Gear and Gear Unit Design: Theory and Practice
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Lecture - 13
Gear Unit Design – Failure of Gear Tooth (Probable Dynamic Load and Wear Load Capacity)

In module 3, we are continuing with design of a general purpose industrial helical gear reduction unit. We are still in the design of first stage gears; now I shall discuss about the failure of gears and probable dynamic load and wear load capacity.

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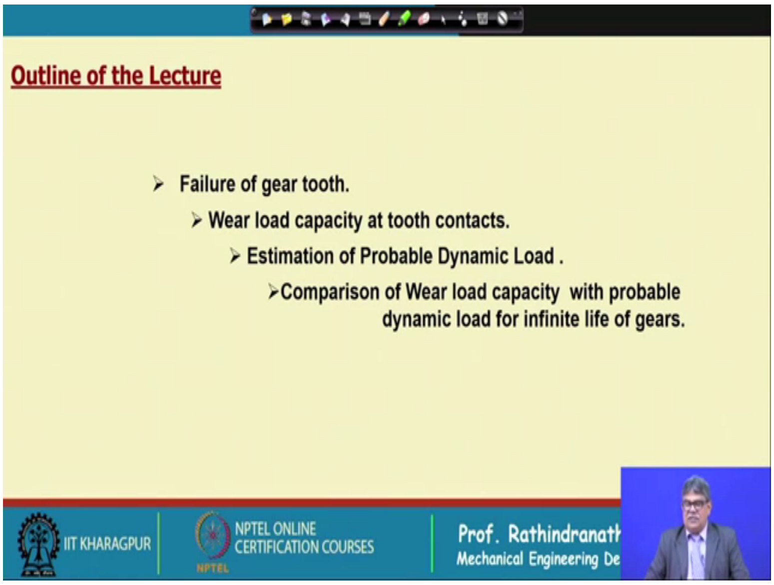

Now, in this lecture I shall cover failure of gear tooth, wear load capacity at tooth contacts, estimation of probable dynamic load finally, we will compare the wear load capacity with probable dynamic load for infinite life of gears. I would like to say that gears are designed; general purpose gears are designed mostly infinite life. That does not mean it will never fail, it will fail at very long run may be 15, 20 years where as if you go for optimum design, then definitely design specification and the parametric values will be different form what we are discussing here and that is; obviously, for a definite life.

Say example for car, for air craft even for machine tools those are of specific life because after certain life the performance depurate because of the wear of the gears and then gears are then avandent; new set of gears are taken. However, in case of industrial gears were increase in dynamic load little increase in dynamic load may not be that savior. And more over if it is designed for infinite life considering, I mean slight over design in that case; we should that gear will last infinitely may be 15, 20 years.

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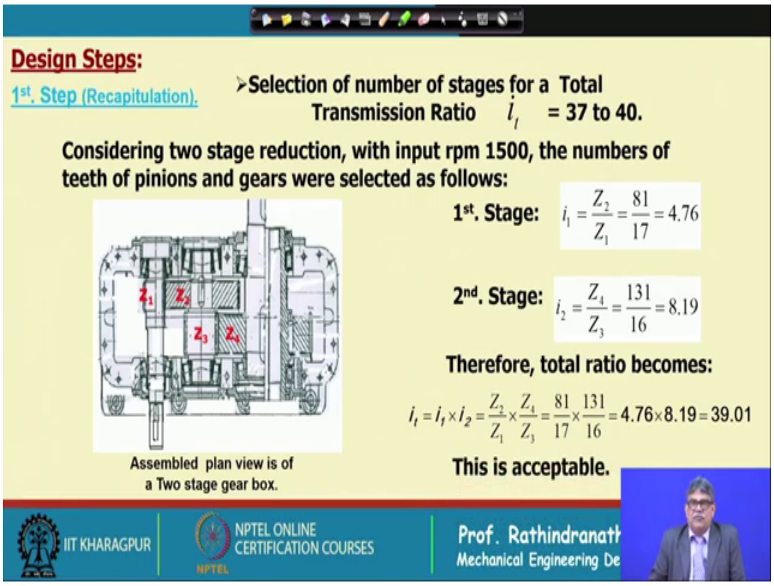

So, again if we come back to the ratios what we have considered; the first stage 4.76 at input RPM is 1500 and then in second stage 8.19.

