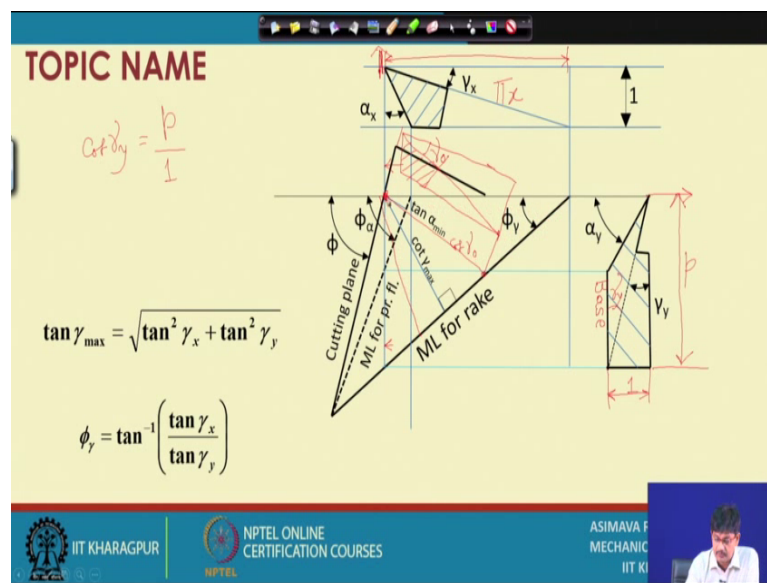


**Metal Cutting and Machine Tools**  
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**Indian Institute of Technology, Kharagpur**

**Lecture - 05**  
**Geometry of cutting tools and numerical problems**

Welcome viewers to the fifth lecture of metal cutting and machine tools. So, in this one, we will be discussing about different cutting tools taking of numerical problems, and also one thing that I have left behind that is in geometry of cutting tools, I will just touch the subject of master line system of you know tool angle reference, master line. This sometimes referred to as or other it is more often referred to as the maximum rake system. So, to start within the in this figure I have drawn the traces of if you look here.

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The traces of the can you see this, of the intersection of the rake surface with the base plane, now what do I mean by the base plane? By the base plane I mean that plane which is you know at the base of the tool and it is necessarily parallel to the reference plane. It might not always be parallel to the reference plane, but in this case we are ah assuming that; which is the base plane here this is the in the sectional view of the tool, in the machine longitudinal plane phi x.

So, let me write down. So, this is phi x. So, in phi x what do we see? We see that gamma x alpha x etcetera have appeared and this is the trace of the base plane so; that means,

that I am assuming here that the velocity vector is sorry the velocity vector is exactly oh just a moment in this direction, it has not exactly fall in there. I can draw it here the velocity vector I am assuming to be like this.

So, that this comes out to be the base plane oh it is difficult to write this way, I have to have more practice base plane. So, if that happens in that case the, this is a sort of representation of the reference plane, and the rake surface as you can see here in this view it is coming down sloping down and intersecting the base plane at this point. So, I take it to the planned view this is the point representing the intersection of the rake surface and the base plane, considered in the machine longitudinal plane.

Similarly, in the machine transverse plane the rake surface is sloping down and it is reaching the base plane at this point, and the corresponding point is here and this is the section that we have considered sorry this is the section that we have considered hence just a moment. So, hence what we have here is that, this line therefore, will represent if we join these two points we it will represent the intersection of the rake surface with the base plane, you might ask me how did you know that is this will be a straight line? It might well be a curved line, here our argument is this that since we are having a plane intersecting another plane the result should be a straight line.

So, this one we call master line for rake what does this mean? This means that each and every point represents intersection of the rake surface with the reference plane, if the reference plane of the base plane they are considered to be the same here. So that means, that from here if we extend lines, these are going to represent this distance for example, if I draw from here and I extend from here. These distances or it is not coming very well in the upper view I can take a take the hell this view, these are going to represent these distances on this particular figure. That is from here to this point is going to be going to represent this one.

