Gear and Gear Unit Design: Theory and Practice
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Lecture – 19
Design Verification of Gear Box Shafts

In module 04, our next lecture is on Design Verification of Gearbox Shafts.

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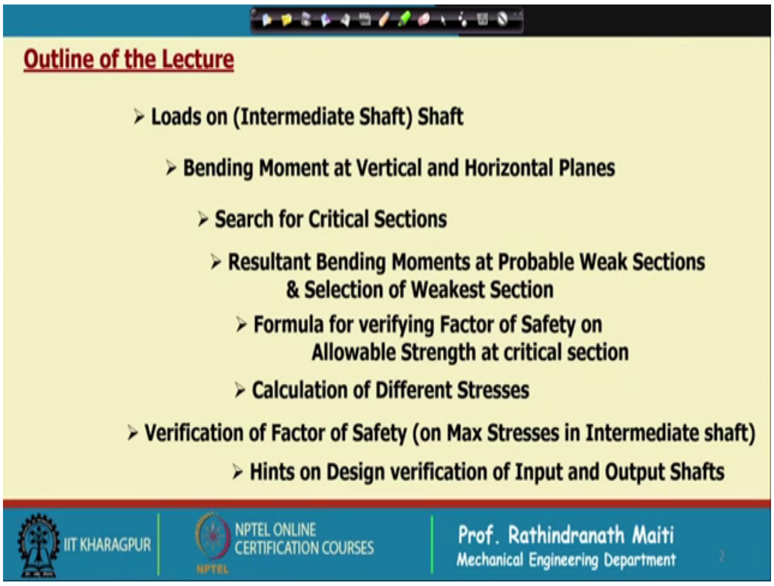

We have already calculated the bearing loads, bearing lives and bending moment and shear force on the shafts. Now in this lecture I shall cover loads on intermediate shaft we have two show again because we are going to calculate the bending moment in details at different section.

So, bending moment at vertical and horizontal plane starts for critical sections resultant bending moments had probable weak sections and selection of weakest section, formula for verifying factor of safety on allowable strength at critical section. This is we shall calculate the stress at weaker section and that we will compare with the allowable strength of the material. Calculation of different stresses for verifying this allowable strength this is where the stresses is within the limit of allowable strength we need to help calculate the stresses at different section.

Verification of factor of safety on maximum stresses in intermediate shafts and next we will also give an hints on design verification of input and output shafts.

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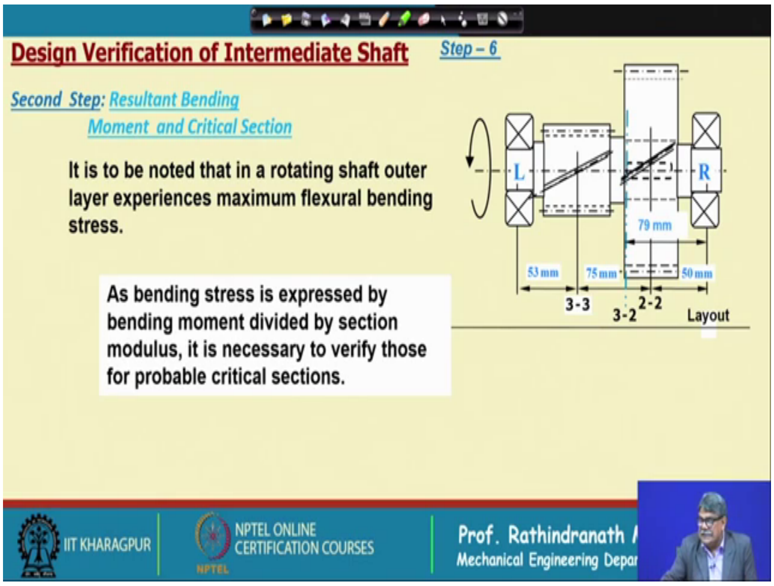

Now it is to be noted that any rotating shaft, outer layer experiences maximum flexural bending stress. As bending stress is expressed by bending moment divided by section modulus it is necessary to verify those for probable critical sections.