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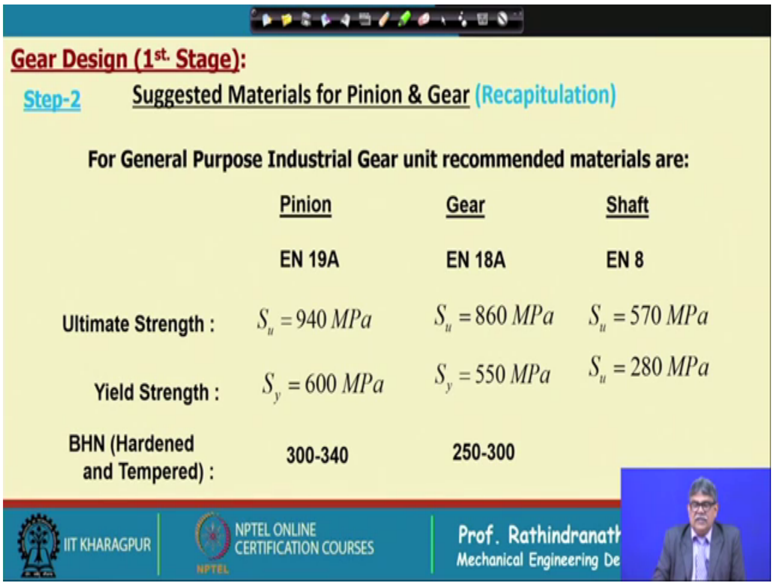

And total ratio is 39.01 which we have accepted for the design; material we have taken EN 19 A for the pinion.

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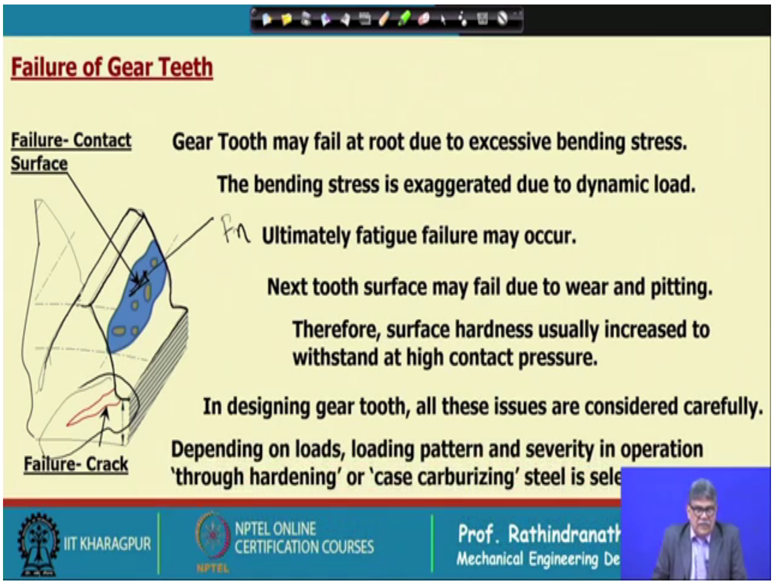

EA 18 A for gear and for shaft EN 8; now let us considered a gear teeth; it is this is the gear teeth, we have considered and load is acting like this Then how this will fail say this is F n normal load; first of all gear tooth may fail at root due to excessive bending stress, it may fail here there a crack will be developed and ultimately it will fail. Now next the; actually bending stress is exaggerate exaggerated due to the dynamic load again.

So, it may fail there if it is not properly designed; ultimate fatigue failure may also occur. Apart from that it is important that this surface fails due to wear and pitting; pitting is a process that when the two materials are in contact and again it is decharge, there is a possibility some portion of the material will be digged out; that normally does not happened if there the contact stress is within the limit; so, we need to verify those.

Therefore, surface hardness usually increased to with stand at high contact pressure; in designing gear tooth all these issues are considered carefully. Depending on loads loading pattern and severity in operation through hardening or case carburizing steel is selected. Now I have discuss a little bit about the hardness; what is called through hardness through hardness is usually done with medium carbon steel of we can see high carbon ordinary steel or allow steel.

