

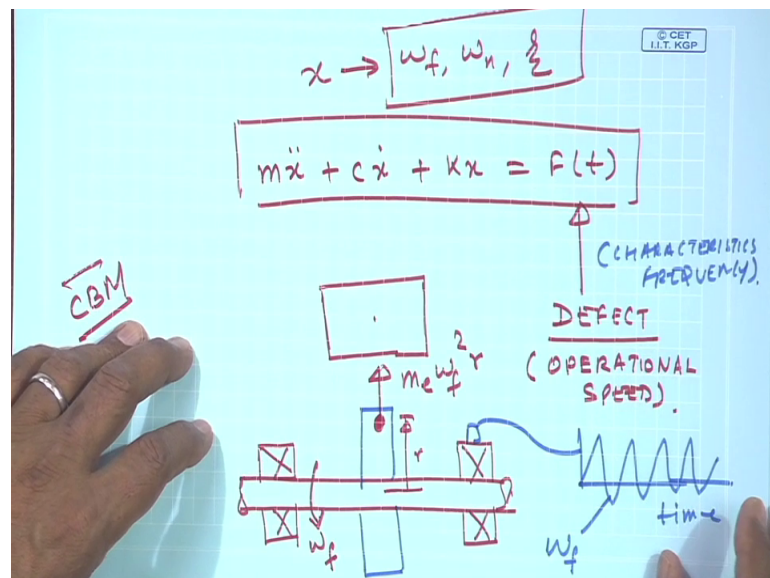
**Machinery Fault Diagnosis and Signal Processing**  
**Prof. A. R. Mohanty**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 07**  
**Free and Forced Response**

In the previous class I had just introduced you to a machinery vibration, and few takeaways from that was you know we discussed about degrees of freedom and then briefly introduce you into free response and forced response and the terms associated with a transfer function in terms of what is mobility, what is impedance; and then explained you or briefly showed you what experimental model analysis. And then towards the end we saw the effect of damping and then what are the different forms of damping, how damping can be measured in the laboratory.

But then there are few other issues in vibration is that is what we are going to discuss now, in this topic on Free and Forced Response of a System.

(Refer Slide Time: 01:17)



When we discussed this body I described it as a by a simple equation  $F(t)$ ; when in condition monitoring I have a body, now question is who is giving this force? So, this force is because of an defect and usually the frequency of this force defect is depending on it is operational speed of the machine. I will give you an example: imagine I have a shaft, it is supported on bearings and this shaft is carrying a disk which could be a pulley

or a set of vanes and imagine it is a big amount of unbalanced mass for some reason because, maybe this it is a set of vanes and something has got stuck in the vane so and this is rotating with a frequency of  $\omega$ .

So which is unbalanced mass was  $m e$ , this would be the force  $m e \omega^2 r$  where  $r$  which is distance from center  $r$ . So, if I was putting a transducer on this bearing. it would capture the response on this bearing. But this response was at this bearing will be predominantly at a frequency  $\omega$ , which is the operational speed of the shaft or the rotor; rotational speed of the shaft.

So, what I demo wanted to prove to you is usually this defects occurs at the operational speed of the machines or sometimes they are also known as characteristic frequencies. And if this response  $x$  will be dependent on  $\omega$   $\omega_n$  damping in the system. So, now in machinery condition monitoring of CBM, all we have is access to  $x$  we may have some information about  $\omega$ , we may have some knowledge about  $\omega_n$  and damping. So, it is indirectly from measuring  $x$  or measuring  $x$  at many instances, we will find out whether a defect as occurred or not.

So, this is what we are going to see. But then there are many instances where I have discussed this for a case of a linear system.

(Refer Slide Time: 05:04)

A handwritten differential equation on a grid background, representing a torsional system. The equation is  $J\ddot{\theta} + C\dot{\theta} + k_t\theta = T$ . Below the equation, three terms are labeled with arrows pointing to them:  $J\ddot{\theta}$  is labeled "ROTATIONAL INERTIA",  $C\dot{\theta}$  is labeled "TORSIONAL DAMPING", and  $k_t\theta$  is labeled "TORSIONAL STIFFNESS". A small circle with a minus sign is written below the labels. In the top right corner, there is a small logo that reads "© CET I.I.T. KGP".

But the same vibration will also hold true for a Rotational system some external torque  $t$ . So, this is the rotational inertia this is the Torsional sorry ct Torsional damping and this is the Torsional stiffness and  $\theta$  is the response in angular domain.

The theory which we understand in a linear vibration is also holds true and like we had linear dampers we will have Torsional dampers, particularly we will see in many of the ships etc or many systems. Where there is a large propeller shaft rotating to reduce the oscillations in the  $\theta$  diamond, people have Torsional put Torsional dampers in the system and now here we will now introduce you to two other concepts, one is this base excitation.