If we take some other view there also the same thing is going to happen. For example, if I take the orthogonal rake then orthogonal rake also will be represented that way, where will the orthogonal rake a view come it will be coming somewhere here that is if I take a section in this particular direction, orthogonal this will read up and this will be orthogonal rake all right. So, in that case what are we going to see? This line is also going to slope down. So, that it will be you know represented on this line somewhere

here; that means, if you join; if you join this one this line is going to represent this very distance this distance.

Now, suppose I say that this height of this tool point, where this point is mean section. So, this point if I consider this height to be 1, in that case  $\cot$  of this angle say  $\gamma_x$ ,  $\cot \gamma_x$  becomes equal to this distance divided by 1. So, let us have a calculation here done quickly. So, let me call this say  $p$ , and let me call this distance I set it to be equal to 1 that is good and this I find is  $\gamma_y$  alternate angles.

Therefore, I can say  $\cot \gamma_y$  equal to what is this equal to this must be equal to? This  $p$  divided by one that is it  $p$  is equal to  $\cot \gamma_y$ . So, I can say that this distance represents  $\cot \gamma_y$  this distance it represents  $\cot \gamma_x$ , this represents  $\cot$  say which one this one yeah this one; this one represents  $\cot \gamma_o$ .

So, just draw a straight line in that particular section in which the rake angle is being considered, these lines will be representing the cotangents of those rake angles, that is good that is interesting. So that means, all these lines emanating from here and coming up to this one let me clean the view of it yeah all these lines are cotangents of the rake angle of that corresponding section, and immediately become to the conclusion in that case there must be a minimum distance here which represents the maximum rake angle this is it this is the one. And incidentally this particular angle, this maximum rake angle is represented by this relation how do we get this relation? We can have a quick look if we take out this particular triangle.

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**TOPIC NAME**

Handwritten notes on the left side of the slide:

$$\frac{1}{2} \cot \gamma_x \times \cot \gamma_y = \frac{1}{2} \cot \gamma_{\max} \times \sqrt{\cot^2 \gamma_x + \cot^2 \gamma_y}$$

$$\tan \gamma_{\max} = \sqrt{\tan^2 \gamma_x + \tan^2 \gamma_y}$$

$$\phi_{\gamma} = \tan^{-1} \left( \frac{\tan \gamma_x}{\tan \gamma_y} \right)$$

Diagram illustrating the geometry of a cutting plane and the relationship between angles  $\alpha_x$ ,  $\alpha_y$ ,  $\phi$ , and  $\phi_{\gamma}$ . The diagram shows a cutting plane, a line of action (ML) for the rake angle, and the angles  $\alpha_x$ ,  $\alpha_y$ ,  $\phi$ , and  $\phi_{\gamma}$ . The diagram also shows the relationship between the angles  $\alpha_x$ ,  $\alpha_y$ , and  $\phi$ .

Handwritten notes on the right side of the slide:

$$1 = \cot \gamma_{\max} \times \frac{\cot^2 \gamma_x + \cot^2 \gamma_y}{\cot^2 \gamma_x \cot^2 \gamma_y}$$

$$\tan \gamma_{\max} = \sqrt{\tan^2 \gamma_x + \tan^2 \gamma_y}$$

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This is cot gamma x is it visible yes, this one is cot gamma y, this one must be root over of cot square gamma x plus cot square gamma y, and this must be equal to cot gamma max. Once we have this relation we will be able to find out tan gamma max to be equal to this how do we do that? We equate the area of the triangles, I mean we equate the areas of the triangle considered from once from half the base into altitude taking this as the base for example, cot gamma x into cot gamma y, being equal to half cot gamma max multiplied by root over of cot square gamma x plus cot square gamma y. Once you have this if you take you know if you divide the two sides by cot gamma x cot gamma y and cancel the half let me cancel the half here itself.