In that case the hardness of both the both at surface and at the core increases, but in some cases it is found that this strength core strength for bending is enough although surface is not that strong. In that case usually either low carbon are medium carbon steel is taken and these surface is added with carbon a small layer of small depth is accepts the carbon from outside.

And then with hid tidment that surface becomes hertz and that case it is essential that we have to go for grinding.

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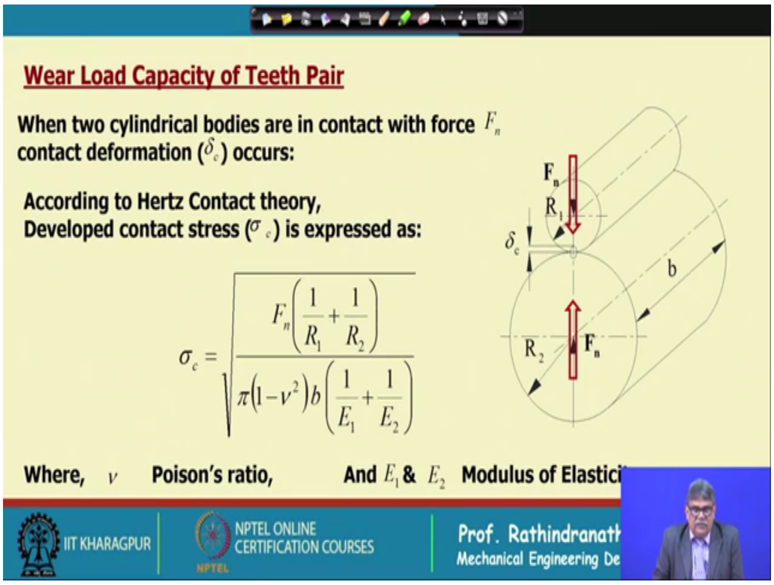

Now, according to hertz contact stress theory; if we consider the two cylinders is coming in contacts, then the contact stress contact stress can be given by normal load; F n into 1 by radius of the smaller or one cylinder plus 1 by the radius of the other in contact divided by pi into 1 minus whole square; new square which is Poisson ratio into the width or length of the cylinder in this case; in case of gears it will be width of the gear into 1 by the hinges modulus of the modulus of elasticity of 1 plus 1 by modulus of elasticity of this second one; now in case of steel both are same.

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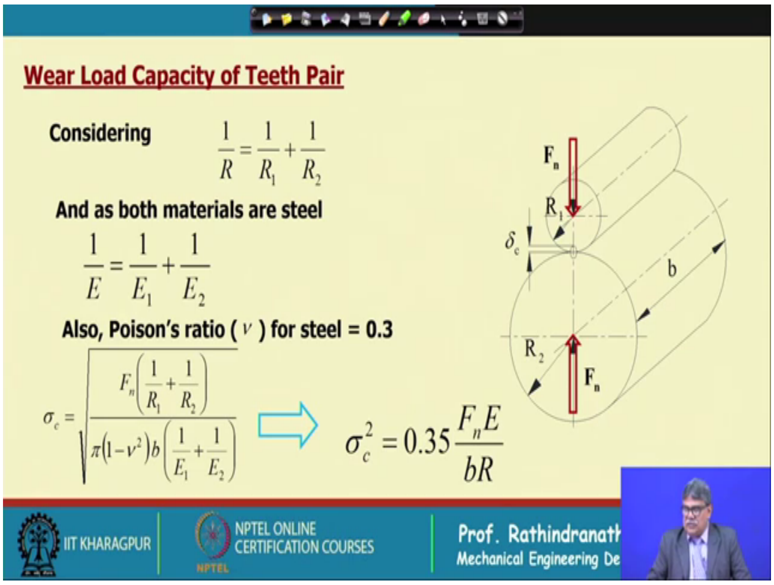

Now, if you consider 1 by R is equal to 1 by R 1 plus 1 by R 2. Then from this equation knowing the R 1 by R 2 we can arrived into a value of R. Similarly we can arrived into a value of the module of elasticity as a E equating 1 by E 1 plus 1 by E 2, but in case of if they are both steel or both same material; then automatically it will come into E.

Also Poisson ratio for steel; if we consider 0.3; then the equation for contact stress will become that sigma c square is equal to 0.35 normal force into modular of elasticity combined divided by length of the cylinder and R the combined radius.

