

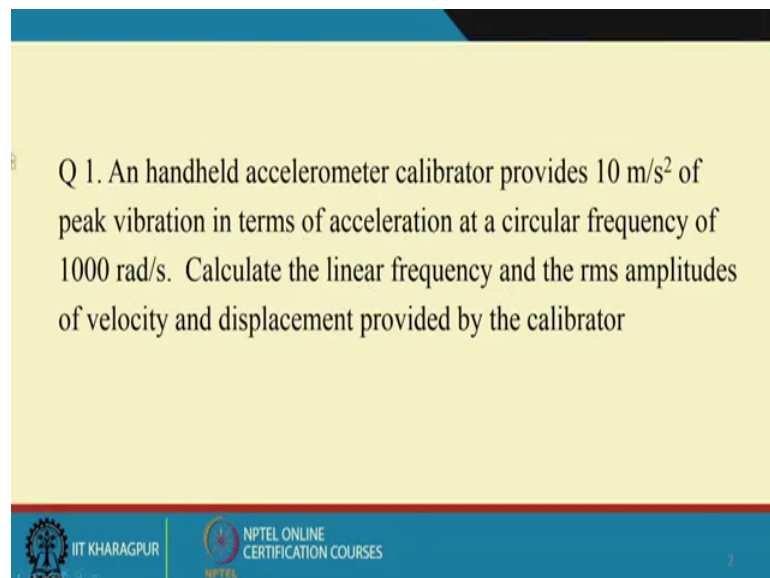
Machinery Fault Diagnosis and Signal Processing
Prof. A. R. Mohanty
Department of Mechanical Engineering
Indian Institute of Technology Kharagpur

Lecture – 23
Numericals in Signal Processing and Data Acquisition

Well, in this week we are discussing certain examples using Matlab and the earlier classes we have seen how Matlab can be effectively used for signal processing and so on. So, in this class I will be discussing on certain numericals on signal processing and specifically on data acquisition. So, just to give you some brief background and practice on what kind of problems we can encounter both in signal processing and data acquisition.

This lecture I will be focusing predominantly on data equation and perhaps in these subsequent lectures I will be demonstrating to you how signal processing is been d1 and certain rudiments of the signal processing in terms of the time domain data, data points, memory storage, number of lines of FFT this is what we are going to see as well.

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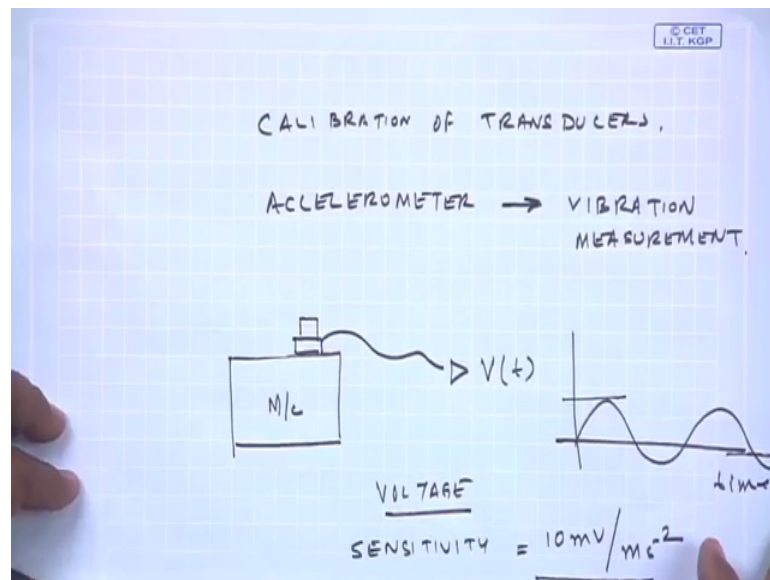


Q 1. An handheld accelerometer calibrator provides 10 m/s^2 of peak vibration in terms of acceleration at a circular frequency of 1000 rad/s . Calculate the linear frequency and the rms amplitudes of velocity and displacement provided by the calibrator

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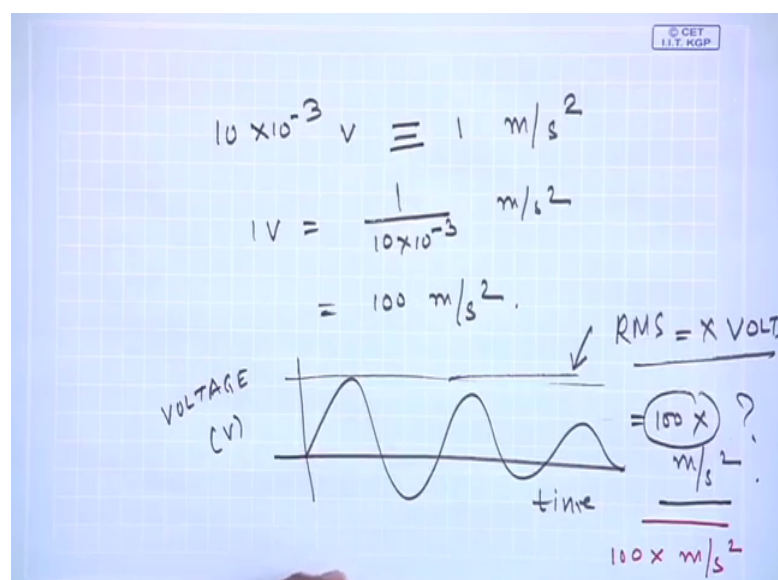
So, if you go to the very first problem we have studied vibration so far, but many a times what happens that in the field, because you know this relates to calibration of transducers.