So, if you divided by cot gamma x. So, on this side you will get one on the other side you will get root over of cot square gamma x plus cot square gamma y divided by cot square gamma x cot square gamma y, which will mean that you will get tan square gamma x plus tan square gamma y. So, if you look at it, it will be you know ultimately from here you will directly get this relationship. So, that ultimately we can say just a movement, here we can consider this way, we can do it even you know in a quicker manner instead of cutting off cot square gamma x. We can send this to the other side we can send this to the other side whichever way you do it, it will come out to be the same thing, that is tan gamma max I will do it here, here we have some space.

So, we have  $\cot \gamma_{\max} = 1$  is equal to  $\cot \gamma_{\max} \sqrt{\cot^2 \gamma_x + \cot^2 \gamma_y}$  divided by  $\cot^2 \gamma_x + \cot^2 \gamma_y$  what do we have from here? We have one  $\cot^2 \gamma_x$  cancelling out so, that this one can be transferred to the other side.

So, that we have  $\tan \gamma_{\max}$  equal to root over of this one, one of one cancels out the other remains and we convert it to tan. So,  $\tan^2 \gamma_x + \tan^2 \gamma_y$  that is it. So, this way we get an expression of a maximum value of rake now how is this useful to us?

This is useful to us in the sense that, we can utilize this as information that if the rake surface orientation has to be expressed, we are generally doing it as a pair of angles that pair might be you know inclination angle and orthogonal rake inclination angle and normal rake, back rake and side rake instead of doing that we can say that the orientation of the rake angle is maximum rake is equal to this much and the other side I mean the ins in the orthogonal side, generally we are expressing the angles in mutually perpendicular directions, in the other direction sorry the rake angle is zero ok.

So, we will say if you consider this particular orientation of the intersection of the base plane and the rake surface, in this direction it is having no angle with the reference plane, but perpendicular to it is having  $\gamma_{\max}$  as the inclination of the rake surface with the reference plane. So, if  $\phi_{\gamma}$  and  $\gamma_{\max}$  they are expressed, we get the orientation of the rake surface in terms of you know maximum rake and 0 as the two angles made by the rake surface with the reference plane in a mutually perpendicular direction.

So, that helps us during operations like grinding; when we take a pull grinding this will be made use of and you will really appreciate this particular application. And this  $\phi_{\gamma}$  this particular angle which depicts the orientation of the in this section of the rake surface with the reference plane, this  $\phi_{\gamma}$  can be found out very easily in this particular triangle that we were considering this one. It is simply  $\tan^{-1} \frac{\tan \gamma_y}{\tan \gamma_x}$ , I am not working it out you can surely follow it ok.

So, with this part we come to the, you know come to understand the usefulness of just a moment. You come to understand the usefulness of the master line method.

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**TOPIC NAME**

The master lines for rake and clearance

$$\tan \gamma_{\max} = \sqrt{\tan^2 \gamma_x + \tan^2 \gamma_y}$$

$$\phi_r = \tan^{-1} \left( \frac{\tan \gamma_x}{\tan \gamma_y} \right)$$

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**TOPIC NAME**

The master lines for rake and clearance

$\tan \lambda + \tan \gamma_x \cos \phi = \tan \gamma \sin \phi$

$$\frac{1}{2} \cot \gamma_x \times \cot \gamma_y + \frac{1}{2} \cot \lambda \times \cos \phi \times \cot \gamma_y = \frac{1}{2} \cot \gamma_x \times \cot \lambda \times \sin \phi$$

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And I will just give you one example through which you will be convinced that yes it has some other uses also. All the tool angle calculations can be done very elegantly with the help of this master line principle. Let us have a quick look how this can be done for example, we can simply go back to the calculation of areas of triangles and make use of it in order to find out the relations between different angles. We do not have to do with that altitude method that we have discussed previously for example. Suppose I can identify here some basic triangles let us let me draw it, this is one triangle this is another one this is right.