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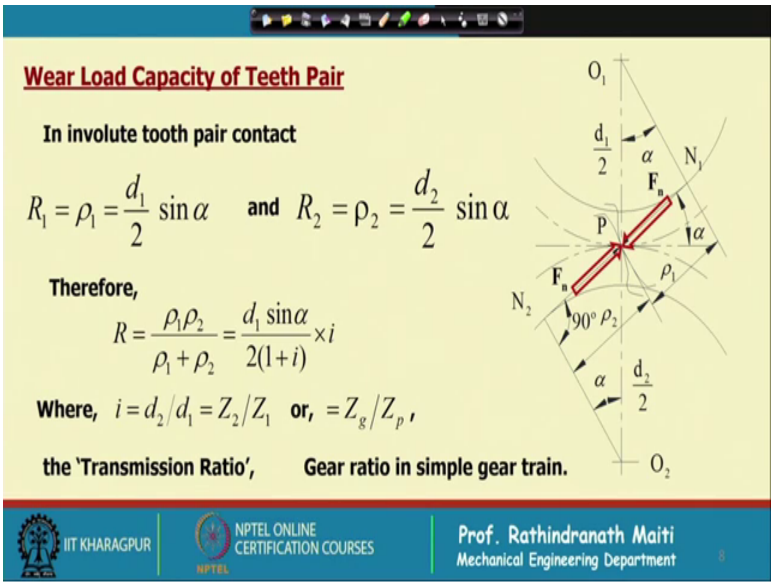

Now, this is normally withdrawed the two cylinders in case of gears what we need to consider? We need to estimate the radius at the contact point; here I have shown the contact at the pitch point and the radius of curvature of this two carves are nothing, but their the distance from that point to the tangents; point of tangency on the base circles; that means, for gear 1; it is rho 1 and for gear 2; it is rho 2; this is for involute teeth.

And now we consider at point P; the forces are acting then we shall consider R 1 in earlier equation is rho 1 is equal to d 1 by 2; that is pitch circle radius of the gear 1 into sin alpha; alpha is the pressure angle.

Similarly, R 2 is equal to rho 2 is equal to d 2 by 2 into sin alpha; alpha is the pressure angle. And R will become rho 1 into rho 2 by rho 1 plus rho 2 is equal to d 1 sin alpha divided by 2; 1 plus i; where i is the into i where i is the d 2 by d 1 which is again can be equated as the number of teethes of gear 2 by gear 1 and which is equal to number of teeth of the or we can express in terms of number of gear and number of teeth of gears; divided by number of teeth of pinion.

And it is called transmission ratio; if we consider the single set we would call that is the transmission ratio of the gears and in simple gear we can call also it is the gear ratio; that means, i is the gear ratio and R capital R that is the mind effect of the that radius can be taken as pitch diameter into sin of the pressure angle of gear 1 divided by 2 into 1 plus the transmission ratio or gear ratio whole into again the transmission ratio.

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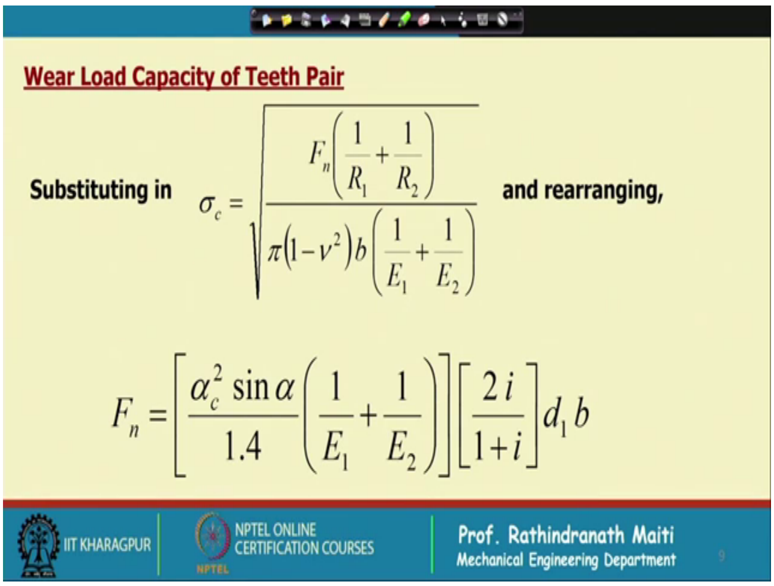

So, again if we consider the formula for the contact stress considering the hertz contacts that becomes which is expressed by that sigma c is equal to normal load into 1 by R 1 plus 1 by R 2 into pi into 1 minus mo; pi generations square into width of the gear into 1 by modulus of elasticity of gear 1 plus 1 by modulus of elasticity of gear 2 and whole under the square root.

