, this will be discussed a backlash. What is backlash and how it is given? That will be also discussed and tooth thickness at any radius, how it can be calculated also a an example will be discussed and finally, minimum number of teeth to avoid undercut that is under discussion.

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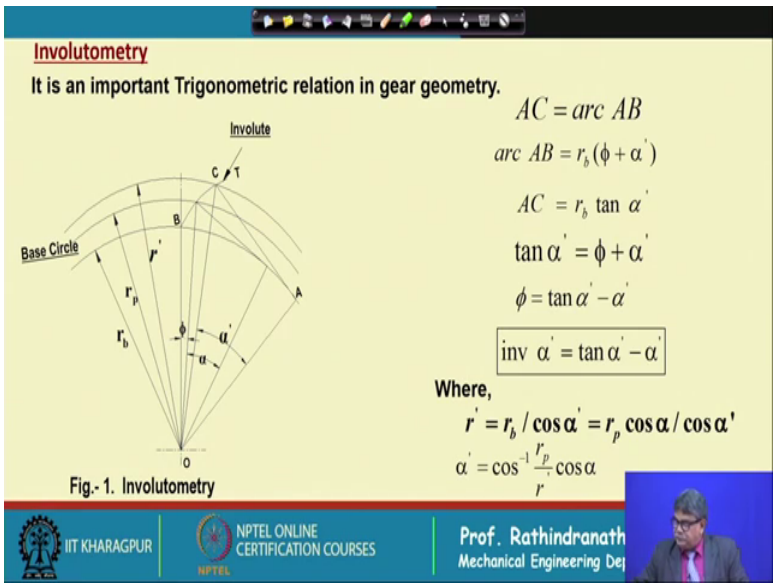

Now, let us consider the involute tooth profile, if you take a circle this is the base circle on the base circle, if we consider a chord like this and if you take a point here on the circle and on the chord and then gradually if we open this chord, unwind this chord on the drum, this point will move here; here location of these points will give us, then involute teeth in this case; in this figure, it is shown that an involute profile B to T has been generated and then when it has this in on the profile, it has reached C, we can consider if we consider the length A, C A is the point of tangency of this chord.

When the C point is generated or the point generating point has reached C. So, we consider that length is C that must be equal to this arc length A B arc length A B. So, we can write A C is equal to arc A B. Now arc A B can be written as the base circle radius r b into this small angle plus alpha dash. So sorry, this is this can be written as we will make this correction again here. So, AC is equal to r b, A C can be written again the r b into tan alpha this r b into tan alpha dash.

Therefore, we can write tan alpha dash is equal to phi plus phi dash, again, I repeat that phi is this angle. So, from this relation, you can write phi is equal to tan phi dash minus alpha, sorry, tan alpha dash minus phi dash alpha dash and this angle is now expressed as involute alpha dash which is nothing, but tan alpha dash minus alpha dash where r dash means this radius is equal to r b by cos of this angle cos alpha dash divided by 2.

So, module into teeth number divided by 2 into cos alpha divided by cos alpha dash; that means, if on the involute profile, if we know the standard pressure angle, then a and teeth number from there at any radius, we will be able to calculate this angle alpha dash which is given by this relation alpha dash is equal to cos inverse r p by r dash into cos alpha.

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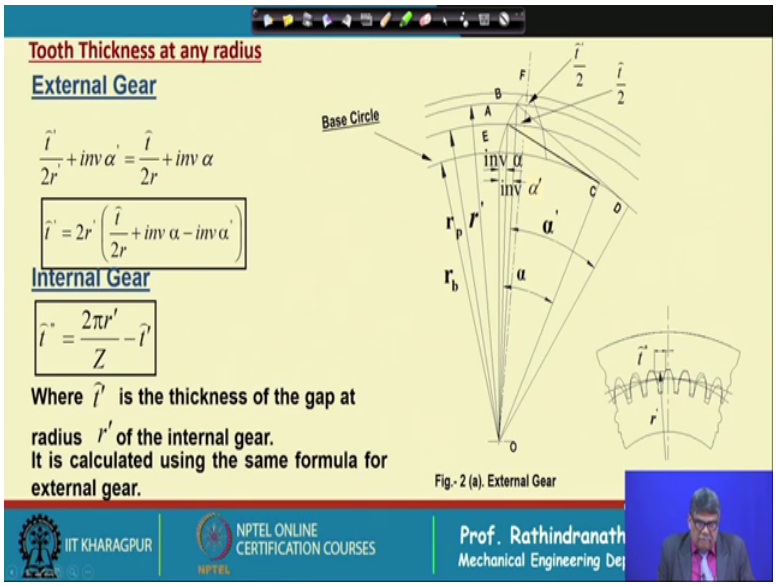