(Refer Slide Time: 06:26)

### Base excitation

- Excitation is applied to the support or base.
- The inertia force is due to mass,  $m\ddot{x}_2$
- Spring force is equal to  $k(x_2 - x_1)$
- Damping force is  $c(\dot{x}_2 - \dot{x}_1)$

• When summing up the forces, we get

$$m\ddot{x}_2 + c(\dot{x}_2 - \dot{x}_1) + k(x_2 - x_1) = 0$$

If  $x_1 = X_1 \sin(\omega t)$   
Assuming solution of the form  $x_2 = X_2 \sin(\omega t - \phi)$

On substitution and solving for  $X_2$ , we get

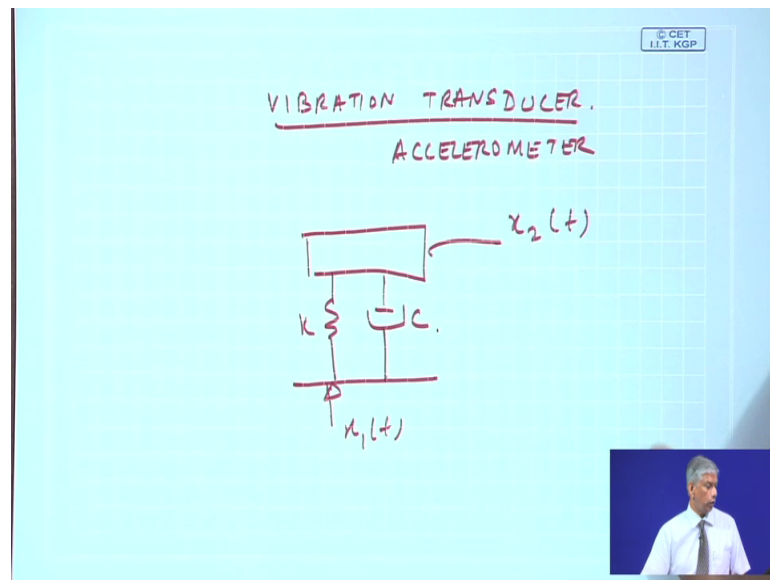
$$X_2 = X_1 \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{[1 - r^2]^2 + (2\zeta r)^2}} \quad \text{----- (12)}$$

The diagram shows a vertical mass-spring-damper system. At the bottom is a box labeled 'Base' with a double-headed arrow and  $x_1(t)$  indicating its vertical displacement. Above the base are a spring (k) and a damper (c) connected in parallel. A mass (m) is attached to the top of the spring and damper. A double-headed arrow next to the mass indicates its displacement  $x_2(t)$ .

Suppose I have a body of mass  $m$  supported on stiffness  $k$  and damper  $c$  and it is subject to the motion  $x_1$  at the base. So, this body  $x_2$  will react with a response  $x_2$ .

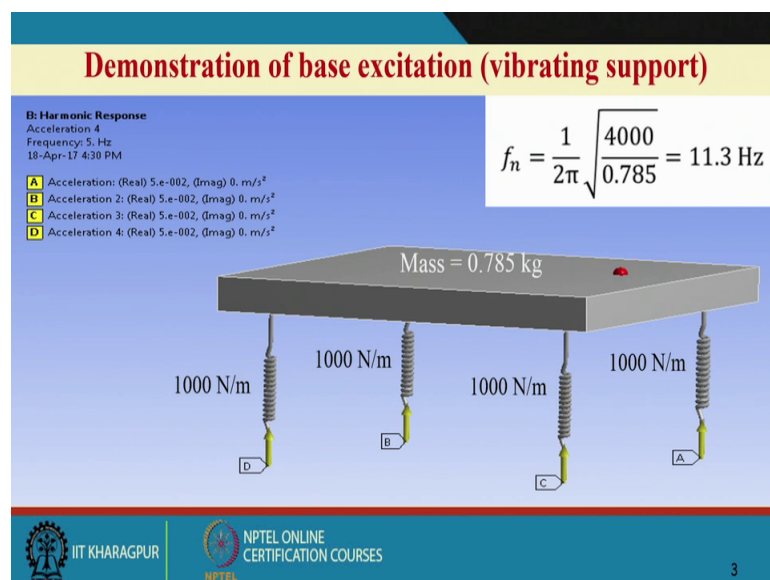
So, if I look into the equations of motion of this, we will get something this expression. Now we will see the response  $x_2$  is proportional to  $x_1$  times this term and this is the equation behind the basics of vibration transistor like an accelerometer.

(Refer Slide Time: 07:17)



So, there is some motion in the base, how this body responding is what we are going to look into. So, you will see in vibrations you will always come across this term, you know  $1 + 2\zeta r$  whole square  $1 - r^2$  plus  $2\zeta r$  square and square root ok. Many a times in you know force excitation base excitation vibration isolation you will come across these terms and we will see what needs to be done.