So, what do we have here? We have here this to be  $\phi$ , this to be  $\cot \lambda$  this to be  $\cot \gamma_x$ , this to be  $\cot \gamma_y$  I am sorry  $\gamma_x$  let me use the eraser that is, it  $\cot \gamma_y$ . In that case suppose I want to establish the relation between  $\gamma_x$   $\gamma_y$  and  $\lambda$ . So, what do we do? We see that the area of this triangle say I mean this  $e$  and I mean this  $b$  the area of this particular triangle is simply half the  $e$  I have already written here. Half  $\cot \gamma_x$  into  $\cot \gamma_y$  that is it I think you have absolutely no problem in you know accepting this  $e$  area  $a$  is equal to this, plus suppose I add area  $b$  how much is area  $b$ ? Area  $b$  is equal to this being the altitude, this altitude is nothing, but  $\cot \lambda \cos \phi$ .

So, we use  $\cot \lambda \cos \phi$  into  $\cot \gamma_y$   $\cot \gamma_y$  is the base, base into altitude into half. So, that way we get this expression  $b$  now if I add  $e$  and  $b$  I must be getting the area of this triangle, what is the area of this triangle? The base is  $\cot \gamma_x$ . So, I write just moment what have I written here, it should be please correct this  $\cot \gamma_x$ . So, sorry, but you know this is now correct. So, I have the bases  $\cot \gamma_x$  and the altitude as this one. So, this must be  $\cot \lambda \sin \phi$  and hence from here if you if you multiply the equation I mean if you divide the expression by  $\cot \gamma_x$ ,  $\cot \gamma_y$ ,  $\cot \lambda$  let us see what we will get let me utilize this space.

So, first of all half cancels out. If half cancels out and we divide it by that thing we get from here  $\cot \lambda \cot \gamma_x$ . So, we are dividing this by  $\cot \gamma_x$ ,  $\cot \gamma_y$ ,  $\cot \lambda$  all of them. So, here only  $\cot \lambda$  will survive and I write  $\tan \lambda$  here because it is in the denominator. So, in the numerator I would like  $\tan \lambda$ , plus here  $\cot \lambda$  will vanish  $\cot \gamma_y$  will vanish and we will only have  $\tan \gamma_x$  surviving.

So,  $\tan \gamma_x \tan \gamma_x \cos \phi$  equal to. So, here in the same way  $\cot \gamma_y$  will survive. So, we will have  $\tan \gamma_y$  into  $\sin \phi$  therefore, we have established a relation between  $\lambda$   $\gamma_y$   $x$  and  $\gamma_x$  and  $\gamma_y$ , using this method and totally circumventing the method that we had learned the other day and this seems to be more straightforward extremely simple.

So, with this knowledge there will be some other exercises which I can offer you regarding you know mask line method, which I will be putting in the multiple choice

questions or sharing with you through some you know uploaded multiple choice questions, let us now move on to the next topic right.

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**TOPIC NAME**

Inclination angle of the main cutting edge is  $0^\circ$  and Orthogonal rake is  $7.5^\circ$ . Principal cutting edge angle =  $75^\circ$  and auxiliary cutting edge angle =  $15^\circ$ . Auxiliary inclination angle is (in degrees)

a. 5  
b. 7.5 ✓  
c. 2.5  
d. 3.74

$\lambda = 0^\circ$   
 $\gamma_0 = 7.5^\circ$   
 $\phi = 75^\circ$   
 $\phi_1 = 15^\circ$   
 $\gamma_0 = \lambda_1$

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A multiple choice question, what does it say? It says inclination angle of the main cutting edge is zero degree. So, we write the information which is given here is that is right we were discussing this particular problem. So, here first of all what is given is lambda equal to 0, orthogonal rake gamma o is equal to 7.5 degrees, principle cutting edge angle is 75, this is the main cutting edge and auxiliary cutting edge is having an angle of 15 degrees.

Once we have this auxiliary inclination angle; that means, what is the value of lambda dash this is the question. So, I know lambda I know gamma o, I know phi, I know phi one can I find out lambda dash. The answer to this question is very interesting, we need not go into any calculation whatsoever we can argue that since phi is 75 degrees phi 1 is 15 degrees.

