

**Gear and Gear Unit Design: Theory and Practice**  
**Prof. Rathindranath Maiti**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 12**  
**Gear Unit Design - First Stage Pinion and Gear Design – 1**  
**( Module on Beam Strength Basis )**

Design of General Purpose Industrial Helical Gear Reduction Unit; Part- II; this is module – 3 and in this lecture, I shall discuss about the Gear Unit Design first stage pinion and gear design part – I. Basically, I will we shall calculate the module for beam strength.

(Refer Slide Time: 00:41)

**Outline of the Lecture**

- Selection of Stage Ratios , Gear and Pinion Teeth Numbers
  - Determination of Design Torque
    - Material properties and material selection
    - In a gear and pinion set which one is weaker and to be designed
  - Choosing other parameters of Lewis formula
  - Module Selection (Preliminary / 1<sup>st</sup> Attempt).

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Maiti  
Mechanical Engineering Department

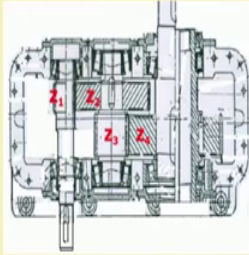
Now, in details in this lecture I shall cover selection of stage ratios, gear and pinion teeth numbers, determination of design torque, material properties and material selection, in gear and pinion set which one is weaker and to be designed, choosing other parameters of Lewis formula, module selection in first attempt or preliminary whatever you like to say.

(Refer Slide Time: 01:29)

**Design Steps:**

**1<sup>st</sup>. Step (Recapitulation):** > Selection of number of stages for a Total Transmission Ratio  $i_t = 37$  to 40.

Considering two stage reduction the numbers of teeth of pinions and gears were selected as follows:



1<sup>st</sup>. Stage:  $i_1 = \frac{Z_2}{Z_1} = \frac{81}{17} = 4.76$

2<sup>nd</sup>. Stage:  $i_2 = \frac{Z_4}{Z_3} = \frac{131}{16} = 8.19$

Therefore, total ratio becomes:

$$i_t = i_1 \times i_2 = \frac{Z_2}{Z_1} \times \frac{Z_4}{Z_3} = \frac{81}{17} \times \frac{131}{16} = 4.76 \times 8.19 = 39.01$$

This is acceptable.

Assembled plan view is of a Two stage gear box.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Mechanical Engineering De

Now, selection of transmission ratio earlier; however, again I repeat. Considering two stage reduction the numbers of teeth of pinion and gears were selected as follows. Now, first stage the gear is a 81 teeth number 81 and pinion is 17 and transmission ratio is coming 4.76. We have to reach close to 37 to 40. Now, the I repeat again why we have taken that teeth ratios in a fraction; to avoid or reduce the dynamics a which is called tooth hunting. If the ratio is an integer suppose there is a is a three ratio completely then every after three cycle the same pair will come in contact.

Now, there are two possibilities or among other possibilities two are prominent; one there are might be pitch error this means that pitch along the (Refer Time: 02:57) may not be equal that happens due to the extensity in mounting while gear is being cut. Although it is a minor, but still each and every gear that might have some pitch error.

Secondly, the surface roughness; it might be every after three cycles when the ratio is three the pair coming in contact they have some roughness due to which again there will be increase in dynamics to avoid that these are made fraction and same teeth meeting repeatedly after a few cycles is called tooth hunting. To avoid the tooth hunting we make the ratio fractional.

So, second stage what we have taken 131 by 16; 131 the gear and the 16 is the pinion, 8.19. Earlier, I mentioned that in no stage it is preferable that ratio should not be more than 6 and to optimize the size again it should not be equal see for example, if you want

to reach at 36, we may think of 6 into 6, but that will make the gear box very large. There will be if we consider the figure then there will be use gap to avoid that we make the ratio in such a way that this gap is minimum. Again, it is not an easy task to select the ratio. So, that it will be optimum design in a single attempt.

However, from practice it is seen that usually if the ratio is kept in this way it might be of optimum close to optimum design that depends on experience, later one can go for optimization if the production is large. Now, in second stage 8.19 justification is that for such a ratio like 37 to 40, we have taken this example intentionally. Probably one can try for three stage, but three stage will be always expensive. So, second stage even if it is slightly larger say 8, 9 it is very often, it is done and as the pitch line velocity is less there. So, second stage we can go a slightly higher than 6. Now, total ratio is coming in that way 39.01, which is acceptable.