(Refer Slide Time: 08:12)

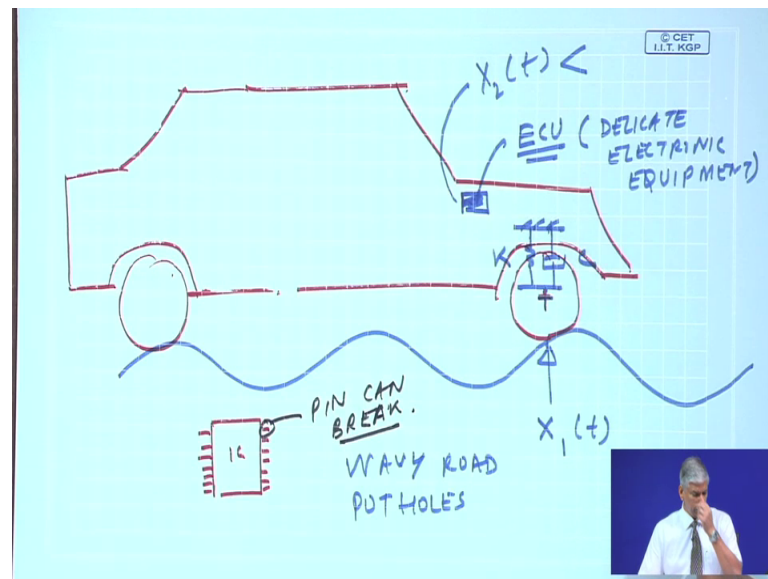


This is just to give you an example, where is a body of mass  $m$  supported on 4 springs and then they are all in and then we have this mass of 0.78. So, effective  $k$  and then we

can find out the linear natural frequency and certain conditions were given of acceleration to this body and then you will see how this will react.

So, this is you can be numerically done and then we can solve it and we will give you an example wherein this scenarios will come in many cases for example, I have a Car.

(Refer Slide Time: 09:02)



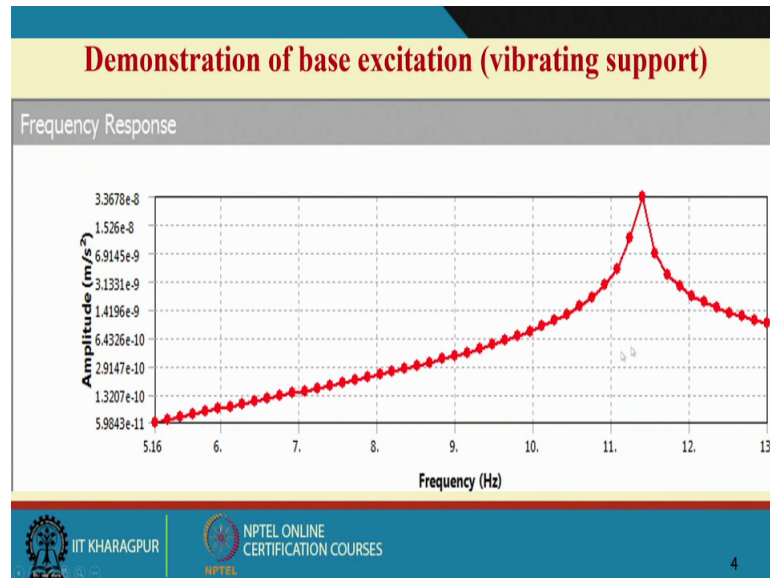
I am not drawing the entire car, which is here suppose I have 1 electronic module which is the engine control unit ECU; it is an very delicate electronic equipment, imagine this vehicle is now going on an undulating road ok.

So, this vehicle suspension system you know there is if you look into the suspension system, there is certain stiffness and damping present in the system and it is there in all the wheels, I am just showing you 1 wheel. So, any motion coming from the road because of the road profile there could we have wavy road or road full of pothole? So, as a designer you have been asked to optimize the stiffness and the damping in the suspension system. So, that the motion here  $x_2(t)$  is the least. So, that those delicate equipment is not filling, imagine I will give you an example for example, we have you know small pins in the ECU; if we look at an ECU on an IC chip there are small pins coming out.

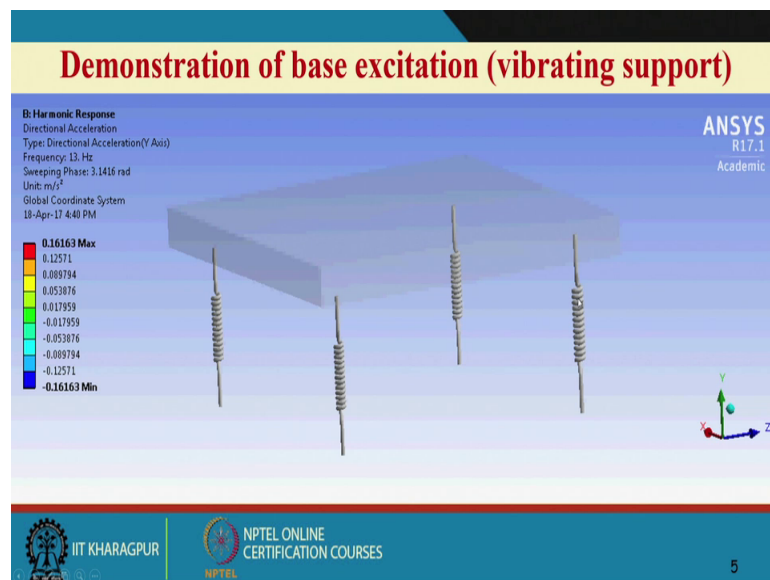
Imagine if this chip which is the resident on this board here, if it is subjected repeated vibrations there may be a fatigue failure and this on the pin can break. So, we will see an

example how what is the allowable motion of this and so on. And this was the response of the previous case where in at resonance the amplitude suits of.