So, if we ref rearrange this equation and consider the Poisson ratio of steel and some other factors it is exact derivation is not given here. We can write that normal load F n can be expressed as the; square of the stresses where not alpha this will be the sigma; sigma square c sin alpha divided by 1.4 into 1 by modulus of elastisity plus of gear 1 plus 1 by modulus of elasticity of gear 2; whole into 2 into transmission ratio divided by 1 plus transmission ratio into pitch circle diameter of the gear 1 into width of the gear.

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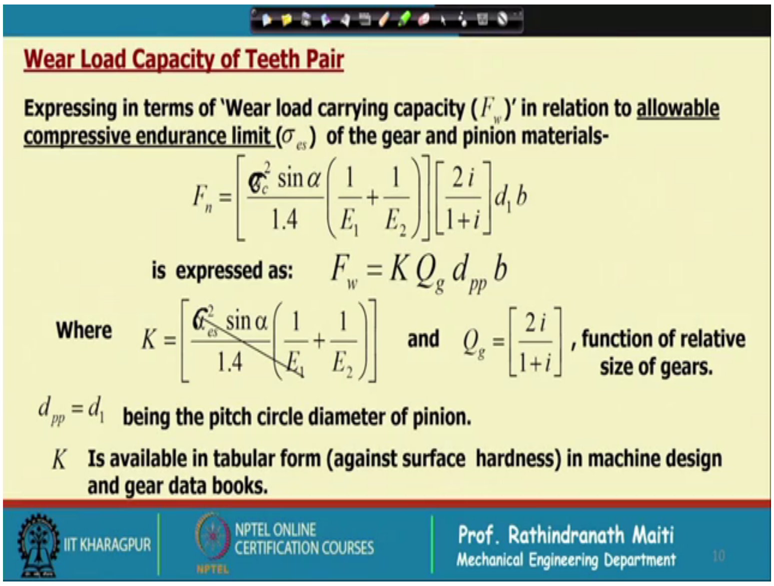

Expressing in terms of wear load carrying capacity F w; the F n we have earlier formula we have shown the normal force that can be withstand at a certain stress. So, now if we express that load is wear load capacity in relation to allowable compressive endurance limit of the gear and pinion materials, then this expression will become that F n is equal to again here sorry this formula; this was the sigma is expressed as F w is equal to K Q g dpp into b.

What are these? K can be expressed as sigma square es we have considered sigma s sin alpha divided by 1.4 into 1 by E 1 plus 1 by E 2 and Q g is given by 2 by i 1 plus I two into transmission ratio divided by 1 plus transmission ratio function of relative size of gears. And dpp or that is that replaces d 1 that is pitch circle diameter of the pinion and K; so, we can express that F w in terms of gear ratio.

And the diameter pitch circle diameter of the pinion; now this K allow that can be calculated from the allowable wear load capacity; allowable compressive endurance limit we can calculate that knowing the pressure angle and modulus of elasticity of the material; however, it is also available in book or gear data books and this is against the hardness; that means, with the increase in hardness; this compressive endurance limit increases as well as the cave allow increases.