Next, if we consider this geometry, we have considered a full teeth, we have considered a full teeth and we have shown here that this is equal to involute alpha dash; that means, or this angle total this plus this is phi or phi angle from this line to this line ok. So, that is nothing, but involute alpha dash.

Similarly, for standard pressure angle, this will be involute alpha and here we consider this distance; that means, from this point to this point is arc t by 2 that is half to thickness at pitch circle and from this point to this point is D dash by 2 arc D dash by 2; that is circular half of the circular is thickness at that radius. Now, if we consider the angle, then t by 2 arc, t by 2 divided by the radius of this radius will give us this angle half to thickness angle at pitch circle.

Similarly, arc t dash by 2 divided by r dash, we will give the angle of this angle the small angle at a radius r dash. Now, what we do? We add the angle respective angle involute alpha dash with that. So, this means that this is giving the angle for the point of generation to the mid of the teeth at two different radius in terms of the half to thickness there and the involute of the corresponding angle.

So, we can write at this angle will be same, we can write this relation arc t dash by twice r dash plus involute alpha dash must be equal to t dash by, sorry, t arc t by twice r plus involute alpha this means that simply, we are considering this angle for in terms of teeth thickness are two different radius, therefore, we can write at radius r dash the arc tooth

thickness is equal to twice of the radius r dash hole into the 2 thickness at arc tooth thickness at pitch circle divided by twice into pitch circle radius plus involute of the standard pressure angle minus involute of the pressure angle at the new point through which the radius r dash is drawn or in other words, if arc to thickness at which circle is known standard pressure angle is known radius is known then at any radius r dash, we will be able to calculate the 2 thickness ok.

So, this is the 2 thickness of standard gear tooth at any radius, now for internal gear. So, this is the internal gear here it is shown in the internal gear teeth is sorry teeth is like this these are the teeth; that means, you can we can consider. So, this is the teeth or in other words, what is the gap there that is the external tooth; whatever the gap is there that is the external tooth.

So, at any radius, if we would like to find out the 2 thickness, what we need to do? We will consider that is given by D double dot t prime double dot r thickness is nothing, but twice pi r dash into z that is the total pitch circular pitch at that radius minus the tooth thickness of external gear teeth; that means, in this case also we can calculate this one at any radius and then from this relation, we can find out, what will be the tooth thickness at this radius.

So, it is not difficult to find out tooth thickness at any radius r just for internal gear also the same method we can follow. So, here it is given the nomenclature.

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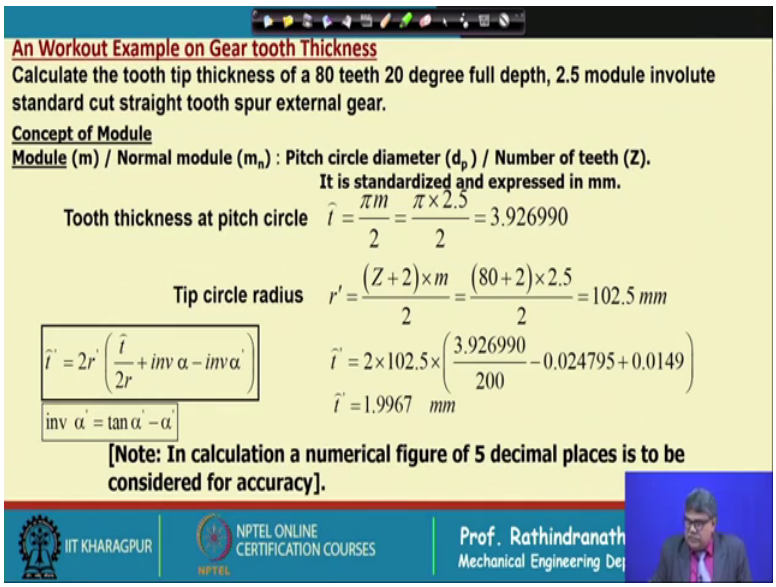

