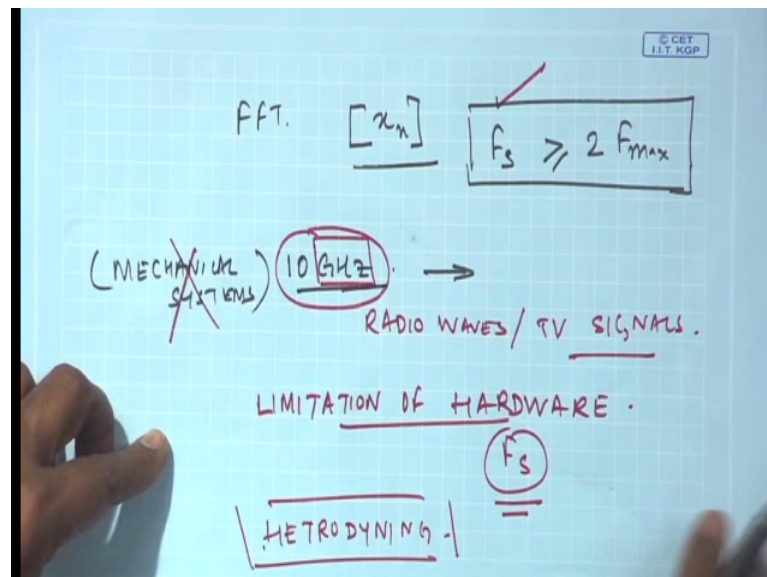


**Machinery Fault Diagnosis and Signal Processing**  
**Prof. A. R. Mohanty**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture – 24**  
**Signal Heterodyning**

Hello well, so far we have discussed various aspects of signal processing and you would have realized that the frequency content in a signal is very important to relate to the characteristic faults in a mechanical system. Well in this lecture we are going to find out what are the methods to find out the frequency contained in a signal other than the conventional FFT and one being actually this signal heterodyning which is useful for, a technique which is useful for finding out frequencies in very very high frequency signal, where actually we are not able to sample the signal at that higher frequency. So, we will see how signal heterodyning helps, because if you realize if you want to numerically do FFT I need to have an array of number  $x$  and this array or this signal has to be sampled at a particular sampling frequency  $F_s$  where  $F_s$  should be equal to twice  $F_{max}$  as per the Shannon sampling theorem.

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But question is when we have a signal so for example, 10 gigahertz for example, a very very high frequency signal which is not true in mechanical systems, we do not have such signals in mechanical systems, but perhaps some electromagnetic waves like you know

radio waves TV signals etcetera very high frequency signals. So, question is if my signal is at such high frequency I need to have a sampling frequency higher than that.

So, this is the limitation of the traditional hardware in finding out such high sampling frequencies. So, it is not possible to get cards or data acquisition systems with the high sampling frequency, they are not available because the sampling times intervals are so small. So, there is another method which is signal hetrodyning which we will discover or try to understand in this lecture, but before that let me try to kind of quickly tell you one of the other methods available to determine the frequency of a signal.

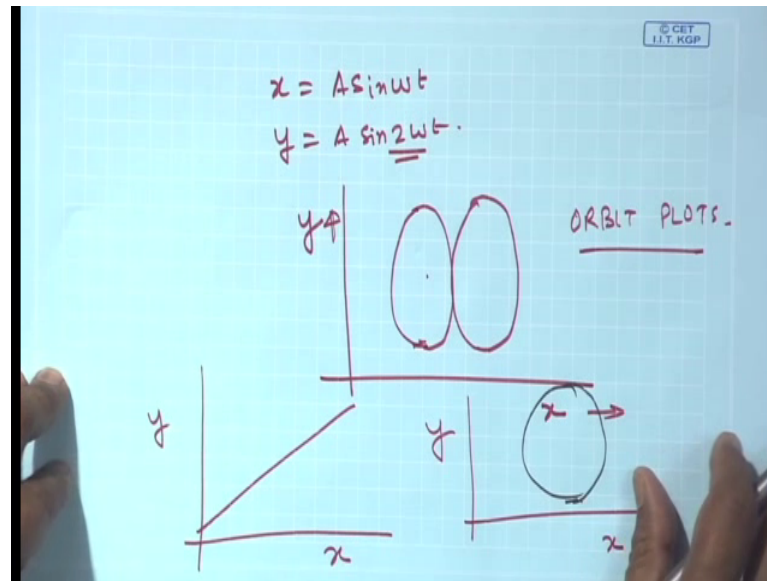
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**Methods to determine frequency of a signal**