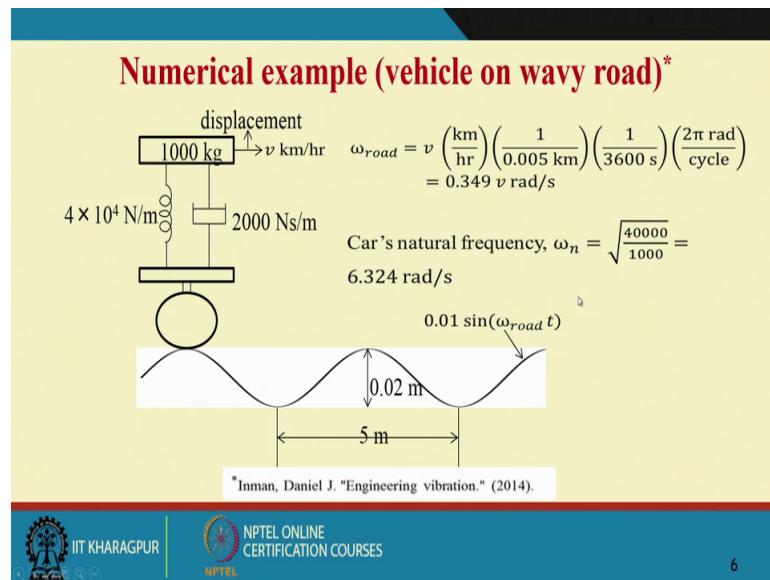
(Refer Slide Time: 11:46)



(Refer Slide Time: 11:50)



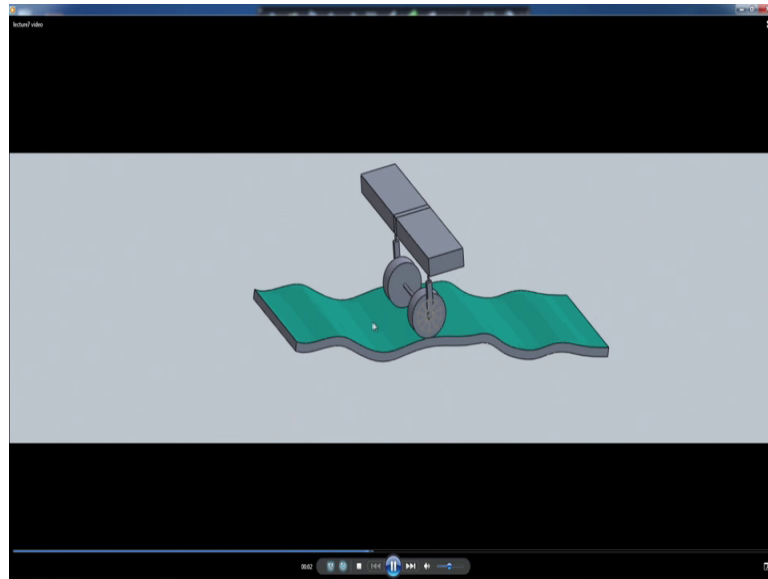
(Refer Slide Time: 11:52)



This is the example which I was going to show you this is from the book by Inman, there is a wavy road and the distance between these valleys is 5 meter and the peak to peak amplitude is 0.02 meter.

If the vehicle is going at certain  $v$  kilometers per hour and the vehicle has a mass of thousand kgs and effective stiffness of  $4 \times 10^4$  Newton per meter and damping of 2000 Newton second per meter, the cars natural frequency in  $\omega$  and is given by this, but look at the base excitation. The base excitation in the right unit is 0.01 is the amplitude  $\sin \omega_{road} t$ , your  $\omega_{road}$  is dependent on the velocity  $v$  right. So, vehicle on a wavy road prior to going to this, I will show you an example wherein we will see how this looks like.

(Refer Slide Time: 13:13)



So, this is a vehicle going on an wavy road and you will see if my ICU was here, what kind of motion it is being subjected to and imagine if you are going in a wavy road and if your vehicle ICU subjected to such large motions, it will lead to any fatigue failure and this is what. So, we can design by effectively deciding on the  $k$  and  $C$  of such a system. So, you know you could have a different road profile, you could you know this example we have taken a sine wave it could have been portals or it could have been a trapezoidal waves. In fact, in actual proving grounds in a test tracks, which I will show you there are many such a road scenarios wherein people. So, we will continue on this problem.

(Refer Slide Time: 14:12)

**Numerical example (vehicle on wavy road)**

If car is running at  $40 \frac{\text{km}}{\text{hr}}$ ,  $\omega_{road} = 0.349 \times 40 = 13.96 \text{ rad/s}$   
Now,  $r = \frac{\omega_{road}}{\omega_n} = \frac{13.96}{6.324} = 2.207$   
 $\zeta = \frac{2000}{2\sqrt{4 \times 10^4 \times 1000}} = 0.158$

According to Eq. (12), car's deflection will be  
$$0.01 \times \frac{\sqrt{1 + (2 \times 0.158 \times 2.207)^2}}{\sqrt{[1 - 2.207^2]^2 + (2 \times 0.158 \times 2.207)^2}} = 0.003 \text{ m}$$

*What if car's speed is 18.11 km/hr ?*

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES | NPTEL

7



See if a car is running at 40 kilometers per hour  $\omega$  here comes on to 13.96 radians per second and  $r$  is equal to  $\omega$  road which is the external force in frequency by  $\omega$  and it comes out to be 2.207 and  $\zeta$  is equal to this and if you go to the very first equation, we will see if this is my  $x_1$  I substitute this. So, this is the displacement of 0.003 meters a couple of takeaways from this expression.