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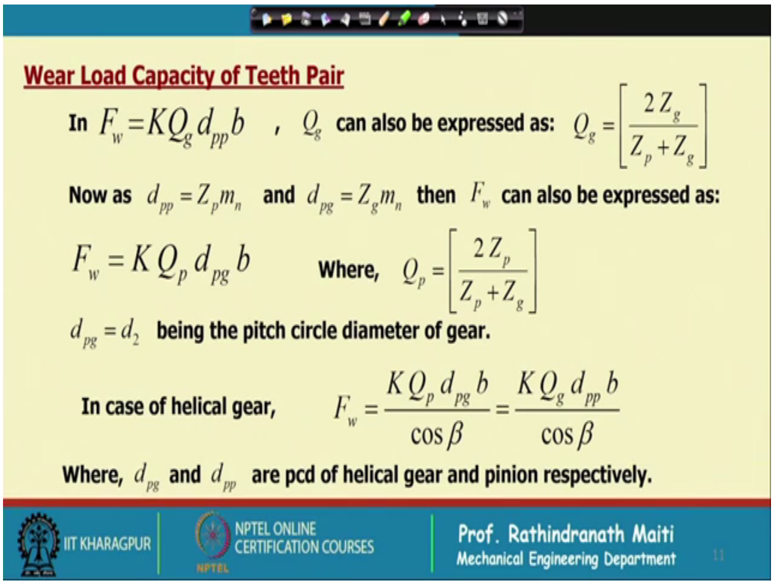

Now, F w again that K Qg dpp into b; Q g also we expressed as twice into teeth number of gears divided by total number of teeth that is the summation of teeth number of pinion and gear.

Now, if this is the expression then dpp; again we are still considering this formula against the stress tooth spur gear. So, pitch diameter of the pinion is given by the teeth number of pinion into normal module. Again the pitch circle diameter of the gear is expressed by teeth number of gears into normal module. Then F w can also be expressed as K into Q pd pitch circle diameter of the gear into the width of the gear where Q p is two s Z p into total number of teeth that is teeth number of pinion and teeth number of gear gears summation of teeth number of pinion and teeth number of gear.

So, this means that we can express in a the F w in either the top one formula or bottom one. Only we should take care what should be the pitch circle diameter we should consider and what should be the ratio for Q p; Q p or Q g. Now in case of helical gear, you can see this if you compare this formula with state tooth spur gear and helical gear; then first of all pitch circle diameter or pitch circle diameter in case of helical gear; the formula will be Z into number of teeth into module divided by cos of helix angle.

So, we consider if we consider in the formula; this pitch circle diameter whether it is pinion or gear is calculated for the helix gear or spur gear. Then only difference will be in width in case of helical gear width will be more because in the it is in the helix direction and that should be b by cos beta. So, if we use the same formula for helical gear a cos beta term will come or in other words we can say the last formula which I have written F w is equal to K Q p into dpg into b by cos beta or equal to K into Q g dpp b by cos beta; this formula should be used for both helical and step tooth spur gear, incase of step tooth spur gear beta will be 0; so, cos beta will be 1.

So, we have so, far arrived into a formula by which we can calculate what would be the load carrying capacity of the gear.

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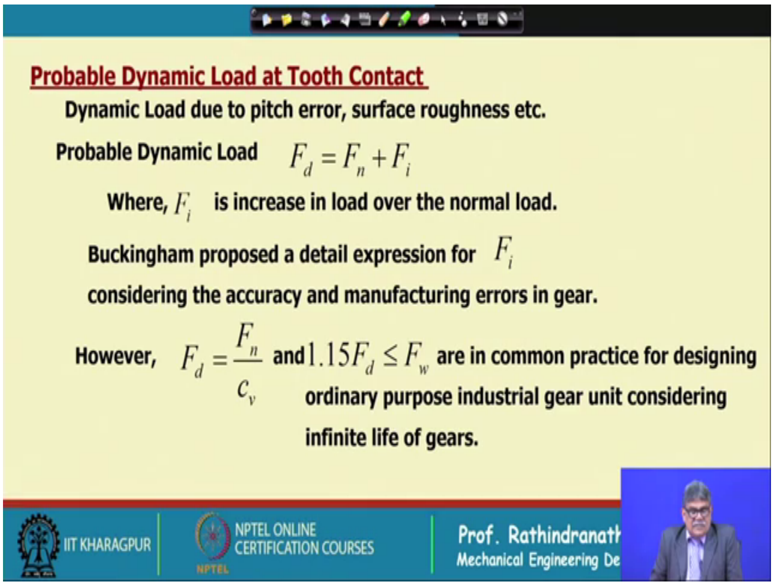

Now, we shall consider what is the probable dynamic load at tooth contact dynamic load due to pitch error or surface roughness etcetera' this probable dynamic load we may consider F d is equal to normal load plus a F i. Now F i is increase in load over the normal load due to the dynamic effect. And as I have mentioned if there is pitch error or due to the surface roughness also there are other factors that bearing has become weak; bearing life has reduced. So, there will be vibrations there will be increase in dynamics which can be estimated.