- Comparison (Orbit Plots)
- Time Period estimation
- FFT
- Filtering
- Hetrodyning

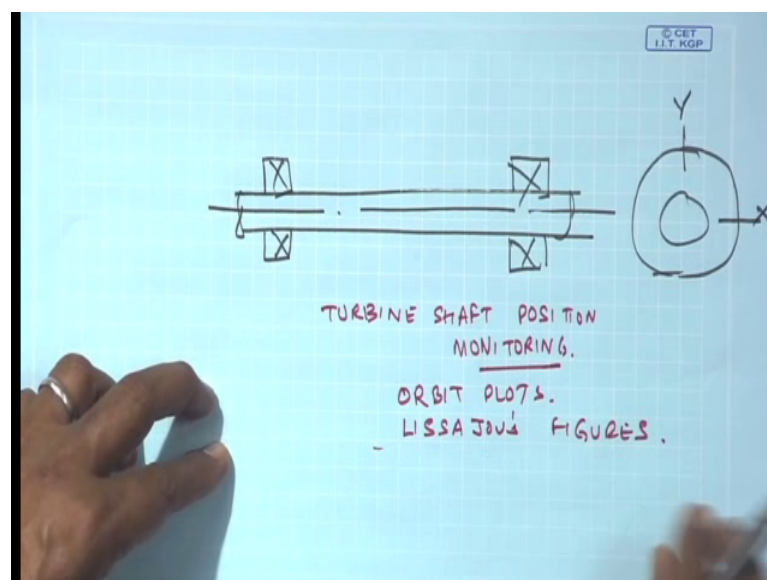
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You would have seen if I compare known signal say for example  $x$  is equal to  $A \sin \omega t$  and  $y$  is equal to  $A \sin 2\omega t$  and if I plot  $x$  versus  $y$  we will see that there will be loops and so on. So, if one signal is known the other signal can be founded and so on. This kind of orbit plots we have seen, if  $x$  is equal to  $y$  I will get a straight line like this, if  $x$  and  $y$  are 90 degree phase difference I will get a plot like this.

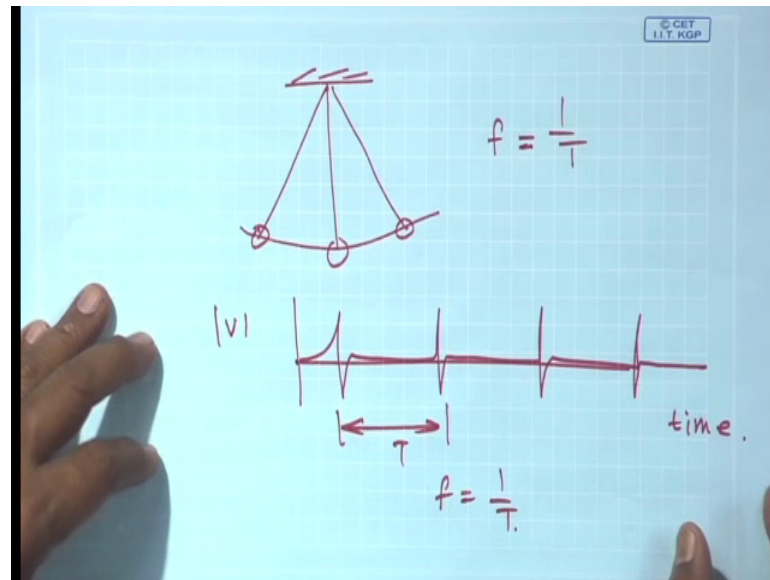
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So, this kind of orbit plots we have seen are helpful in finding out the displacements or amplitudes in a shaft, so which is rotating supported on bearings and if I look at this end