(Refer Slide Time: 14:55)

$$x_2 = x_1 \sqrt{\frac{1 + (2\zeta r)^2}{[1 - (2\zeta r)^2]^2 + (2\zeta r)^2}}$$

$\zeta$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$m = 1000 \text{ kg}$   
 $m = 10000 \text{ kg}$

$k, c$   
 $x_2 < 0.003 \text{ m}$

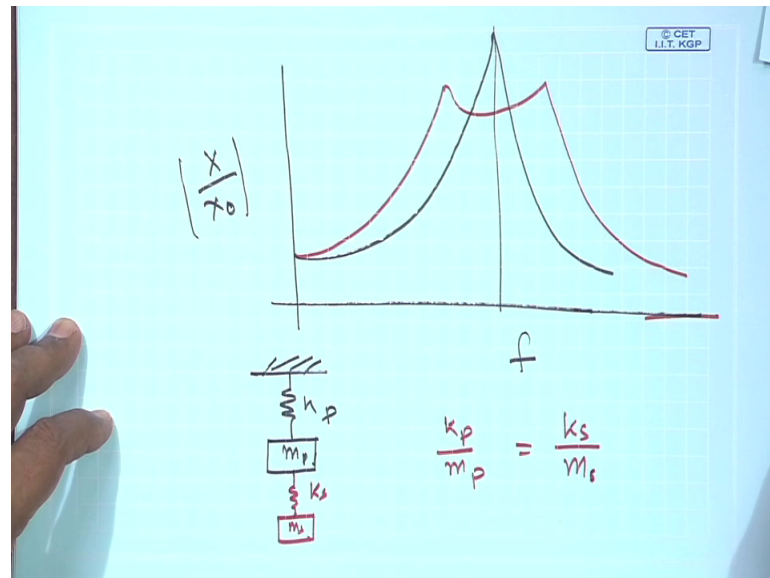
Here  $x_2$  is  $x_1$  times  $1 + 2\zeta r$  whole square and square root or I can refer you to the previous equation.

This is the expression here so in the case of the numerical example  $\zeta$  is known to us  $r$  is known to us and then  $x_1$  is known to us we will find out  $x_2$ . So, as a vehicle designer you will see that if we can play around with  $\zeta$ , I can decide on the damping in the dampers of the suspension system to play around with the stiffness  $k$  I will and your natural frequency  $\omega$  and root over  $k$  by  $m$ , I do not have any goal on this scope to change mass, but I can change  $k$  and I can change  $c$  and for every any given scenario I can find out  $x_2$ .

So, as a numerical example all of you could do is we took  $m$  is equal to 1000 kg for the same road; say for example, if you are going on a bus here maybe  $m$  is you know 10 times that 10000 kg you will see that  $x_2$  will be significantly less than 1.003 meter and all of you would have realized, going on a wavy road on a large heavier bus or a heavier vehicle you will not realize this kind of displacements.

So, this is just to give you an example as to how I can play around with the damping and the stiffness to control the response. Now another example which we will come across is what is known as a tune mass damper system by now you get a feeling.

(Refer Slide Time: 17:25)



That whenever resonance occurs there is a peak in the amplitude. So, a Tune mass damper system is such that I can attach another body. So, this is my primary mass I can attach a secondary mass, where  $K_p$  by  $m_p$  is equal to  $K_s$  by  $m_s$ .

(Refer Slide Time: 18:19)

**Tuned Mass Damper (TMD)**

- TMD for a single dof system:** amplitude of vibration of a single dof system may be minimized by adding additional sdof system.

$$\text{amplitude of main system, } X_1 = \frac{F_0 (K_2 - M_2 \omega^2)}{(K_1 + K_2 - M \omega^2)(K_2 - M \omega^2) - K_2^2} \quad \text{----- (9)}$$

amplitude of 'M<sub>1</sub>' is zeros if  $F_0 (K_2 - M_2 \omega^2) = 0$  ----- (\*)

if  $K_2$  and  $M_2$  are selected according to Eq. (\*), it acts as a dynamic vibration absorber.