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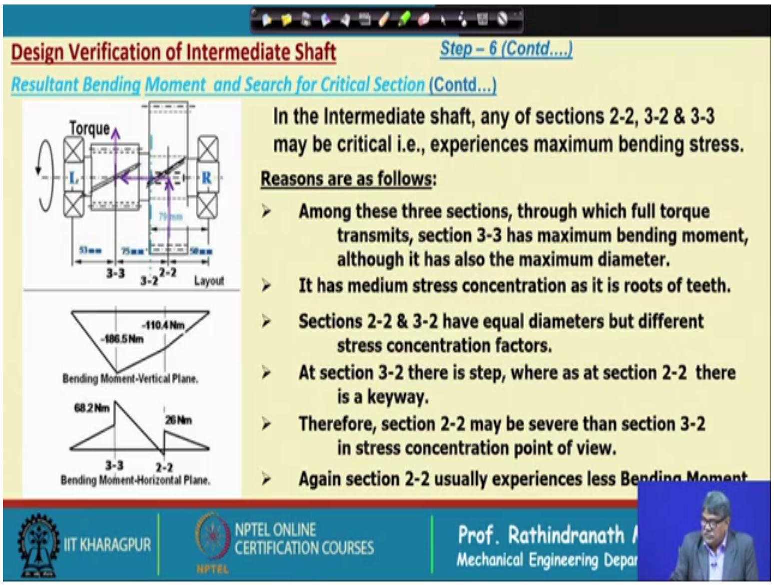

Now, in earlier in previous lecture I have already shown that how to calculate bending moments at two planes. And in this figure it is shown on the shafts in two planes what are the bending moments are acting; as well as it is also shown that how the torque is being transmitted from the input stage gear to output stage pinion. We have considered as if the torque is being transmitted in the concentrated way.

But actually, if we consider then we must say the torque is from 0 to gradually increasing here and in this directions while it is transmitting to the final stage gear; the output shaft then torque is being reduced in this way; however, if I consider the torque here that torque value remains same this torque is input torque divided by the first sorry multiplied by the first stage ratio the torque of the intermediate stage. However, this is good enough if we show this torque is being transmitted in this way.

In the intermediate shaft any of sections 2-2, 3-2 or 3-3; that means, it is shown on the diagram that of 2-2 is the section in the middle of the gear, 3-3 is a section on the middle of the pinion where the bending moments are maximum and 3 mine 3-2 section is the gears left hand end that is the step that may be also a weak section.

This is from the experience these three sections will be weak than the other section and stresses will be higher than any other sections. But among these three sections only one section is critical that we will find; now why these three sections we are considering? Among these three sections through which full torque transmits section 3-3 has maximum bending moment although it has also the maximum diameter. It has medium stress concentration as it is the root of teeth, if it is without any tooth or any other keyway and other things; it could have list stress concent concentration, but as this is with the gear teeth you will have to some extent stress concentration, but not as in case of steps or QA.

Section 2-2 and 3-2 have equal diameters, but different stress concentration factor section 3-2 there is step whereas, at section 2-2 there is a keyway. Therefore, section 2-2 may be severe than section 3-2 in stress concentration point of view. Again section 2-2 usually experiences less bending moment; in section 2-2 stress concentration is more, but bending moment is less then what is in section 3-2.

So, these two also need to be verified to find out which is less than stresses at which section will be less. And also separately 3-3 although it is having higher diameter, but bending moment as it is high we need to check the stresses there. From the experience it can be concluded that among these three sections, if we check the stresses at these three sections we need not check stresses in other sections flexural stress.

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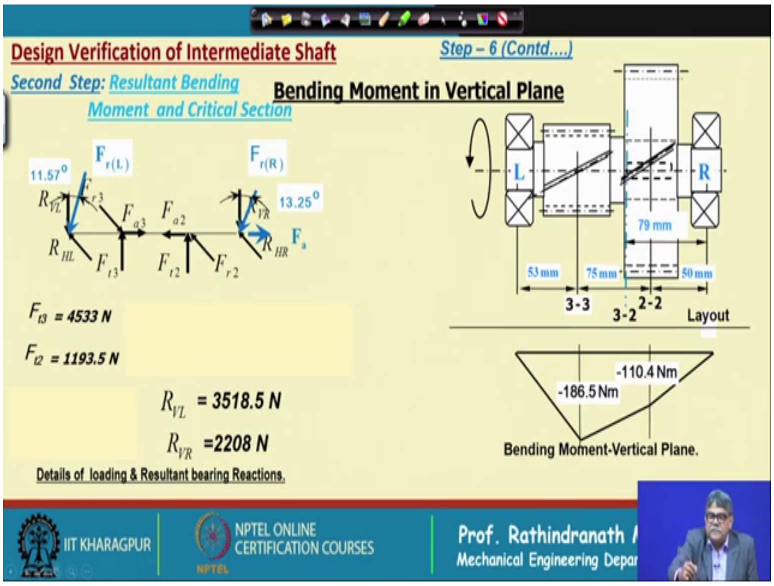

Now, bending moment in vertical plane; we have already shown, but here again I can show you that how it can be calculated? What we can do say this is on the vertical plane, this vertical plane means we are considering this load, we are considering this load these two loads and we are considering the bearing loads this end and this end R VL and R VR.

And then we can calculate from one side bending moment the R VL into a distance will give the bending moment. And if we consider the R VL and this distance which is in meter it is a 0.053 meter. So, 0.053 meter into 3318.5; this value into this distance will give this much of bending moment. And why we have considered it is negative, because this is a convention that as the shaft is bending in the upward direction. So, you have considered this has the negative bending moment.