(Refer Slide Time: 06:16)

**Gear Design (1st Stage):**  
**Step-2** In next step gears are designed:

**1st. Stage**  
**Module Estimation on Strength Basis:**

$$m_n = \sqrt[3]{\frac{2T \cos \beta}{S_o c_v \psi Y Z c_w}}$$

Input Torque (Nominal):

$$T_i = \frac{Power}{2\pi \omega}$$

Assembled plan view is of 2-stage gear box.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Mechanical Engineering De

Next, we shall consider the design of gears. Now, first stage if we consider the formula it is modified Lewis formula for bending strength. We find this module can be estimated by considering the torque helix angle and alloy will strength of the material, width factor, form factor and the number of teeth. We have consider the helical gear, so, we will now consider the other values.

Now, if the power is given in that case we can calculate the torque from the power which is power divided by 2 pi into angular speed of that shaft.

(Refer Slide Time: 07:14)

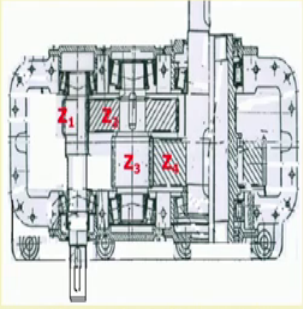
**Gear Design (1<sup>st</sup> Stage):**

**Step-2** In next step gears are designed:

**1st. Stage**

In this problem input continuous torque = 30 Nm.  
*i.e.*  $T_i = 30 \text{ Nm}$

As starting torque is 200%  
the design torque may be taken as:

$$T_{id} = 2 \times T_i = 60 \text{ Nm}$$


Assembled plan view of 2-stage gear box.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Mechanical Engineering De

In this case our torque is given. So, given torque is 30 Newton meter. Now, here is a question shall we design the gears considering the torque is equally to 30 Newton meter or we should take some other higher value because there is a question of safety. Now, there is a no clear cut methods by which we can we can say that we should take this much torque, but in this case as at the starting torque 200 percent and if we consider the this gear box will be started now and then; that means, in a day it might be starting 10 times or 8 times, in that case it is better to go for designing the gears considering the torque is equal to 60 Newton meters that is 2 times, twice, 200 percent torque.

Now, this will give you can say this is a fail shaft design, moreover unless there is a question optimization and if such gear box are in the production line of a gear box manufacturer who offers such gear box gear box for a certain range of operation in that case whether the starting torque is 200 percent and 150 percent probably same gear box can be used in one in 150 percent, it will be slightly over design.

(Refer Slide Time: 08:58)

**Gear Design (1<sup>st</sup> Stage):**  
**Step-2** • **Suggested Materials for Pinion, Gear & Shaft**

	<u>Pinion</u>	<u>Gear</u>	<u>Shaft</u>
	<b>EN 19</b>	<b>EN 18A</b>	<b>EN 8</b>
Ultimate Strength :	$S_u = 940 \text{ MPa}$	$S_u = 860 \text{ MPa}$	$S_u = 570 \text{ MPa}$
Yield Strength :	$S_y = 600 \text{ MPa}$	$S_y = 550 \text{ MPa}$	$S_u = 280 \text{ MPa}$
BHN (Hardened and Tempered) :	<b>300-340</b>	<b>250-300</b>	

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Mechanical Engineering De

So, we have considered the torque is double or designing the gears. Now, material proposed is EN 19 and for pinion for gear it is EN 18A and for shaft EN 8. Now, from the material specification the ultimate strength is 940 mega Pascals for EN 19 and for EN 18A it is 860 mega Pascals and for EN 8 it is 570 mega Pascal and yield strength is 600. In case of pinion EN 19, in case of gear it is 550 mega Pascals, in case of EN 8 it is 280 mega Pascals and BHN or the BHN means benien harvest number, it will be 300 to 340 in case of pinion material, in case of gear material it is 250 to 300.

Now, I would like to say at that hardness if the usually this blanks are forged and then machined, so, after forging if it is heat sheeted to raise the hardness 300 to 340 and in case of gear 250 to 300 still that is machinable with the (Refer Time: 10:49) cutters and there is no necessary to grind it. So, that is why it has material has been proposed like this.