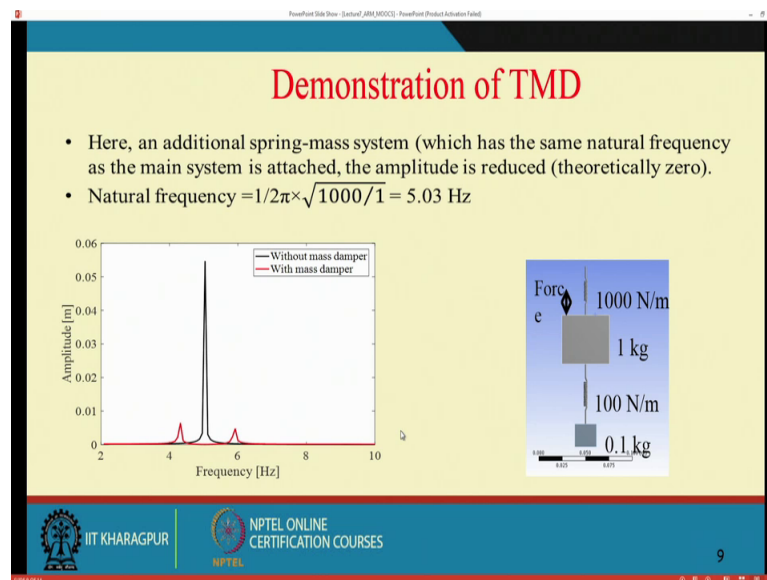
IIT KHARAGPUR

NPTEL ONLINE  
CERTIFICATION COURSES

8

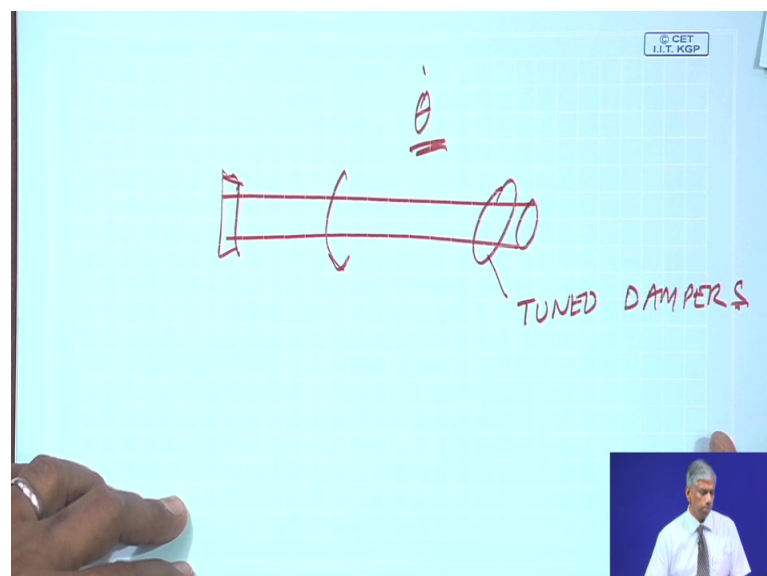
So, that the now it becomes a 2 degree freedom system. So, this amplitude will reduce in natural frequency shift and this has many practical examples in which you will see and it is for a single mass system, where we can work on the expressions here and this is another example without mass damper.

(Refer Slide Time: 18:44)



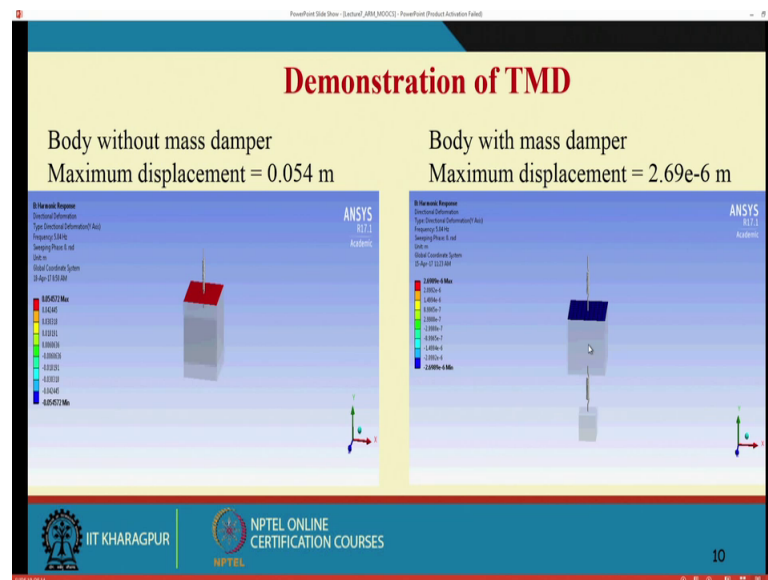
You see the natural frequency is shooting up with mass dampers, the amplitudes have come down and frequencies have shifted. So, this is a very easy way to control vibrations and resonances many a times in.