So, this must be a right angle. If this is the right angle in that case it is given that the tool is having zero degrees along this, and the tool is having how much seventy 7.5 degrees along this direction. In that case this must be you know how did I find out 7.5? Because since this is the cutting edge, if this is the cutting edge orthogonal rake I mean orthogonal plane must be at right angles to it. So, we find that the orthogonal plane and the auxiliary cutting plane, they are becoming coincident. So, if that be so, orthogonal rake must be



equal to lambda dash, and hence auxiliary inclination angle or lambda dash is equal to 7.5 I just repeat lambda is given to be zero degrees.

So, along this direction it is zero degrees. Gamma o is equal to 7.5 degrees. So, gamma o is inclined this way 7.5 at 90 degrees, but as phi is 7.5 and phi 1 is 15 degrees hence this must be 90 and this must be the auxiliary cutting edge as well. So, this is the trace of the auxiliary cutting plane. So, orthogonal plane and auxiliary cutting plane they are becoming coincident and therefore, this angle gamma o must be equal to lambda dash, these must be equal and therefore, lambda dash is 7.5 I have a question for you. If I had given lambda to be nonzero, would the answer have been the same that is, is this redundant please think and then come to a conclusion and then we will share you know our opinions in some later lecture. So, let us go move on to the next one.

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**TOPIC NAME**

In a single point turning tool, the orthogonal rake angle is equal to the normal rake angle. In that case,

- The inclination angle must be necessarily zero
- The inclination angle must be necessarily non-zero
- The orthogonal rake must be necessarily zero
- None of these

Handwritten notes and diagram:

Diagram showing a single point turning tool with angles  $\gamma_o$  (orthogonal rake angle),  $\gamma_n$  (normal rake angle), and  $\phi$  (inclination angle). The rake surface is labeled. Handwritten equations include:

$$\tan \gamma_n = \frac{a}{x} = \frac{c}{x} \cos \lambda = \tan \gamma_o \cos \lambda$$

$$\tan \gamma_o = \frac{c}{x}$$

$$\tan \gamma_n = \frac{a}{x}$$

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Just one moment in a single point turning tool, the orthogonal rake angle is equal to the normal rake angle in that case the inclination angle must be necessarily zero, the inclination angle must be necessarily nonzero, the orthogonal rake must be necessarily zero none of these. So, first of all let us see what is given. We are given that sorry we are given that the orthogonal rake.

So, let us draw the relation between orthogonal rake and the normal rake. If you have orthogonal rake coming out this way, this is the reference plane this is the rake surface in the orthogonal plane, this is let me this is gamma o, in the same way the inclination angle

is creating another intersection, I mean not intention angle this is the incline in the normal plane. So, this is orthogonal plane this one is the normal plane this one is the rake surface etcetera. On the rake surface orthogonal plane has come straight vertical while the normal plane is likely inclined. Now if you look at the you know the planned view the rake surface comes has the cutting edge this way.

So, that if I go on you know reproducing the cutting plane on the rake surface, they will all come to be parallel and the cutting edge here makes right angles with the normal plane. This is the reference plane, in this particular view the sorry this is the orthogonal plane in this view the normal plane would not be visible.

So, we are looking at like this. So, in that case we have a small triangle formed here if we have a cut by the, you know cutting plane a small triangle will be formed here and in this small. So, this is the intersection that we are talking about this line  $x$  is produced here. So, in this line what is this particular angle it must be 90 degrees, because the cutting plane intersecting the rake surface will always produce intersections which are you know exact orientation of the cutting edge on the rake surface, these lines are all cutting edges.

So, if this is the cutting edge, this must be 90 degrees because the cutting edge is always 90 degrees to the normal plane. So, this is a 90 degree. So, let us give it some names  $a$  and  $c$ . In that case we can quickly define  $\tan \gamma$  equal to  $c$  divided by  $x$ , and  $\tan \gamma_n$  equal to what do we have here?  $e$  divided by  $x$  now what about this angle? This must be  $\lambda$  because the inclination angle and the rake angle they are having this particular angle between them sorry the normal plane I am always using wrong terminology the normal plane and the orthogonal plane are having angle  $\lambda$  in between them hence this must be  $\lambda$ .