It could be taken another way also and then after this point this moment is there and then this load into this distance. So, from this point we shall consider first this load into distance here say suppose here and then this load into this distance they in are in opposite direction. So, gradually it is being reduced and we will reach at this point with this value, if we calculate and then finally, we will find the bending moment is 0 here. So, in that way we calculate the bending moment of the in the vertical directions.

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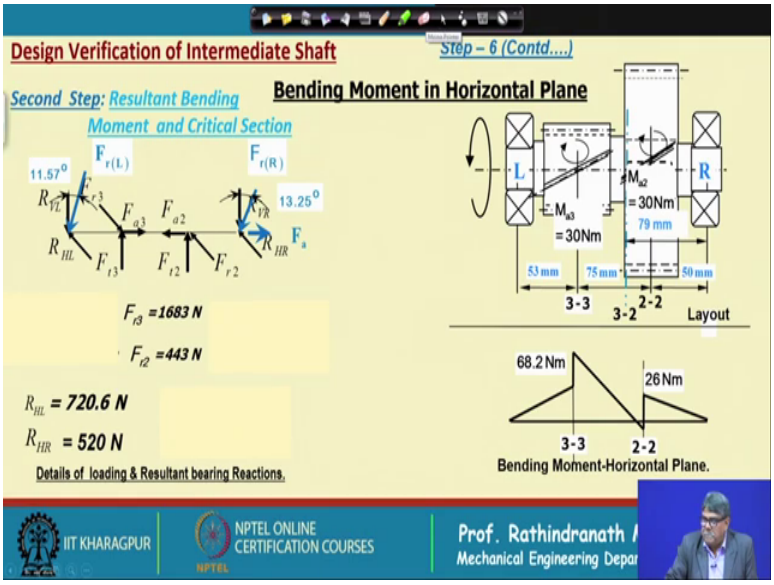

Now, we shall consider the horizontal plane also in horizontal plane what are the loads? You can find out the loads here in the horizontal directions.

But more clearly these are the loads in horizontal plane say two radial loads; one radial load is acting on the gear that is from this side to towards the axis. Whereas, the radial load on the pinion is acting on the opposite side, it is acting from the opposite side as shown in the figures left hand figure. But along with that there is another two moments which we should need to consider along with the moment diagram; this has these two moments due to the axial load at say this moment due to the axial load here and this moment due to the axial load here.

They are of equal amount as you find here interestingly these two moments has been calculated to find out the reaction loads; now these two moments need to be considered while we are making the bending moment diagram. If it is neglected from the very beginning, there is no harm because this amount is very less, but it is better to consider these two moments also. Now here also we will calculate the bending moments in the same way and this diagram will become like this say.

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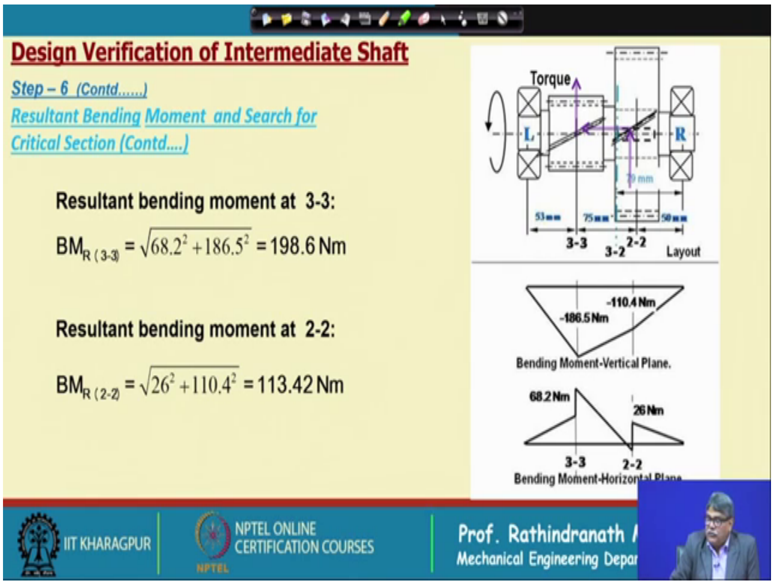

If we consider now what we are doing? Now at these three section what we have considered section 2-2 section 3-2 and section 3-3; we are calculating the resultant bending moment at these points. Now I would like to I again I would like to focus that at section 3-3 and section 2-2; there is the change in bending moment due to the additional moment there which is present at these two sections, these are simply added or subtracted suitably and as you see that bending moment in horizontal plane is somewhat complicated than the bending moment diagram in the vertical plane although magnitudes are higher there.