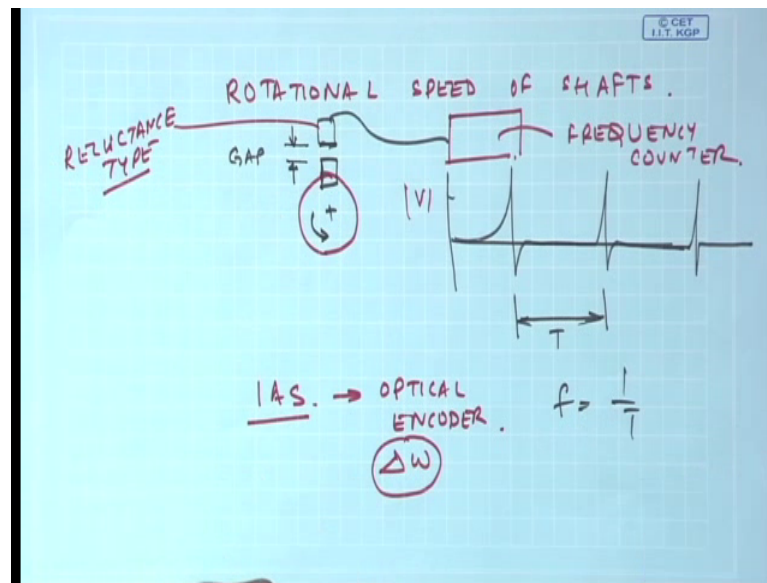
view here, if I measure an x here with the transducer y with the transducer I can see if x and y are same or different my ploy just plotting the orbit plot. So, physically for turbine shaft position monitoring such plots are used orbit plots or they are also in the one of lissajous figures.

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Well another very easy we are finding out the frequency content of a signal if it is a very freely slowly moving phenomenon. For example a pendulum which is you know oscillating, just measuring the time period of oscillations how many swings it passes by in a given time I can find out frequency is nothing but 1 by T. Many a times in many slowly changing events just by measuring the change in the signal maybe takes, this is in time at some voltage, so inverse of this time period. In fact, in the industry will you see you will see that many times we need to monitor the rotational speed of shafts how do you do that.

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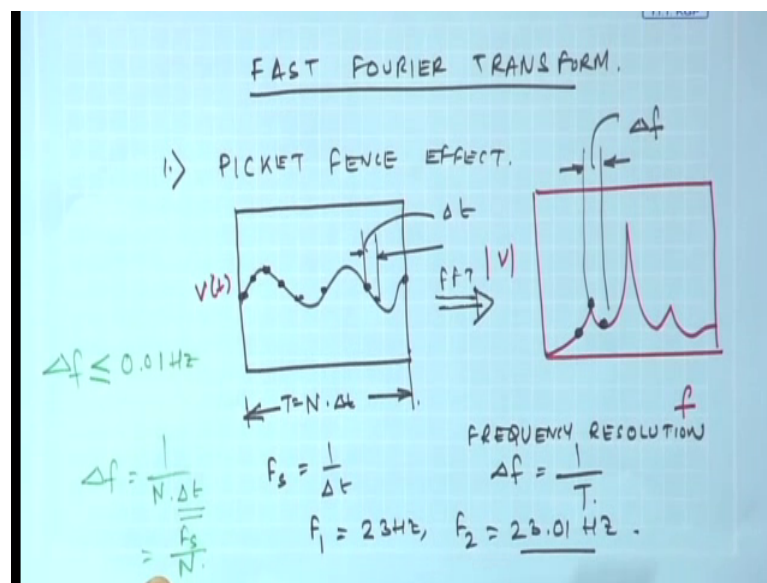


So, for example, a shaft is rotating essentially a shaft has a key shaft is rotating. So, if I put a probe here or a reluctance probe, which senses the gap between voltage produced is proportional to this gap here. So, every rotation of the shaft I am going to get a pulse. So, this again the inverse of the time taken or just by having a an  $f$  is nothing but  $1$  by  $T$ , just by having a frequency counter I can get the rotational speed of the shafts.

In fact simple device is used in many industries where online we can on the run measure the rotational speed, but then this gives you an average rotational speed. Later on we will we have seen how through this optical encoders we had find out the instantaneous angular speed using optical encoders. The difference between an optical encoder and such probes is the fact that in the optical encoder the sensitivity is so high that within a rotation it has a small change in the speed I was able to sense it, but in such reluctance type pick up because of the change in the gap the magnetic flux changes.