Now, what you consider? We have considered a tooth of 18; 80 number of teeth 20 degree full depth 2.5 module involute standard cut state tooth spur gear external gear full depth means the addendum is one and addendum is 1.25.

So, we have to calculate the tooth thickness at some point since in this case, we have to find out the tip thickness at the tip. Now concept of module as a hints is given, but to make the teeth equi spaced and one should not follow to the other the we define the gears by its module and the base circle number of teeth the module or it is sometimes called normal module its pitch circle diameter divided by number of teeth, it is standardized and expressed in millimeters.

So, this means that if we calculate the module by the teeth number or teeth number by the module, we will get the pitch circle diameter standard with circle diameter and 2 thickness is nothing, but the standard circular pitch which is pi into module divided by the two because at standard circular pitch which is pi m the tooth thickness is equal to the gap thickness therefore, the 2 thickness can be written as pi m by 2 and in this case numerically, we can calculate pi into 2.5 is the module. So, it will become 3.926990. Now, in this calculations, if it is done by calculator, we should consider at least fifth the decimal place to find out the 2 thickness or any other value reason is that usually corrections amount of corrections are not very high.

So, if we would like to find out the change in teeth thickness. So, it will be differences will be very small may be in the fourth or fifth places that is why it is taken fifth places also to get the value of involute of any angle, we will find the differences in 4-3 places that is why we take we consider fifth places in numerical calculations. Now tip circle radius, we have already developed the formula in terms of r dash. So, we will consider that r dash must be equal to z plus 2 2 is here in this 2 is the twice into addendum factor addendum factor we have taken one.

So, z plus 2 into module divided by 2 which comes eighty plus 2 into 2.5 divided by 2; 102.5 millimeter, next tooth thickness has given this formula, we have written already, we have derived this formula in this formula, we the involute alpha dash can be calculated from the alpha angle, if we know the alpha and radius, we can calculate also alpha dash and from there, we if we substitute the values it comes like this.

So, this is the involute alpha dash which is for the alpha at the tip circle radius and this is standard if one can remember this value that involute 20 degree is equal to point naught 1 4 9 and fifth place, sixth place is 0 0. So, we can remember this value point naught 1 4 9 and other values from say t ash, we have already calculated and r is equal to 100. So, to r is equal to 200 and this thickness this radius is this much we have already calculated. So, we find out t dash which is equal to t dash r thickness 1.9967 millimeter it is quite interesting because this is 2 millimeter close to 2 millimeter and this means for 2.5 module straight tooth spur gear with standard addendum the tip thickness will become 2 millimeter.

So, again I please note that in calculation of numerical figure of 5 decimal places is to be considered for accuracy.

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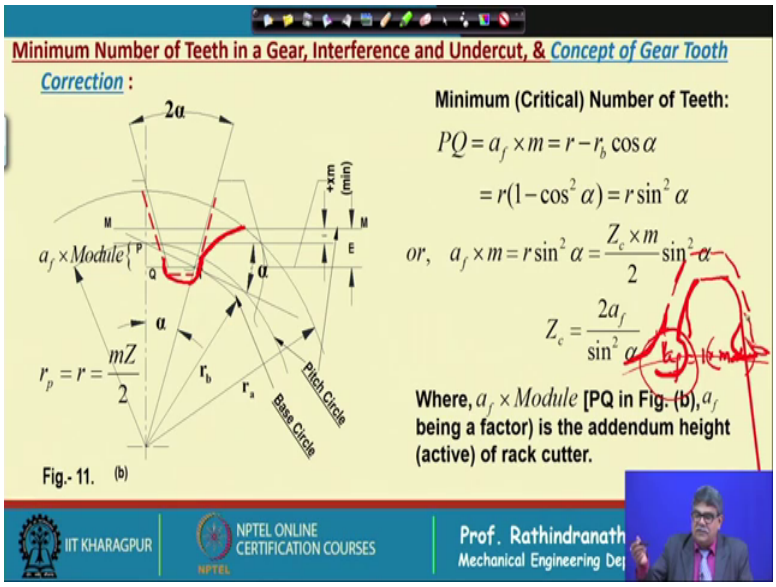