Anyway for any section say section 3-3 in that section that in the horizontal plane; the moment is 68.2 and the vertical plane the magnitude of moment is 186.5 Newton meter. So, resultant bending moment is 198.6 Newton meter. Now similarly we will find out bending moment at section 3-2 and 2-2, but we can if we if you study a little more in this direction; we will find this bending moment as acting at different planes as the load are different. So, that is why we calculate the bending moments and different section resolving into the components of loads.

Now, result in bending moment at section 2-2 is 111 sorry 113.42 Newton meter that is again if we consider that section this is 26 Newton meter that is 26 square plus for vertical load 110.4 that is 113.42 Newton meter.

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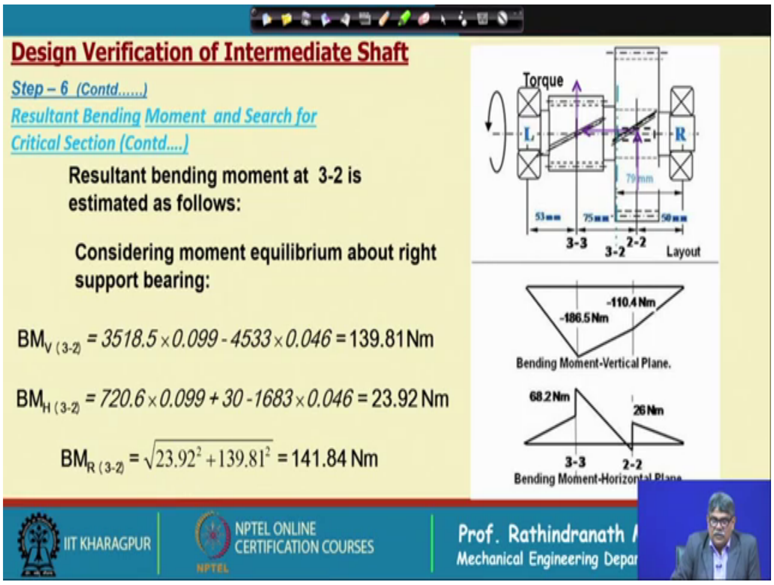

Now, for the section 3-2 first of all again from the moment calculation; so, we need to calculate because from the graph of course, we can take we can measure, but we separately we calculated calculate it for section 3-2 that is from the left hand load to this distance of this section which is 0.099 meter and minus the load acting at the pinion into the distance up to that point.

So, this gives us in vertical plane 139.81, in horizontal planes plane 23.92 Newton meter. So, we again find out the resultant bending moment at that section which is 23.92 square plus 139.81 square under the square root which gives us 141.84 Newton meter.

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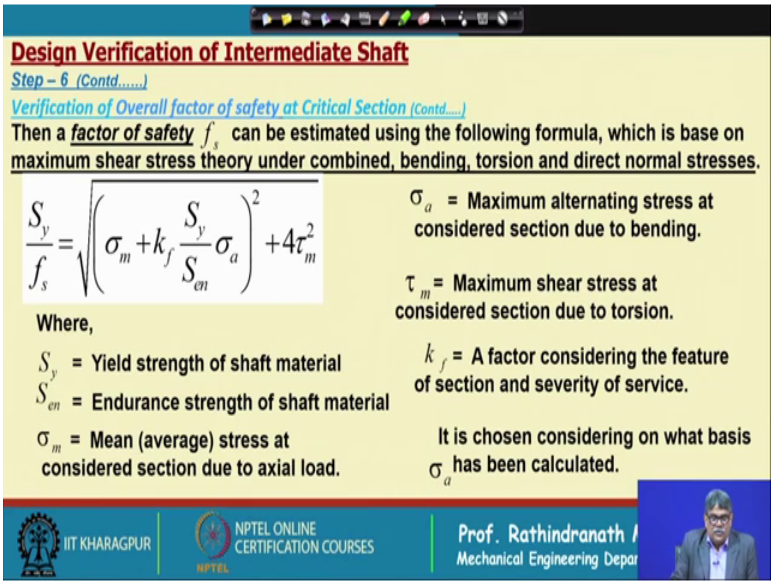

Then what we do a factor of safety can be estimated using the following formula which is based on maximum shear stress theory; under combined, bending, torsion and direct normal stresses. The formula is proposed here it is not proposed I would say this is a modified form of maximum shear stress this stress formula which is subjected to bending moment and the torsion.

Now, this is given by S y by fs is equal to square root of sigma m plus k f into S y by S en into sigma a whole square plus 4 T m square where the S y is the yield strength of shaft material S en is the endurance strength of S not n S en is the endurance strength of the material and then sigma m being average stress at consider section due to axial load sometimes this can be neglected because this is this is usually less.