(Refer Slide Time: 19:05)



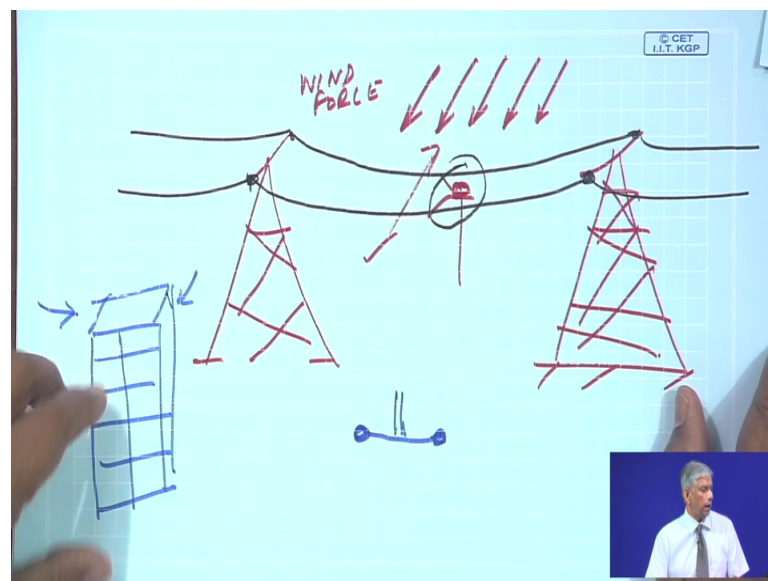
Personal systems particularly in drive lines of vehicles, you know when there are excessive Torsional vibrations  $\theta$  or  $\ddot{\theta}$  people have attached Torsional tuned dampers always as a fix to reduce this  $\dot{\theta}$  ok. And, I will give you an example also which is known as this is the demonstration here.

(Refer Slide Time: 19:37)



If which you can see, but I will give you another example you would have seen.

(Refer Slide Time: 19:47)



Height transmission towers power lines and then there are electrical catenary cables, now imagine if and then this continuous and so on with another pole and so on. Imagine if

there was a large wind force. So, what would happen these electrical conductors would sway in the wind and it so happen their amplitudes of oscillation will be so high that 1 may touch against another.

So, some sort of a touching can occur and then you can see the consequence of a major contact of these 2 conductors. So, to prevent these vibrations of these cables, because of large oscillations what 1 does is 1 uses what is known as the stock damper, it looks something like this and they attach it to this ends like this kind of a dumbbells.

(Refer Slide Time: 21:22)

**Tuned Mass Damper (TMD)**

- A tuned mass damper is a device that is attached to a structure in order to reduce its response.
- The frequency of TMD is tuned to a particular structural frequency.
- Tuned mass dampers are widely used in transmission line, automobiles, and buildings.




Figure: Stockbridge damper

IIT KHARAGPUR | NPTEL ONLINE CERTIFICATION COURSES

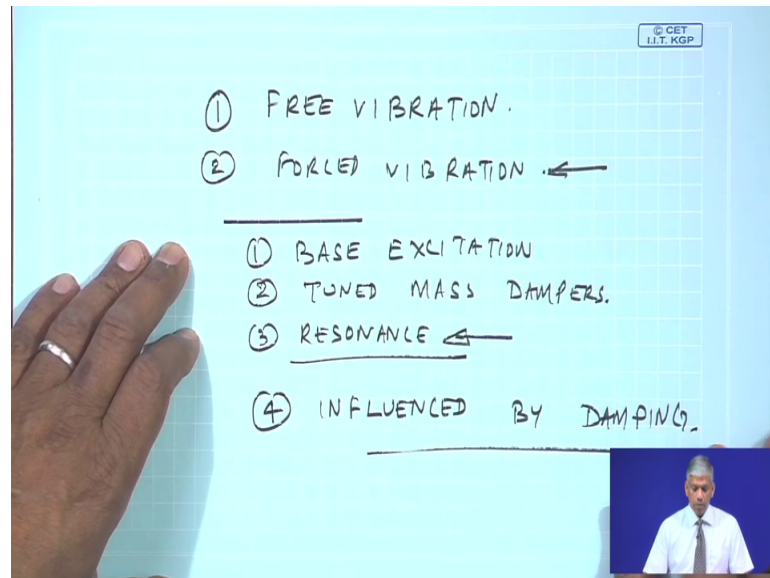
11

Now, if you look here this is which Stockbridge damper, sometimes known as a moose damper; it looks like the horns of a moose. So, there is a mass here each 1 of them and you can see an electrical cable here and this is actually put onto the main conductor at 1 of these pores.

So, in a effect what happens because of a large wind force, the cable would not vibrate rather these would vibrate at the resonance frequencies and these are easily design for frequencies you know which is predominantly when somebody talks about wind forces, predominantly frequencies anywhere from 8 hertz to over 30 hertz or 40 hertz. So, tune mass dampers this is one example, other one was I was giving you the example of a driveline of an automobile, even you would have seen in large tall skyscrapers. If because of large motions, they want to move if these motions are not desirable we can have tuned mass dampers attached to such units ok.

So, as I was telling you vibrations have many effects and if I was to list down I was you know this free vibration.