(Refer Slide Time: 10:54)

Alloy Steel Equivalent					
International Standard	BS	DIN	IS	EN	SAE/AISI
<b>EN18A</b>	<b>530A40</b>	<b>37Cr4</b>	<b>40Cr1</b>	<b>EN18</b>	<b>5140</b>
EN18	530A40	37Cr4	40Cr1	EN18A	5140
<b>EN19</b>	<b>709M40</b>	<b>42Cr4Mo2</b>	<b>40Cr4Mo3</b>	<b>EN19</b>	<b>4140,4142</b>
EN19C	709M40	-	40Cr4Mo3	EN19C	4140,4142
EN24	817M40	34CrNiMo6	40NiCr4Mo3	EN24	4340
Carbon Steel					
<b>EN8</b>	<b>080A40</b>	<b>CK45</b>	<b>45C8</b>	<b>EN8</b>	<b>1040,1045</b>

Now, again if we look a quickly into the little specification of the materials there is EN 18A, what we find that BS standard for EN 18A, EN 18 these are more overly same. So, we can consider that these are same material, we have we have taken 18A and DIN standard is also as we find it is same 37 Cr4 and Indian standard IS we find 40 Cr1 in both cases in 18A or 18 and EN is European norms that is again either EN 18 or EN 18A these are specified there is are small differences which are not projected in other specifications. Similarly, as we see EN 19 that is according to Indian standard 40 Cr4 Mo 3 and EN 8 is 45 C8 you can simply call 45 C.

(Refer Slide Time: 12:16)

**Gear Design (1<sup>st</sup> Stage):**

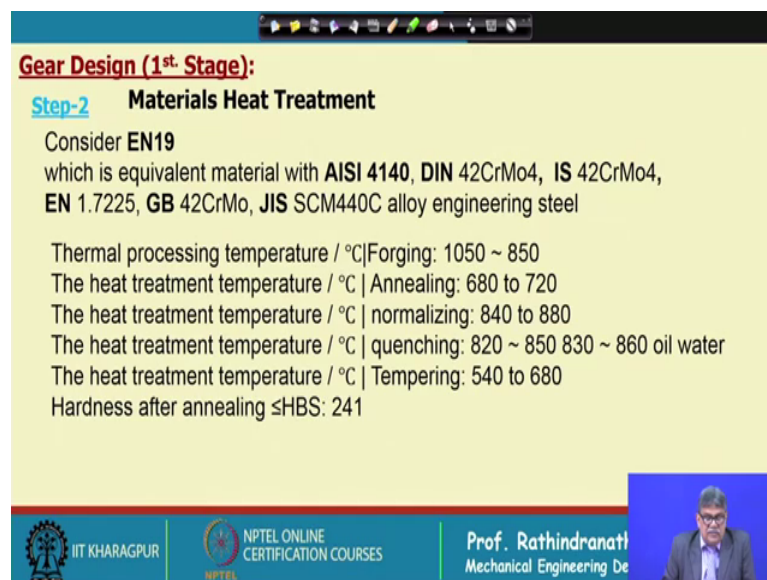
**Step-2 Materials &- Chemical Composition**

EN 19	C(%)	0.35~ 0.45	Si(%)	0.10~0.35	Mn(%)	0.50~ 0.80	P(%)	≤0.035
	S(%)	≤0.050	Cr(%)	0.90~1.50	Mo(%)	0.20~ 0.40		
EN18	C	.35 – .45	Mn	.60 – 1.00	Si	.10 – .35	S	.050 MAX
					P	.050 MAX	Cr	-
							Ni	-
Standard	Grade	C	Mn	P	S	Si		
BS 970	EN8/08 OM40	0.36- 0.44	0.60- 1.00	0.05	0.005	0.10-		

Now, what it is, if you go to the next slide for EN 19 composition chemical compositions are like that carbon is 0.35 to 0.4 percent and silicon is 0.10 to 0.35, manganese 0.50 to 0.8, phosphorous less than 0.035, sulphur less than 0.050, chromium 0.9 to 1.5 and Molybdenum 0.2 to 0.40 and as you see and it is called 40 Cr it is called 40Cr4Mo3; that means, 40 means here 0.4 percent carbon, Cr means 0.4 percent chromium, Mo3 means 0.3 percent Molybdenum.

So, in case of 18 the specification as you see there you will find it on the printed note and for EN 8 as you find it is simply a medium carbon steel, it is not we should not call it an alloy steel and carbon percentage is 0.36 to 0.44, it is very good for shafts of medium load also it can be used for gears.