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We will see later on that accelerometers are used for vibration measurement, but as you would have seen when I have a machine if I put a transducer I get some certain signal voltage as a function of time, looking at this voltage level depending on the sensitivity of the transducer say for example, this sensitivity is 10 millivolts per meters per second square there is a typical sensitivity of an accelerometer.

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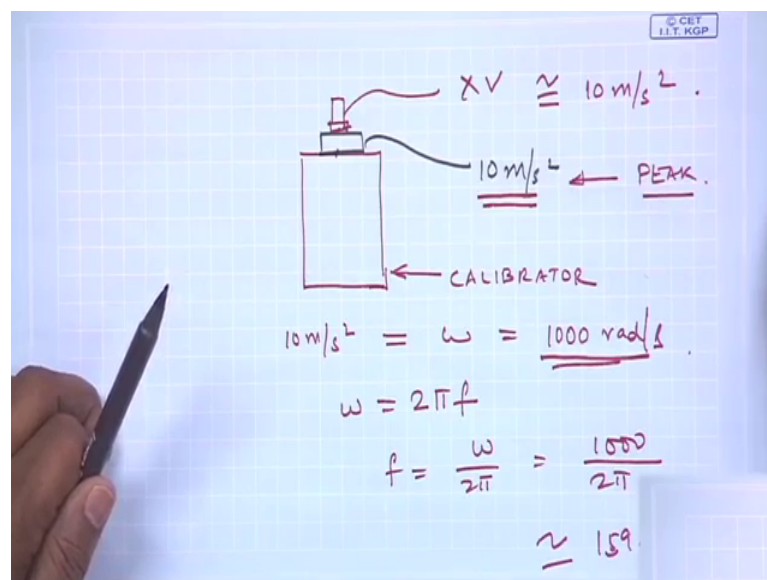


So, whatever voltage I get I know it corresponds from for example, out of the transducers 10 millivolt or 10 to the power minus 3 volt corresponds to 1 meters per second square.

So, 1 volt corresponds to 1 by its 100 meters per second square. So, in my readout unit which is in voltage say v I get I can measure some value maybe you know from signal processing the RMS value is some x voltage.

So, I know 1 volt corresponds to 100 meters per second. So, x volts corresponds to 100 times x meters per second square. The reason now this whether this is correct or not how do I know whether this value which is 100 x meters per second square is correct or not is a problem which I will be encountering because the data which we relied on was the sensitivity, this sensitivity here. Now this sensitivity is given by the manufacturer, so when we look into the specification sheet of the transducer we took this value and we assume that this is the correct value, but that may not be always correct.

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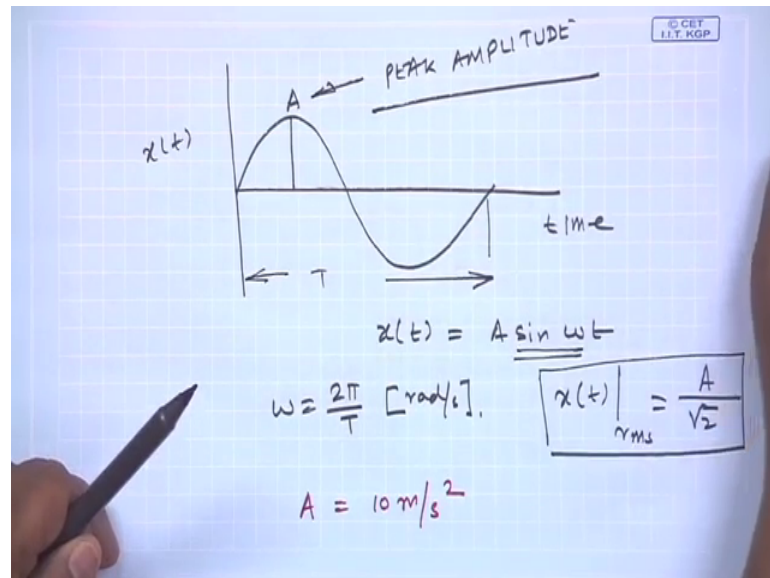


So, to ensure that such mistakes do not happen in the field actually there is an calibrator, this is a calibrator which gives right here on this platform a value of 10 meters per second square. So, any accelerometer which I use to measure the vibration whatever x volt it gives I know this x volts corresponds to 10 meters per second squared. So, this problem here is relating to a calibrator which gives an acceleration of 10 meters per second square at ω is equal to 1000 radians per second as you know ω is a circular function frequency so ω is $2\pi f$.