However for accurate calculation we need to consider and sigma a maximum alternating stress at consider section due to bending. Then a tau a is the maximum shear stress at considered section due to torsion and kf is a factor considering the feature of section and severity of service. This means that here at the section where the pinion is there we will consider one value where there is a step we shall consider another value and where there is a keyway we shall consider another value.

These are usually determined from the experiment and these are recommended by the designers I mean sorry by the organization; who are designing of course, there are standard recommendations available in the book, but in many cases these values are recommended by the manufacturer of gear cut cutting machines. It is chosen considering on what basis sigma a has been calculated; that is this we have to sometimes while we are considering the shaft say for example, keyway where the keyway is there the idea of keyway is not considered in case of calculating area moment of inertia. Then this factor will be slightly different and if we consider that area in the calculation of the section modulus or area moment of inertia.

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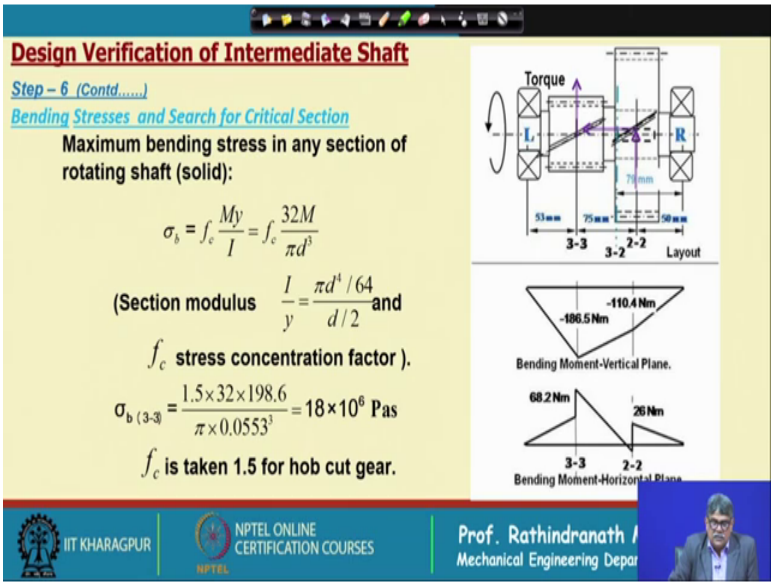

So, but these are minor differences; now maximum bending stress in any section of rotating shaft it is solid shaft the sigma b into fc my by I which becomes for solid circular shaft 32 M; M is the bending moment by pi d cube, I is the area moment of inertia and y is the distance from the neutral axis where the maximum stress is there. And in case of shafts definitely this will be at the outer layer of the shaft; that means, at the maximum radius at that section and f c is a factor of safety for calculating the bending stress at that section.

Now, section modulus I by y is pi d 4 by 64 by dy 2 that is why it is 32 has come over there and f c is the stress concentration factor which we have considered 1.5 for the root of the pinion. And the stress found there 1.5 into 32 into moment we have calculated resultant moment and the section we have calculated 198.6 and diameter there is root diameter of the pinion 0.0553. So, cube of that that gives us 18 into 10 to the power 6 Pascals.

So, 18 into 10 to the power 6 Pascals; so, bending stress at section 3-3 is 18 into 10 to the power 6 Pascals. Now 1.5 has been taken for hob cut gear now if it is ground if root could be ground this should be slightly less, but if it is more rock cut gear it would be more. So, 1.5 is taken a good finish expected from of cutting.

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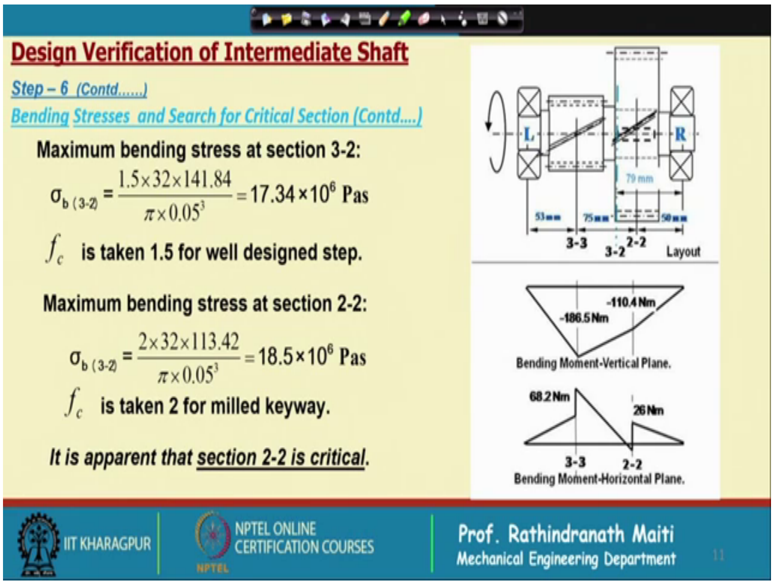

Now, consider section 3-2 in that section in the same way we will find the bending stress is 17.34 into 10 to the power 6 Pascals here also we have taken 1.5 for the steps it is recommended 1.5 can be taken and resultant bending moment 114.34 there and the diameter is 50 millimetre at that section; we need to consider the diameter lower diameter at that section. This is we can say it is just right side of that step. So, 1.5 for well designed step; well design means when we will make the detail drawing I will show you that how these steps are designed and machined.