So, we have a relation now between  $a$  and  $c$ , I can write finally, let me write in bold here  $\tan \lambda$  sorry where is this eraser right  $\tan \gamma_n$  equal to,  $e$  by  $x$  equal to now let us replace  $e$  this  $e$  must be you know component of  $c$ ,  $c \cos \lambda$  equal to  $c \cos \lambda$  by  $x$  and  $c$  by  $x$  is nothing, but  $\tan \gamma$ . So, we have this as  $\tan \gamma = \cos \lambda$ . Once this is proved we can tackle this question. Let us see the first one the inclination angle must be necessarily 0.

So, you know in a single point turning tool the orthogonal rake angle is equal to the normal rake angle the orthogonal rake angle where is it this one is equal to normal rake angle. So,  $\cos \lambda$  seems to be one if  $\cos \lambda$  is one then  $\lambda$  has to be a particular value now what's how much is that  $\cos \lambda$  is one. So,  $\lambda$  must be 0, but there is a big but there must be necessarily 0 not so. If the orthogonal rake and the normal rake they are both 0; that means,  $\tan \gamma_o$  is 0, whatever be the value of  $\lambda$   $\tan \gamma_n$  will be 0. So, they will be equal.

So, it is not correct if rake angle is orthogonal rake angle is 0, normal rake angle will be 0, but inclination angle might not be 0 it can be any other angle. So, therefore, the first one is wrong, the inclination angle must be necessarily nonzero this is also not correct because once again this might be having a value say  $\tan$  degrees. So, this will be having  $\tan$  degrees if this is 0. So, it cannot necessarily be nonzero. So, this can be zero this is also not correct the orthogonal rake must be necessarily 0 this is also not correct; orthogonal rake might be  $\phi$  degrees normal rake might be  $\phi$  degrees and inclination angle might be 0 therefore, this is also wrong hence we come to the conclusion none of these.

Actually it constitutes to this particular fact that is these angles are equal, either the orthogonal rake is 0 and the normal rake is 0 and the inclination angle is just anything or the inclination angle is 0 orthogonal rake is nonzero normal rake is 0 I sorry nonzero so, but they are equal or another case might be their orthogonal rake is 0, inclination angle is 0 and normal rake is 0. So, the answer to this question is none of these. So, I did have some other questions also, but time is drawing near let me see I think we can manage another one just one moment just one minute.

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**TOPIC NAME**

In a single point turning tool, the principal cutting edge angle is  $90^\circ$ . In that case

- a. The orthogonal rake angle and the back rake angle are necessarily equal
- b. The normal rake angle and the side rake angle are necessarily equal
- c. The orthogonal rake angle is necessarily zero
- d. The orthogonal rake angle and the side rake angle are necessarily equal

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In a single point turning tool, the principle cutting edge is 90 degrees in that case. So, let us draw of a figure of a tool this is it sorry this is it a tool of this type it is moving this way and this is 90 degrees that is it.

So, in that case the orthogonal rake angle and the back rake angle are necessarily equal now what is this mean? Orthogonal rake is you know 90 degrees to the cutting plane. So, orthogonal rake must be in this this section and the back rake angle is in this section. So, they are not going to be necessarily equal their completely different. The normal rake angle and the side rake angle are necessarily equal normal rake angle and the side rake angle. Normal rake angle we do not have any clue, because inclination angle is not given how much it is we do not know. So, this we cannot say they are necessarily equal, the orthogonal rake angle is necessarily 0.

Now, why should it be 0? We have not given any values. The orthogonal rake angle and the side rake angle are necessarily equal this is correct. So, we put sign like this this is correct. So, I think we have reached our limit. So, we will be I will put some I will discuss some questions in the third sixth lecture, but unfortunately you would not be having access to that for answering the first assignment of the first week. So, in that case I will upload them also so that, if you want to have a quick look at them you can do that in the first week itself.

Thank you very much.