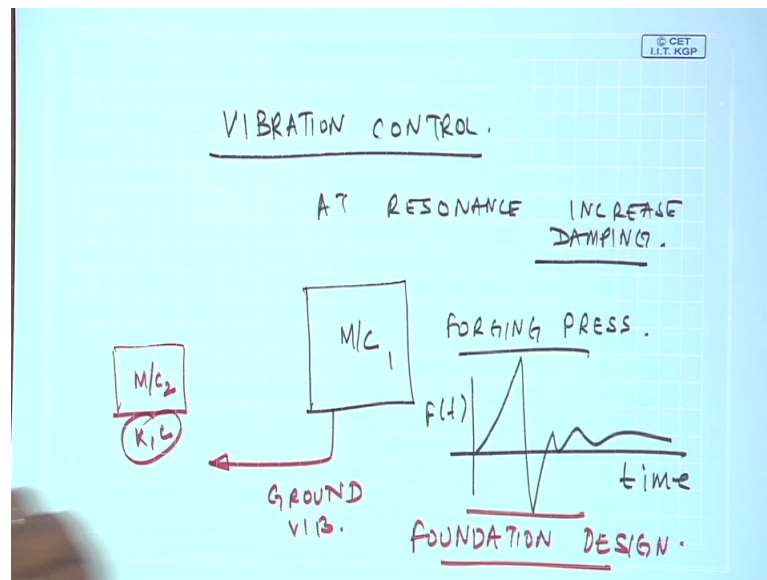
(Refer Slide Time: 22:52)



Free vibration occurs when there is no external disturbance or there is an initial disturbance, but it is this forced vibration which is giving an energy because, there is an external force and the frequency of this external force is responsible for making the system run at that external frequency or the forcing frequency and particularly in CBM, whenever we do the vibration measurements from any machine we will see the frequency of this force vibration being reflected in the measure time history and then few other cases I was this base excitation. Then we have the case of the tuned mass dampers, of course the condition of a resonance.

So, we have to avoid resonance at all costs and all of them are influenced by damping, but then we will see as I was telling you vibrations is undesirable, but what if vibrations does exist. So, we need to see how can I control the vibration?

(Refer Slide Time: 24:37)

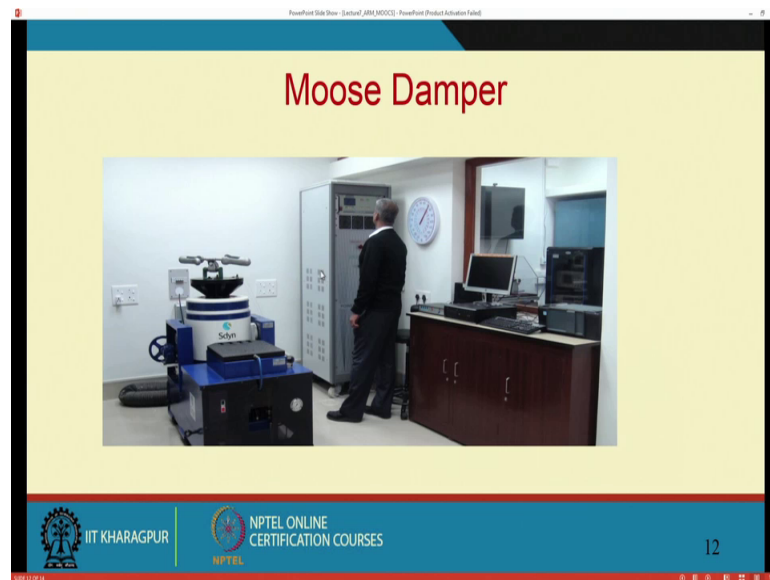


So, at resonance we can increase damping to reduce the vibrations, but we will see in many cases what if there is the machine like a forging press or hammer. So, each time the forging press comes down, it gives an impact force like this ok.

Now, imagine this force will get transmitted to the ground and it will influence another machine 2. So, this ground vibration of this machine has to be reduced by increasing the force by reducing the force. So, we need to play around with again with  $k$  and  $C$  and that is what is known on the effective foundation design. And in the next class we will talk about vibration and shock isolation.

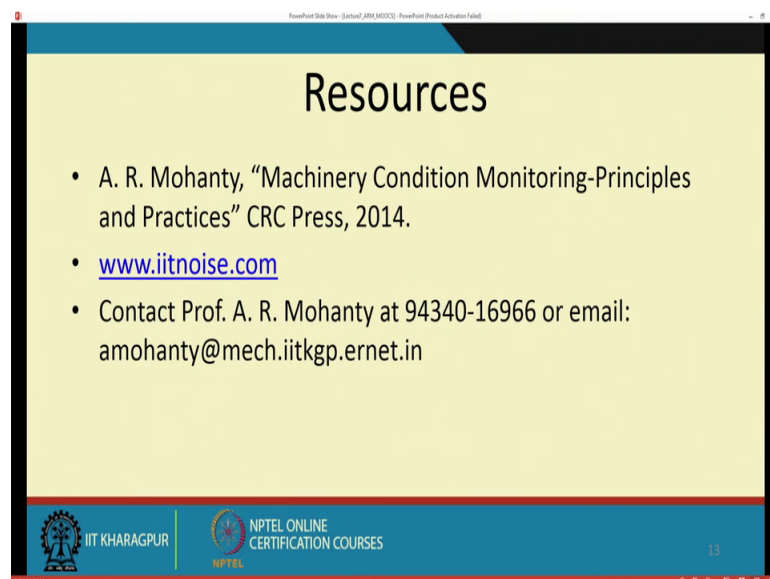


(Refer Slide Time: 26:16)



So, this is a view of this moose damper being tested in our laboratory, where we have a electromagnetic shaker wherein we can generate forces. And then see we will later on show you a video of this as well.

(Refer Slide Time: 26:33)



Thank you.