Now in the lastly we can consider the section 2-2 where there is a keyway and for the keyway this factor of safety we need to have more. But as we as already mentioned that we have considered the area moment of inertia of that portion neglecting the area of the keyway that groove for the keyway; if you have idea about the keyway this is usually in this figure what it is shown this is the mail cart gear by the in cutter type keyway cutter. So, this is a groove on the shaft and if we consider that shaft at that section; that means, that this section the shaft looks like this. So, in calculating ; the area moment of inertia we could have considered this area also, but this is not considered as we are taking this directly 0.053 and the factor of safety to is reasonable for such considerations.

. So, now, we compare the which section will be weak; it is apparent that section 2-2 is critical because the stresses what we have calculated that is more in section 2; that means, now we need to verify that at section 2; whatever what is the factor of safety over the yield strength of the material.

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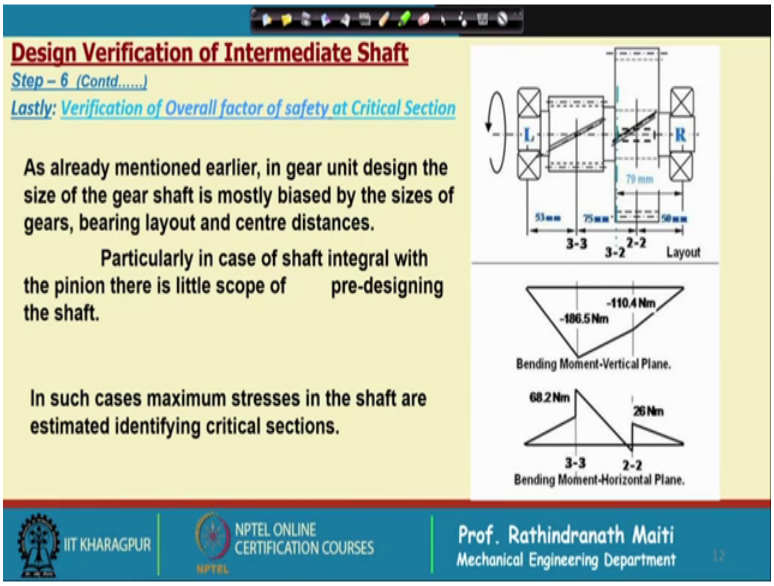

As already mentioned earlier in gear you need design the size of the gear safety is mostly based on by the size of the gears bearing layout and centre distance. Particularly in case of shaft integral with the pinion there is the there is little scope of redesigning the shaft.

In such cases, maximum stress in the shaft are estimated identifying critical sections.

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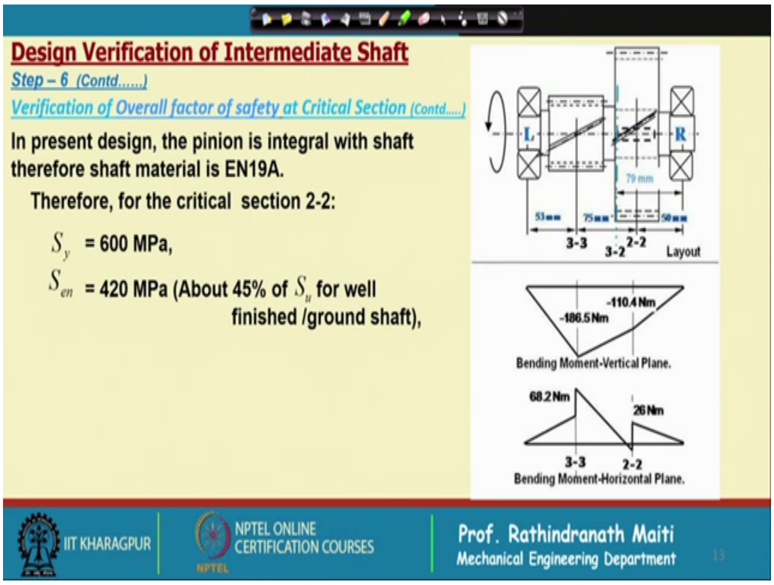

In present design, the pinion is integral with shaft therefore, the shaft material is EN19A which is the material for the pinion. Therefore, the critical section 2-2 for that critical section we will consider the yield strength is 600 mega Pascal which is given, endurance limit is 420 mega Pascal which is if not given can be taken as 45 percents of the ultimate strength for well finished and ground shaft; shaft may be ground if it is heat treated after the gear hobbing.