Now, I shall discuss about the minimum number of teeth in gear interference undercut concept of gear tooth correction when I shall come into the concept of gear tooth corrections gear corrections in module seven we will discuss more about that corrections, but here I would like to mention that let us consider, this is the rack basic rack, this one is the basic rack and this is the pitch line of the basic rack and this is a pinion which is being cut for which this is the base circle and this is the pitch circle ok.

So, the contact can start at this point at this point t when it is machine with the basic rack and if now we consider that the extra portion of the cutter that will go inside the base

circle or below the base circle or even this point moves here that will go below the base circle, but that will not affect the gearing motion as the contact will always remain on the involute portion.

Now, if we consider this PQ portion which is addendum factor into module this must be equal to this pitch circle radius minus this Q to this distance which is equal to nothing, but this r b into cos of alpha. So, well finally, we get r b is equal to r 1 minus cos square alpha that is r into sin square alpha now if we equate then addendum factor into module must be equal to r sin square alpha that is equal to z C a critical number of teeth for which contacts begin at t with addendum of rack is equal to 1 module and we finally, get the relations twice a f by sin square alpha is equal to minimum number of teeth.

If we take the teeth number below this then this t point will become below the base circle. So, here I have given a of the module. Now, if it becomes below the base circle what will happen. So, this means it has come over here below the base circle this point mind it this a f; a f we have taken is equal to 1 into module sorry addendum we have taken one into modules. So, addendum factor must be equal to 1 ok.

So, this means that this red line which we have considered still it is having the addendum is equal to 1, but as it has come down then this tooth profile will be something like this and if we notice it that not only here is a extra chord, but also this profile this available profile involute profile is above the base circle not from the point of base circle. So, therefore, what will happen with teeth number less than this critical number tooth form will be something like this and this portion is called undercut this is also called as the cutter is interfering with the route it is called tooth interference in case of external tooth interference means the cutter is going below the base circle for active profile generation.

So, this will be the undercut and if there is the undercut that need to be modified it is possible that we can modify the teeth and we can have the we can have heard this undercut in case of modification say, for example, if I consider this teeth itself we cannot allowing the cutter to go to the full depth or we can maintain the depth at least for which this involute will be generated from the base circle say, let us consider this is the base circle. So, this means that we can give we can withdraw the cutter a little bit and then we will get a profile something like this and for which will be something like this and this is

of course, modified teeth which is the profile shifted teeth and it will it will it can be used for the gearing purpose.

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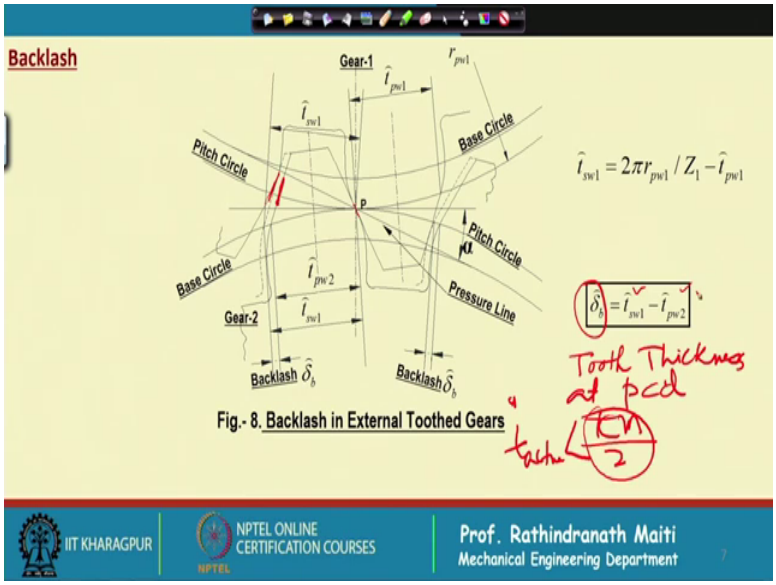