Thus we will have a voltage corresponding to this change voltage induced voltage and by sensing this voltage I am able to get this peaks and the inverse of this peaks is nothing but the time period and that is what we have got here at the rotational speed. So, inverse of the time period is method to measure the rotational speed when phenomena are very slowly changing you know you can just have a stopwatch and measure the events, how frequently they are changing and so on.

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Of course, the next technique is FFT. By now we have you all must be knowing for this fast Fourier Transform is I will come to this filtering and heterodyning a little later, but let me tell you 2 most possible errors in FFT because whatever be it. My purpose is to accurately estimate the frequency contained in a signal and the amplitude at that corresponding frequencies.

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## POSSIBLE ERRORS IN FFT

- Picket Fence Effect (Inadequate Frequency Resolution)
- Leakage Error (Windowing)

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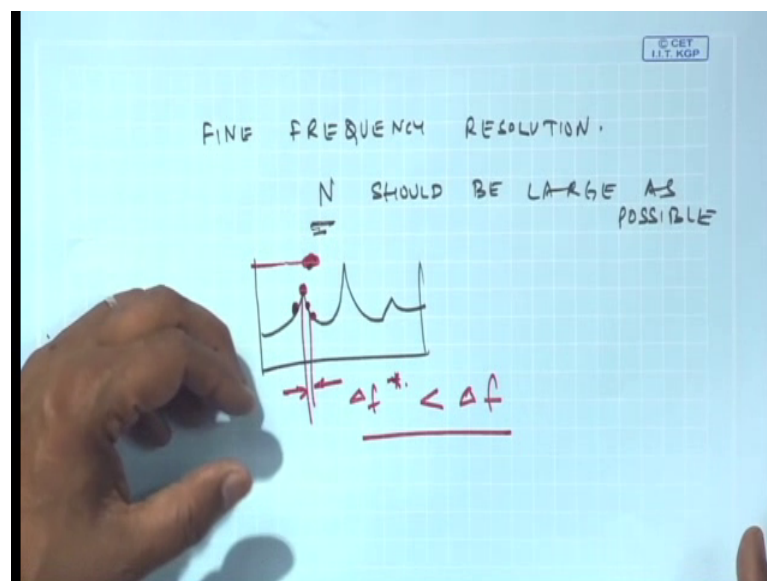
Now, to do that if I do some error in my frequency estimation I will have a problem. The first one is what is known as the Picket Fence Effect. This happens because of inadequate

frequency resolution let me show you by an example. So, this is my time window of some signal so, I have taken  $N$  points where our  $T$  is equal to  $N$  times  $\Delta t$   $N$  number of points and the time interval between any such point is  $\Delta t$ , where  $F_s$  is nothing but  $1/\Delta t$ .

If I do an FFT I will look at the magnitude of the frequency spectrum and  $e f$  is some voltage in time domain this is a voltage absolute value I may get some signal like this, where the spacing between 2 successive point is  $\Delta f$ , where the frequency resolution  $\Delta f$  is nothing but  $1/T$ .

Now, question is in a practical signal, suppose my signal has a frequency  $F_1$  is 23 hertz and  $F_2$  as 23.01 hertz. So, in such a case my  $\Delta f$  has to be  $\Delta f$  should be at least less than 0.01 hertz right, but you see  $\Delta f$  is nothing but  $1/N$  times  $\Delta t$ .

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If my sampling frequency is fixed  $F_s$  I can only increase  $N$  so that means, to have fine frequency resolution your  $N$  should be large as possible. There are of course, there are limitations why  $N$  cannot be large because of the memory requirement because every data point requires a storage space or a memory space. So, I cannot make  $N$  very large because that will be limitation of my storage space.