(Refer Slide Time: 14:09)



**Gear Design (1<sup>st</sup> Stage):**  
**Step-2 Materials Heat Treatment**

Consider **EN19**  
which is equivalent material with **AISI 4140, DIN 42CrMo4, IS 42CrMo4, EN 1.7225, GB 42CrMo, JIS SCM440C** alloy engineering steel

Thermal processing temperature / °C | Forging: 1050 ~ 850  
The heat treatment temperature / °C | Annealing: 680 to 720  
The heat treatment temperature / °C | normalizing: 840 to 880  
The heat treatment temperature / °C | quenching: 820 ~ 850 830 ~ 860 oil water  
The heat treatment temperature / °C | Tempering: 540 to 680  
Hardness after annealing ≤HBS: 241

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Mechanical Engineering De

Now, with heat treatment these materials are further improved. So, let us consider EN 19 which is equivalent material with AISI 4140 that is American industrial standard or something like that DIN is the German standard, IS the Indian standard, EN is the European Norm standard and GB is the Chinese standard GB and DIN and IS more or less same thing and Japanese standard is SCM4406. They are different all countries are having their own standard although there is an international standard also.

Now, these are heat treated materials properties can be improved by heat treated. Now, thermal processing that is temperature if you raise if you go for forging it is 1050 to 850 degree centigrade and if you go for annealing then 680 to 720 and annealing means the

bring the material into normal after forging and also this normalizing is a also after heat treatment it is 840 to 880. Then quenching; quenching is that after heating the material is put either into water or in the oil it is for the hardening or quick cooling it is done and by that the material is hardened and tempering is called heating for the process of increasing the stress level.

So, tempering are quenching they go together to hmmm have the to increase the strength of the material and a hardness after annealing it is usually 241 for usual material. And, as I told this hardening is necessary to increase the stresses usually for gears this materials are hardened and if we raise the hardness upto 300 to 340 stresses strength of the materials will increase, but still it will remain able to machining machinability see for the example hobbling etcetera.

(Refer Slide Time: 16:29)

**Gear Design (1st Stage):**

**Step-2**      **Design of Gears – Module estimation (Contd.):**

**1st. Stage (Cont.)**

In a gear pair which one to be designed?

$$m_n = \sqrt[3]{\frac{2T \cos \beta}{S_o Y \psi Y Z c_w}}$$

Compare  $(S_o Y)_{gear}$  with  $(S_o Y)_{pinion}$

$Z_{Pinion} = 17$        $Z_{Gear} = 81$

Assembled plan view is of 2-stage gear box.

IIT KHARAGPUR      NPTEL ONLINE CERTIFICATION COURSES      Prof. Rathindranath Mechanical Engineering De

Now, we will try to estimate the module. In a gear pair which one to be designed because if we consider the twice T; here, the T is the design torque; that means, in this case it will be 16 Newton meter. If we consider the this part accept the allowable strength S 0 into Y, other part will be same for gear and pinion, because in the we will go to the design of the gear torque will be increased by the transmission ratio and Z also will be increased by the same number, so, this ratio will remain same. Size, the width factor which is active width factor, so, that will remain same also, so, and c v, c w are the same, beta is the helix angle will remain same. So, what we find is 0 Z that will. So, if we consider S 0 Y,



Y is the form factor with respect to the formative number of teeth because we are considering the helical gear which I have already discussed y is against the formative number of teeth which is given by the actual number of teeth divided by cos cube of the helix angle.

So, if we consider that form factor and the allowable strength for gear and if we consider the allowable strength for pinion and pinion form factor then with this 2 values definitely one will be less than the other. So, we should design for that one which is less because that will give the more amount of module in calculation. Now, pinion teeth number is 17 and gear teeth number is 18.

(Refer Slide Time: 19:22)

**Gear Design (1<sup>st</sup> Stage):**

**Step-2** Allowable strength of selected material in gear design.

It is to be noted that in the following Lewis formula for bending strength of gear tooth.

$$m_n = \frac{2T \cos \beta}{\sqrt[3]{\frac{S_o c_v}{c_w} \psi Y Z}}$$

$\frac{S_o c_v}{c_w}$  Is the ultimate allowable strength for the particular gear considering its dynamic and lubrication conditions in operations,

Now  $S_o$  is taken as certain proportion of either ultimate strength or yield strength of the selected material.

$S_o = \text{Yield strength} / 2.5$  for gear design is often considered.

IT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranati Mechanical Engineering De

Now, allowable strength of selected material in gear design; that means, gear pair design. It is to be noted that in the following Lewis formula for bending strength of gear tooth  $S_o$  by  $c_v$  with  $c_w$  is the ultimate allowable strength for particular gear considering its dynamic and lubrication conditions in operations.  $c_v$  factor is taken on the (Refer Time: 20:02) of the gears and the pitch line velocity.