If it is not heat treated if it is well finished the same can be taken; however, if it is not finished well finished in that case it should be slightly less.

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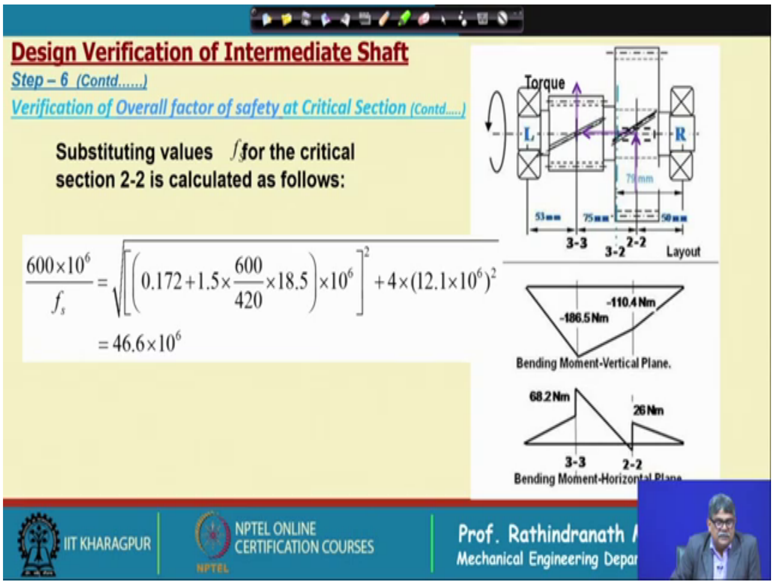

Now, the mean stress due to the axial load is simply may be taken at that section; at the section 2-2 is the axial load because axial load is acting there very much because our axial load is going to the right bearing therefore, at section 2 definitely the axial load is there.

So, what we have considered that axial load divided by the area of the shaft there again we have neglected the small cut out area of the for the key, but we have considered the factor of safety 2. So, it is compensated and in that way we find there the main stress or average stress due to the axial load is 0.172 into 10 to the power 6 Pascals. Now 2-2 because that section is found to be weakest section. So, sigma b 2-2 and bending stress there 18.5 into 10 to the power 6 Pascals and the shear stress due to the torque is given by 16 d into pi d cube this is the known formula and here also factor of safety we have considered 2 and which comes 12.1 into 10 to the power 6 Pascals.

So, here f c is taken two in general for milled single keyway ; if it is say spline in that case it would be different it would be slightly less than a single key way because there the stress constant concentration will be less. But in that case maybe for section modulus calculus the root of the spline is considered. Now substituting values f s for the critical that is factor of safety over the yield strength for section 2-2 is calculated as follows.

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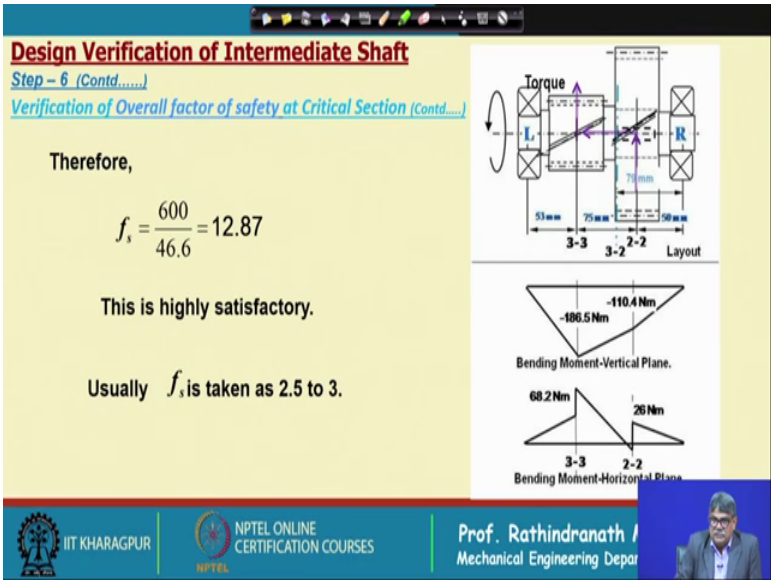

Now, we have substituted all the values we have calculated and then finally, we find f s becomes 12.87; that means, it is the what the stress has been developed there that is the 12.87 times less than the yield strength. So, definitely it is a same design and this is highly satisfactory usually this value should be could be 5.523.