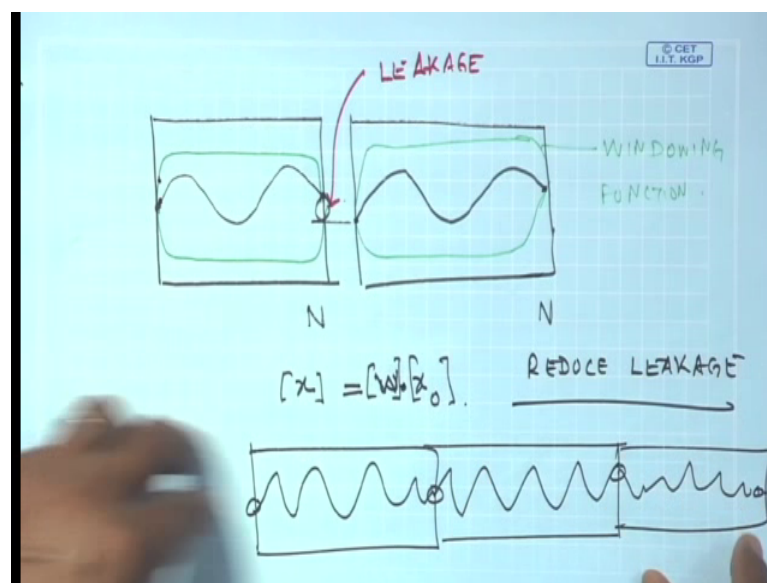
So, this  $\Delta f$  in if it has to be fine I need to make  $N$  larger and thus have adequate frequency resolution so that I can represent a signal at its peak, otherwise you know I



may miss if the peak is somewhere here I will miss it. I am sampling here, here I may have miss this peak and wrongly represented his as this, right amplitude was here because I did not have this was not available to me in  $\Delta f$  star is something much less than  $\Delta f$ , so I am missing this peak and that is what is known as this picket fence effect.

So, to only way I can do for a fixed  $F_s$  is increase  $N$  and we will see in the subsequent class an example how through improving the  $N$  I can have  $\Delta f$  to very very high.

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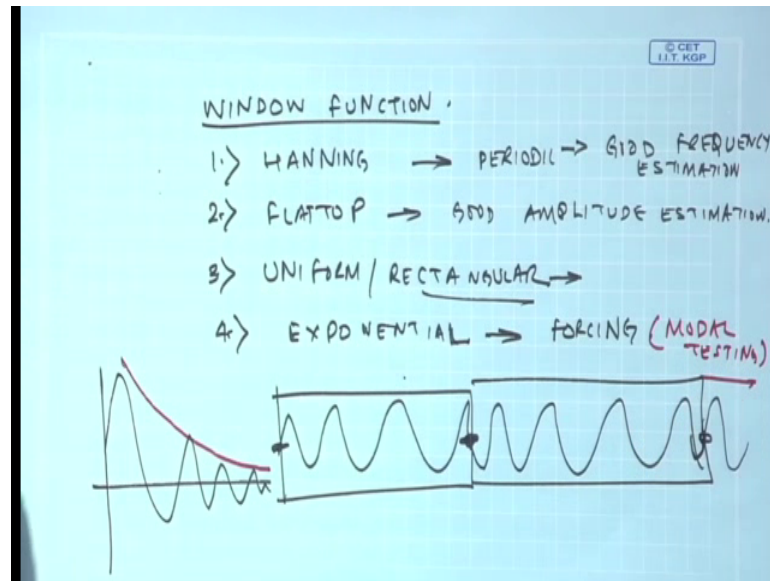


Now another possible error in FFT is what is known as a leakage error see for example, when we assumed or did FFT on a window block, we took a window of  $N$  data points, now signal like this the next signal would assume. So, if I bring this signals closer I will have maybe this kind of a gap, so this is what is known as a leakage.

To reduce leakage what we do we multiply the signal  $x$  with a windowing function, so that windowing function so that the signal  $x$  which we go for FFT is actually dot product with the window function times the  $x$  original,  $x$  original is my sample signal. So, we again so that I multiply  $x_i$  to this window function, so it becomes 0 at the end and thus I have what is known as the continuous signal. So, this continuity is maintained and so on and thus reduce leakage.