So, if we go for hob cutting gears finished by hob cutting, if we go for gears finished by grinding did definitely there will be difference in surface finish and  $c_w$  is the ware load factor or lubrication factor which is considered that 1.25 in case of regular, but not very often lubrication whereas, this can be taken as 1 for the force lubrication. So, this takes

care of the this definitely  $S_0$ ;  $c_v$  by  $c_w$  will reduce the stress value for reach we will get more module and design will be safe.

Now,  $S_0$  is taken as certain proportion of either ultimate strength or yield strength of the selected material. Yield strength divided by 2.5 for gear design is often considered. This will be ultimate strength, ultimate strength we should consider this is ultimate strength. So, we should consider ultimate strength divided by 2.5 for the design. This is not yield strength, ultimate strength.

(Refer Slide Time: 21:45)

**Gear Design (1st Stage):**

**Step-2**

$$(S_o)_{pinion} = (S_y)_{EN 19 A} / 2.5 = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}$$

$$(S_o)_{gear} = (S_y)_{EN 18 A} / 2.5 = 220 \text{ MPa} = 220 \times 10^6 \text{ Pa}$$

**Considering nominal helix angle  $\beta = 12^\circ$**

$$Y_{pinion} = 0.484 - 3.28 / Z' = 0.484 - 3.28 / (17 / \cos^3 12^\circ) = 0.303$$

$$Y_{Gear} = 0.484 - 3.28 / Z' = 0.484 - 3.28 / (81 / \cos^3 12^\circ) = 0.446$$

$$(S_o Y)_{gear} = 220 \times 0.446 = 98.12 \quad (S_o Y)_{pinion} = 240 \times 0.303 = 72.72$$

$(S_o Y)_{pinion} < (S_o Y)_{gear}$  means pinion is weaker and it is to be designed.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Mechanical Engineering De

Now, we take in that case this 940 was the value divided by 2.5 it is around 240 mega Pascals and for gear it is coming 220 mega Pascals. So, considering nominal helix angle now, here at this stage we consider the nominal helix angle is about 12 degree. Then, for pinion the formative number of teeth is given 17 teen by cos cube of helix angle and the formula for the formatives form factor is 0.484 minus 3.28 divided by formative number of teeth; in case of pinion it is coming 0.303, in case of gear it is coming 0.446. Obviously, with the increase a teeth number this form factor increases.

So, we now compare  $S_0$  into  $Y$  for  $Y$  for gear and  $S_0 Y$  for pinion and we find that  $S_0 Y$  for pinion is less than  $S_0 Y$  for gear, this means that pinion is weaker and we should designed the pinion.

(Refer Slide Time: 23:20)

**Gear Design (1<sup>st</sup> Stage):**

**Step-2**      **Design of Gears – Module estimation (Contd.):**

**1<sup>st</sup> Stage (Cont.)**

**Pinion shaft is the input shaft. Therefore:**

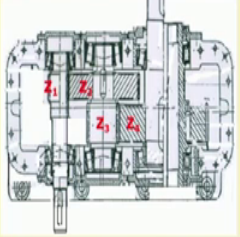
$T = 60 \text{ Nm} \quad \beta = 12^\circ \quad \psi = 20$

**Considering accurately cut gear and assuming module may be 2.5 mm.**

$$V = \frac{\pi(17 \times 2.5) \times 1500}{\cos 12^\circ \times 1000 \times 60} = 3.41 \text{ m/sec}$$

$$c_v = \frac{4.5}{4.5 + V} = \frac{4.5}{4.5 + 3.41} = 0.57$$

Assembled plan view is of 2-stage gear box.



IIT KHARAGPUR      NPTEL ONLINE CERTIFICATION COURSES      Prof. Rathindranath Maiti  
Mechanical Engineering Department

Now, pinion shaft is the input shaft therefore, torque is in this design we shall consider 60 Newton meter and the helix angle 12 degree and width is also width factor is also 20. Here, another thing I would like to mention that, in case of gear we have considered the torque is 16 Newton meter, but in case of shaft we may go for less torque or more torque that depends on what the components is, the what factor will be multiplied in design with the nominal torque that depends on what component it is and what is it is function. In case of gear we have taken say 200 percent torque then the nominal torque, in case of shaft it might be different depending on what is the shape of the shaft and what is the dynamics coming on the shaft. Anyway continuing with the gears we have taken the torque is 60 Newton meter.