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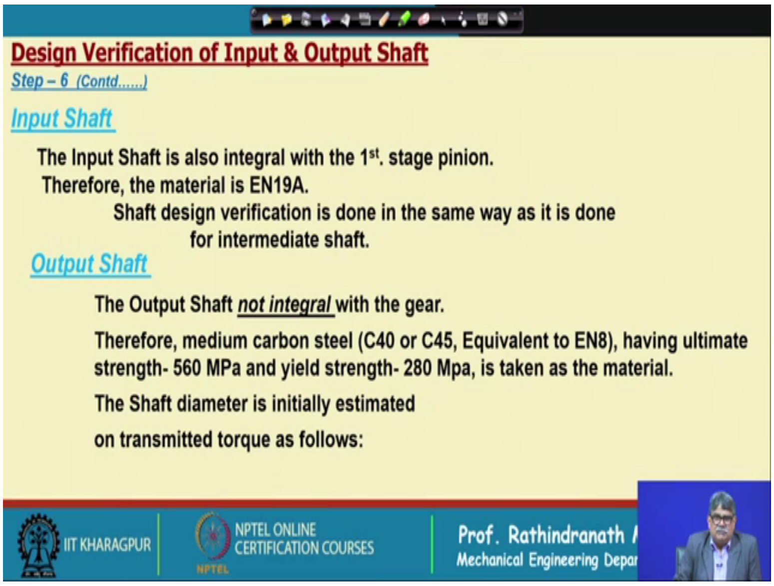

Now, if I consider the input shaft this is also designed in the same way; that means, we have to calculates calculate all loads on the bearings from their bending moment, we have to select the critical section also that is shaft is also having the pinion integral and, but there is no gear. However, there is a keyway at the input, but at the keyway I think input there is no bending moment normally; there is no bending moment bending moment will be between the two bearings only. So, probably the critical section will be there on the middle of the pinion and the same way we can find out the factor of safety there.

Now, in case of output shaft output shaft not integral with the gears. So, there is a scope also gear size is big there is a scope we can go for very high diameter of the shaft also. But we have to keep in mind within the centre distance output bearing and intermediate shaft bearing has to be said. So, from there you again we will find we cannot go for very high shaft design and it is not essential also.

So, what we do in that case we have consider EN8 which is equivalent to C40 or C46 in case of Indian standard having ultimate strength of 560 mega Pascals and yield strength of 280 mega Pascal and the trap diameter is initially estimated on the basis of transmitting torque using the formula which is shown here.

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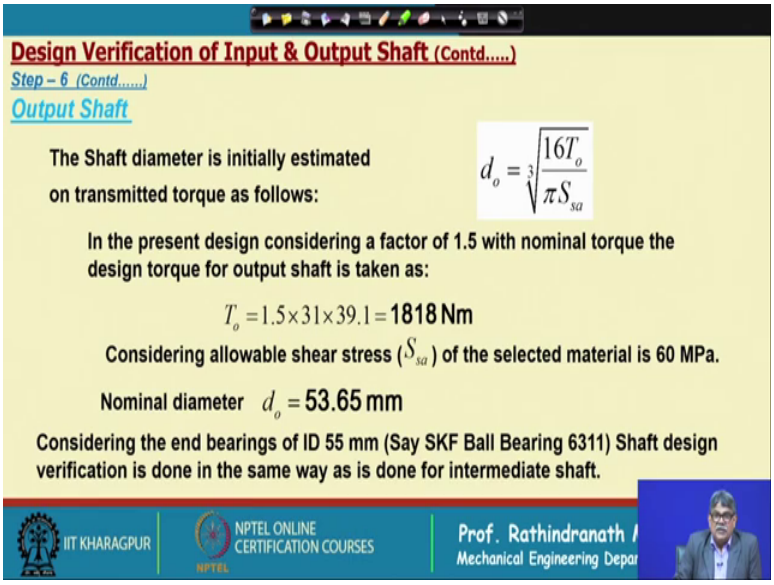

And in the present design the factor of 1.5 with nominal torque the design torque the output shaft is 1.5 into 31 because in designing shaft we have considered the input torque is 31 and the total transmission ratio is 39.1. So, torque is 1818 Newton meter and the shear stress of selected material is 60 mega Pascal, nominal diameter is 53.65 millimetre.

Considering the end bearing of 55 millimetre 6311 SKF shaft design verification is done in the same way as is done the intermediate shafts.

So, if we take the bearing id of 55 millimetre. So, we can go for step of 65 millimetre easily; so, this means that where the gear is sitting the diameter of the shaft is 65 millimetre. And then we calculate the critical section we consider the critical section in this case also again the middle of the gear. Because there will be keyway and although the diameter is hashed there, but there will be the keyway and we verify the factor of safety at that section. This is not shown little calculations are not shown, but why we make the little design through the assignments, we will calculate also that and we will see what are the factor of safety there so.

Thank you.