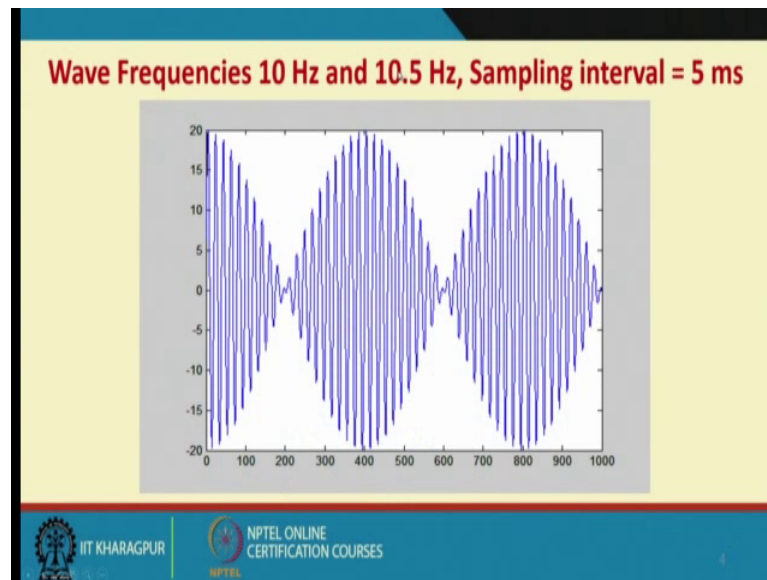
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There are many window functions available. So, that like the handing window, flattop uniform or rectangular window or the exponential window. Now usually for periodic signal for good frequency estimation such signal is used, this is for a good amplitude estimation and uniform rectangular window is actually a no window when signal itself has no continuity like nice sine wave, there is no need to have, because this table end at 0 and start at 0. So, nothing is going to go wrong. In such case you cannot you need not apply any window and exponentially is used for forcing functions or where response is dying out with time, for example such a. So you can use an exponential window usually for model testing we will discover discuss this later on.

So, to reduce errors in FFT one is the picket fence effect we need to have adequate frequency resolution that can be done by increasing the number of data points you sample and to reduce leakage errors or to have a right estimation of the amplitude or the frequency of the signal we need to window the signals.

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Now, I will come to one example wherein we will see when 2 frequencies are very very close by, you can see that the it is the phenomena where the amplitude actually increases and decreases and this is what is known as beating. Now even if this frequency is a very very high, then what happens if the difference is very small I will have this amplitude beating, but it is increasing and decreasing, but once the signal frequency is match what is going to happen I will have no longer increasing or decreasing of the signal, but a same signal.

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**Beat Frequency and Heterodyning**

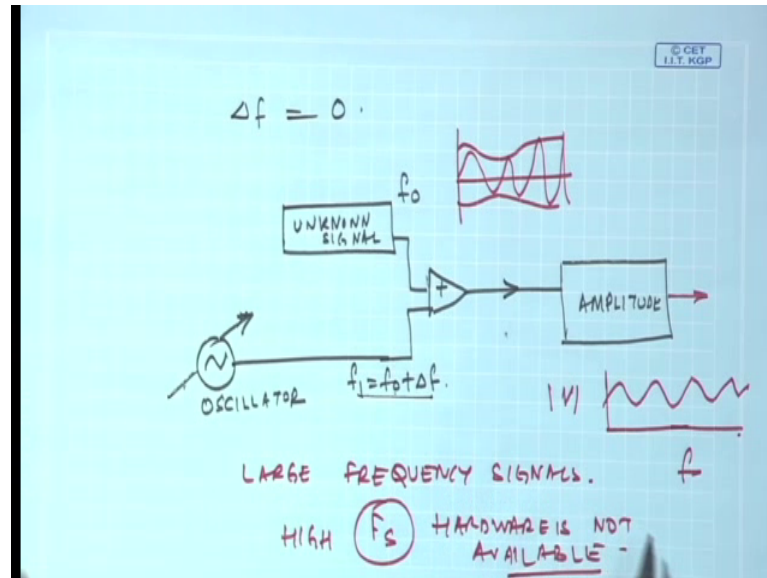
$$y = A \sin(2\pi f_0 t) + A \sin[2\pi(f_0 + \Delta f)t]$$
$$= 2A \cos(2\pi \frac{\Delta f}{2} t) \sin(2\pi \frac{f_0 + f_1}{2} t)$$

Signals are independent of each other |

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So, this is taken care of in signal heterodyning for example, if I have a signal  $f$  and another signal  $f + \Delta f$  at a small change in frequency I will get a signal like this  $\cos \Delta f t$  and  $\sin f + \Delta f$  by 2.