Now, in the formula we have to calculate the  $c_v$  values and in that case we should know the size of the pinion, but before first calculation how we know that what is the size of the pinion so, we have to assume something. From experience, usually it is considered in this case we have considered the module may be close to 2.5, so, on on that value we have considered the velocity pitch line velocity is equal to the diameter pitch diameter of that components.

That means, pinion which is  $\pi$  into  $Z$  into module divided by  $\cos$  of the helix angle into  $\omega_1$  is 1500 divided by 1000 into 60 because we have divided by 1000 to make it meter. So, it comes 3.41 meter per second and the velocity factor in that way we have

considered 4.5 divided by 4.5 plus V which is for accurately cut gear will you consider, although there will be no heat treatment after the after cutting the gear and there will be no grinding, but it will be very accurately cut by the hob, so, we consider this formula and we find that c v is coming 0.57.

(Refer Slide Time: 26:10)

**Gear Design (1<sup>st</sup> Stage):**

**Step-2**

Substituting the values in- 
$$m_n = \sqrt[3]{\frac{2T\cos\beta}{S_o c_v \psi YZ c_w}}$$

$c_w$  Being 1.25 for oil lubrication with regular inspection.

$$= \sqrt[3]{\frac{2 \times 60 \times 0.978}{\frac{240 \times 10^6 \times 0.57}{1.25} \times 20 \times 0.303 \times 17}} = 0.218 \times 10^{-2} m = 2.18 mm$$

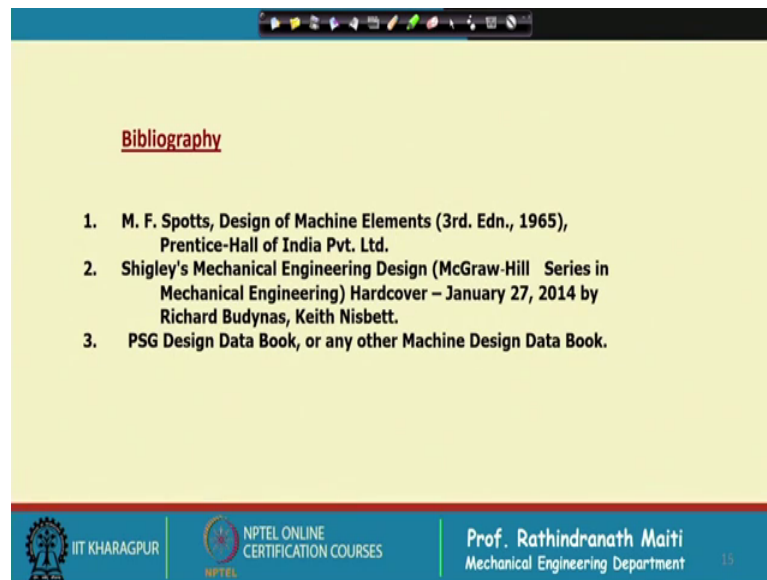
Therefore, 1<sup>st</sup> stage normal module (in 1<sup>st</sup> Attempt / round) may be taken as **2.5 mm**

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Maiti  
Mechanical Engineering Department

Now, with this value and c w we have considered 1.25 that oil lubrications with regular inspection this means that inspection will be regularly, but not may not be very frequent and it is in oil some the gears are in oil some that is just (Refer Time: 26:40) lubrication we have consider this value is 1.25.

Now, we substitute the value the module comes 2.18 millimeter, 2 into 60 is the torque 0.978 is the cos of helix angle 240 into 10 to the power 6 is S 0, 0.57 is c v, c w is 1.25, width factor is 20, modified form factor is 0.303 and teeth number is 17. So, we get 2.18 and here we may conclude that first stage normal module in first attempt or in first round may be taken as 2.5 millimeter. Why I have said it is a first round, because later from other load consideration we may find this module may need to increase.

(Refer Slide Time: 27:38)



**Bibliography**

1. M. F. Spotts, Design of Machine Elements (3rd. Edn., 1965), Prentice-Hall of India Pvt. Ltd.
2. Shigley's Mechanical Engineering Design (McGraw-Hill Series in Mechanical Engineering) Hardcover – January 27, 2014 by Richard Budynas, Keith Nisbett.
3. PSG Design Data Book, or any other Machine Design Data Book.

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | Prof. Rathindranath Maiti  
Mechanical Engineering Department 15

And, again I suggest that any machine design book you will find this type of treatment although this modified form formula may not be available it will not be available. I have suggested to machine design books and also I suggest the PSG Design Data Book, but one can follow any machine design book in this regards.

Thank you.