However we shall learn about more about the gear current corrections in module 7. Now, concept of backlash the teeth what is if it is cut to the exact among; that means, if the tooth thickness is kept at pitch circle by the circular pitch divided by 2 and if they are allowed to meet at the exact center distance in that case we will find that the teeth is touching here; that means, teeth is touching at both the side there is no gap.

Now, there is should not be any harm that the gear is operating at that conditions, but due to the thermal expansion which will be there due to the friction or even increase in temperature then what will happen that 2 thickness will increase a little bit and then gear will jam also if there is small amount of particles debris is inside the oil which is lubricating, then if that touches both the side the wear will be more or there is a possibility of malfunctioning is more, then if there is some gap to fit that what is there usually a gap is provided which is called backlash.

Now, this backlash is usually generated by over cutting a teeth a little bit suppose we are not changing the center distance what has been calculated. So, in that case if we just make the gear for standard gear such that this tooth thickness there at pitch circle say which is pi into m by 2 is the r 2 thickness is there and actual tooth thickness say t, we

put actual its slightly less than slightly less than this value very small amount then this backlash will be provided and it is done for all gear.

However where we are changing the center distance then these calculations come over amount of corrections and normally we need not calculations has backlash and then calculation the depth of chord usually this automatic setting is there or maybe if we cut twice or thrice on the same diameter then automatically this gap will be generated; however, this this is a different issue. So, ultimately this backlash can be calculated by this one which is difference between the gap and the tooth thickness there.

So, which can be calculated by the amount of correction is being provided for the for generating the backlash which is very small here it is the hole the nomenclature is given from which we can have the concept of backlash and we should remember, if the backlash is provided then contact is always at the load point not at the other side other flank.

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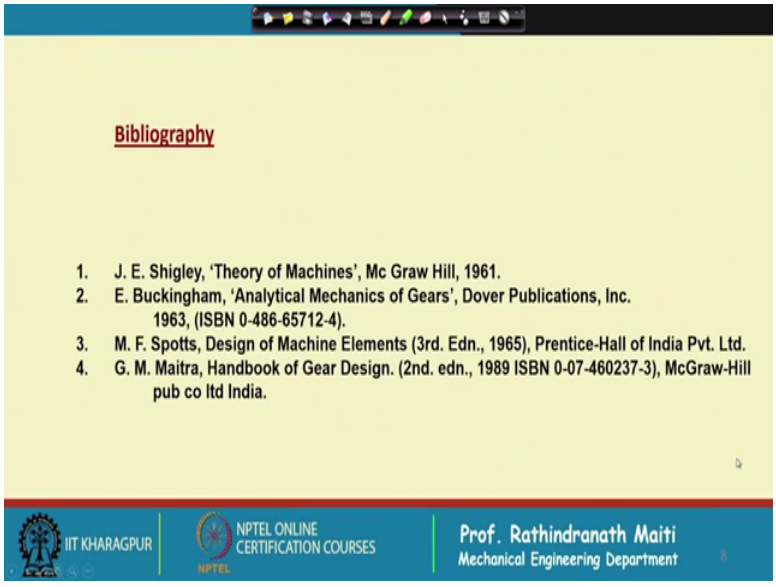

So, we can follow this the following books.

So, in this lecture, we have discussed about the basic geometric properties and laws of some gearings, we have also discussed about the backlash, we have discuss about the what is undercut, if what is the critical number of teeth if the teeth number is less than that critical number there will be undercut there will be interference and also we have

discussed a little bit about the gear tooth correction by which we can avert such an undercut, but the correction another part will be discussed in details later in module 7.

Thank you.