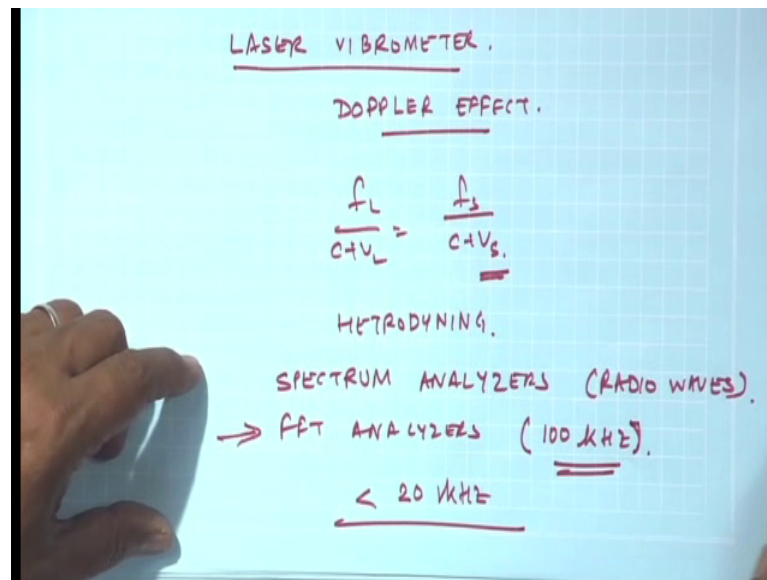
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Now, if this  $\Delta f$  becomes 0 I will just have a plain simple harmonic signal. This is taken care in an method to find out an unknown signal so for example, I have a very very high frequency unknown signal, here I will have an oscillator and then I will sum it up and look at the amplitude of the signal. So, this is my original signal  $f$  and I will create a signal here where  $f_1$  is nothing but  $f + \Delta f$ . So, I will be changing the signal frequencies here in this oscillators and generate the signal.

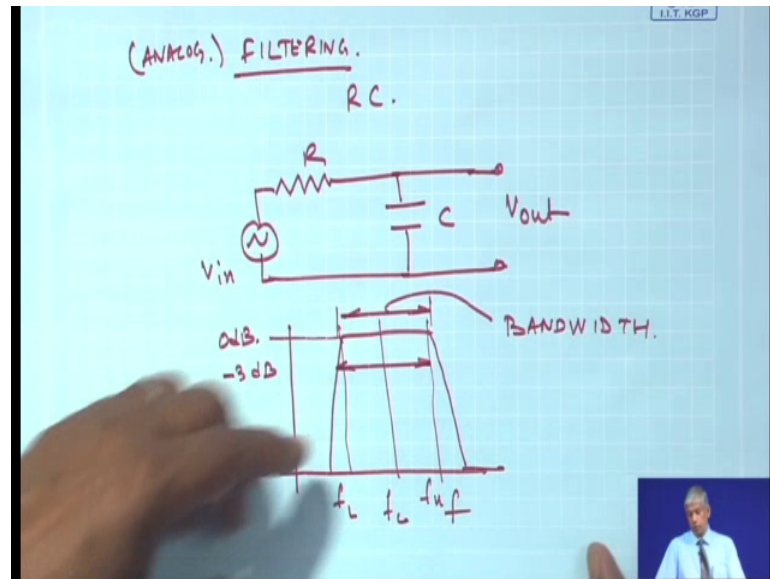
So, whenever  $f_1$  becomes equal to  $f$  whatever  $\Delta f$  becomes 0 and then I will have a single. So, I would be normally having a beating phenomena and but then once heterodyning has then I will. So, such heterodyne and lasers are used to find out frequencies of large frequency signal or high frequency signal because as I was telling right in the beginning when  $F_s$  is hard whereas, high  $F_s$  is not possible. One can do such heterodyning analyzers. Exact example I will tell you, even we are going to study later on about laser vibrometer. Where laser vibrometer works in the principle of you know Doppler effect and laser frequencies are very very high frequency.