Now, Buckingham proposed a detail expression for F i which on the accuracy and manufacturing errors in gears. Now that formula is complicated and conversion to calculate; so, there is a simple approach to estimate what will be the F d; one can consider the F d is equal to whatever R the normal estimated force. normal force acting on the gears divided by c v; the velocity factor.

And then finally, 1.15 into F d that should be less than the allowable load or the wear load carrying capacity; that means, at contact surface what load it can withstand with the load; the dynamic load should be less then that. Again we have multiplied a factor of 1.15 into F d. And if we considers as a design, the gear will be of infinite life; here it is to be remember that this semi factor already we have considered while we were calculating the beam strength of the gear tooth and it was used in Lewis formula.

So, it can be said that if we design a gears where there is no dynamic load; that means, the loading patterns itself is hazard; the torque is always remains constant and even if there is variation of torque that is blurring very slowly and smoothly. So, probably we need not consider that there will be increase in dynamic loads except when that gear box will be old it will be owned out there will be where of the other components probably there will be increase in the load.

This means that if in all design all general purpose design if we consider such dynamic load; in true sense the gears are over designed and that will be for infinite life. In practice, in industry those who are manufacturing gears for a general purpose and may be in mass productions that gear box is are offered for different cases; usually such over designed is done. If it is a taylormate gear box probably we can consider more carefully and for mass protection of (Refer Time: 27:21) nowadays; if we go for finite element techniques designing the gears probably we will arrived into the gears of optimum designed.

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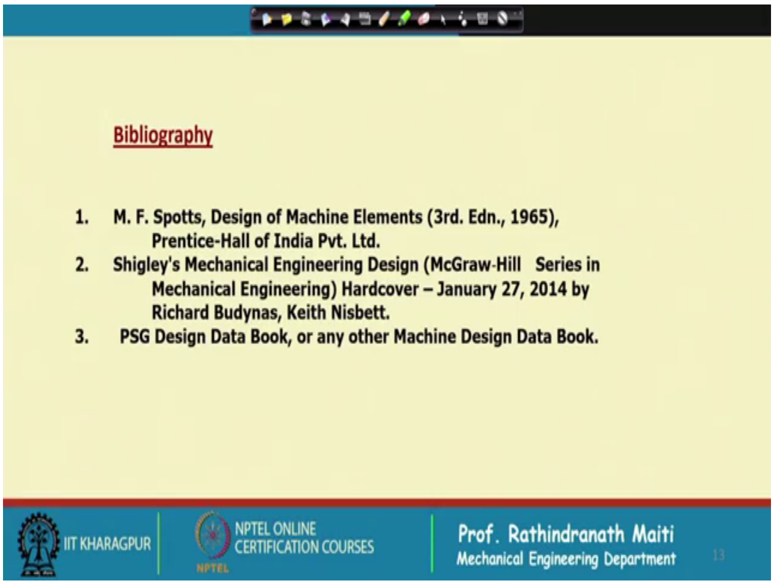

So, again I in bibliography I propose a same books this means that in the gear designed we have to we need to consider, we need to estimate what might be the dynamics load. And what is the actual wear load capacity surface strength of the gear and then we compare that this dynamic load multiplied by 1.15; in this case or if I (Refer Time: 28:08) consider that dynamic load must not be more than this wear load capacity. And after looking into that the teeth is failing not failing due to the bending strength, we can probably increase the surface strength by hardening.

So, in a particular design if we find that the teeth is not failing due to the bending strength, but surface strength is not enough probably we should go for case curveraging steel; that means, we may choose alloy steel with a medium or low carbon, later we can add after the hobbling the gears we can add the carbon to the surface and heat treat it and then ground it and then we can use it without increasing the module.

Alternatively if you go for through hardening probably we have did increase the module to increase the surface area, to increase the surface strength and we will find that din is much stronger for transmitting that load. So, at the design stage designer has to take a decision on that what he should do.

And in next lecture, I will show for our first stage gears what is the probable dynamic load and what is the surface strength of the first stage gear? And then we will compare and we will decide on that; what we need to do whether we go for increasing the hardness or we should go for higher module.

Thank you.