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So, such no I can find out the velocity of the source, depending on the frequency of the incident wave and the reflected wave and these 2 values are very very high. So, conventional FFT will not work. So, I can use a signal heterodyning an optical heterodyner, to find out if 1 is known I can find out the other. So, that is known as the Doppler shift frequency. Once I know the Doppler shift frequency I can find out the velocity of whatever is your source or target it is vibrating and this principle is used in estimation of TV signals on such analyzers are actually known as spectrum analyzers for radio waves.

Our conventional FFT analyzers may be today the maximum bandwidth we can have with such analyzers is 100 kilo hertz ,though mechanical systems were happy with signals less than 20 kilohertz. That is the conveyor FFT analyzers and then we will see in the next class how FFT analyzers can be used to demonstrate to you different signals. (Refer Slide Time: 25:32)



Of course, another technique of finding out frequency content of signal is through filtering. I will name it as analog filter, so the combination of RC circuits, I can get some output depending on R and C I can design filters and I can have series of such filters and then for any signal I can have a filter, which will have certain bandwidth and then this is the floor and  $f$  upper I will have a central frequency and this is 0 d B.

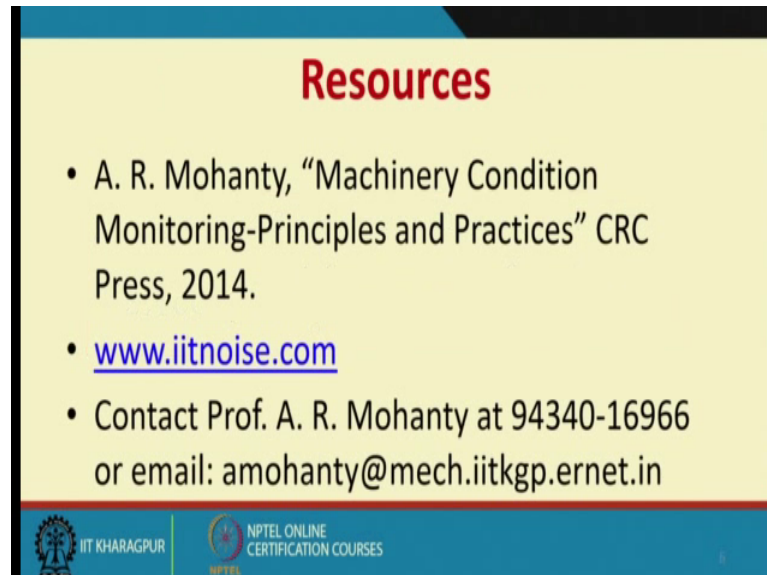
So, they sometimes say this as minus 3 d B. So, any signal which goes into a filter, will come out depending on what kind of frequency were contained in such bandwidths. So, we will discuss about filtering later on in a separate class on the low pass filters, band pass filters, high pass filters, notch filters etcetera and then we will see how such signals can be analyzed through filtering through analog feelings.

So, to summarize today we have discussed the different techniques of finding out the frequency contained in a signal, the first being the orbit plot or the lissajous figures and of course, a simple way of estimating the time period and the inverse of the time period this was the frequency.

We have seen the errors which could happen in FFT like in a picket fence or the windowing and how signal heterodyning can be used to find out the frequency content of signals which are very very high frequency, where the conventional FFT analyzers will not work and such analyzers which work on the principle of heterodyning are known as the spectrum analyzers. Which are used for analyzing signals by a high frequency signals like the TV signals and radio signal and of course, we had and just mention that analog

filters can also be used or a series of such filters could be used for inter to find out difficulty content of a signal.

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The slide is titled "Resources" in red text. It contains a bulleted list of three items: a book by A. R. Mohanty, a website URL, and contact information for Prof. A. R. Mohanty. The slide has a yellow background with a blue header and footer. The footer contains the IIT Kharagpur logo and the NPTEL Online Certification Courses logo.

## Resources

- A. R. Mohanty, "Machinery Condition Monitoring-Principles and Practices" CRC Press, 2014.
- [www.iitnoise.com](http://www.iitnoise.com)
- Contact Prof. A. R. Mohanty at 94340-16966 or email: amohanty@mech.iitkgp.ernet.in

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Thank you.