So, f is ω by 2π and this becomes and this is corresponds to about 159.2 hertz. So, such commercial collaborators are used in the field which are known as the handheld

accelerometer calibrator, which will give you a known level of 10 meters per second square at a frequency of 1000 radians per second. The question is, what is the RMS amplitude of velocity and displacement provided by the calibrator. Now, question is this provides a peak value, so this is peak and the answer is Austin amplitudes of velocity and displacement theory of vibrations, you will realize.

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Suppose I am looking at the displacement as a function of time say certain x t and this value is A . So, x t is equal to A sine ω t , where till the time period the ω is nothing but 2π by T this is in radians per second and A is the peak amplitude. Of course, you know for such a way we can find out the RMS value also, you know x t rms value you will see it is A by root 2 for a harmonic wave like a sine wave, but that is not what the question is, question is we have been given with the peak vibration in terms of acceleration as 10 meters per second square. So, given to us is the value of A which is 10 meters per second squared, no the question who is asked is what is the RMS amplitude of velocity and displacement.

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$$x(t) = A \sin \omega t$$

$$\dot{x}(t) = \frac{d}{dt} x(t) = A \omega \cos \omega t$$

$$\ddot{x}(t) = \frac{d^2}{dt^2} x(t) = -A \omega^2 \sin \omega t$$

$$|x(t)| = A \quad \leftarrow \text{DISPLACEMENT}$$

$$|\dot{x}(t)| = A \omega \quad \leftarrow \text{VELOCITY}$$

$$|\ddot{x}(t)| = A \omega^2 \quad \leftarrow \text{ACCELERATION}$$

So, if I have a signal $x(t)$ as $A \sin \omega t$ the displacement at the velocity $x(t) \times \dot{x}(t)$ is nothing but d/dt of $x(t)$ which is nothing but $A \omega \cos \omega t$ and similarly the acceleration $x(t) \times \ddot{x}(t)$ is you know d^2/dt^2 of $x(t)$ which is nothing but minus $A \omega^2 \sin \omega t$.

So, if I look at the amplitudes of $x(t)$ it is A , the velocity amplitude is $A \omega$ and the acceleration amplitude is nothing but $A \omega^2$. Displacement, this is velocity and this is acceleration. So, but so you see here if 1 is given the other is actually either a multiplication by ω or a division by ω and so on. So, from $A \omega^2$ I want to get $A \omega$ I divided, from $A \omega$ I want to get A I will divide it by ω or subsequently from displacement now I want to get velocity I will multiply. So, this is very useful.

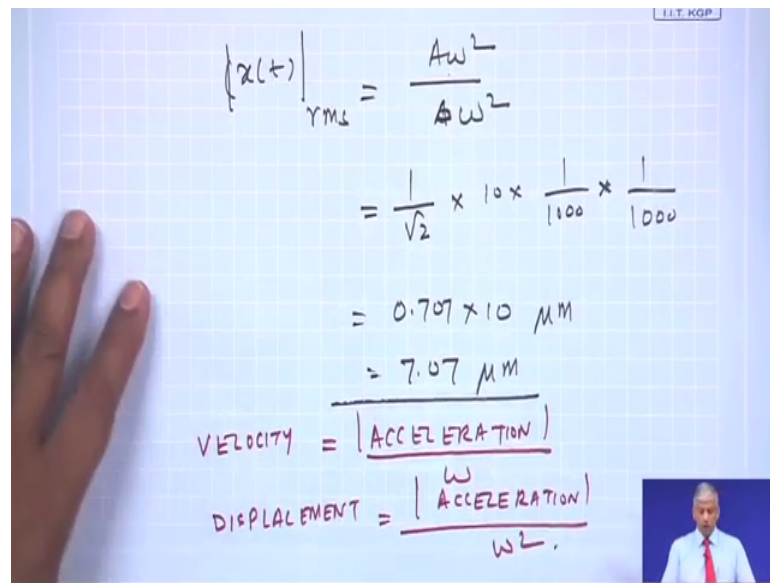
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$$\begin{aligned} |\dot{x}(t)|_{\text{rms}} &= \frac{1}{\sqrt{2}} A \omega \\ |\ddot{x}(t)|_{\text{rms}} &= \frac{1}{\sqrt{2}} A \omega^2 \\ |A \omega^2| &= 10 \text{ m/s}^2, \quad \omega = 1000 \text{ rad/s} \\ |\ddot{x}(t)|_{\text{rms}} &= \frac{1}{\sqrt{2}} \times 10 \times \frac{1}{1000} \\ &= \frac{1}{\sqrt{2}} \times 10 \text{ mm/s} \\ &= 0.707 \times 10 \text{ mm/s} \\ |\ddot{x}(t)|_{\text{rms}} &= 7.07 \text{ mm/s} \end{aligned}$$

Now, the question of the because these are all harmonic waves, so this will be nothing but the RMS amplitude would be x t rms would be nothing but 1 by root 2 times A omega and next double of t rms would be 1 by route 2 A omega square, but so A is given to us as 10 meters per second squared and omega is 1000 radians per second. So, just by substitution x t rms would be 1 by root 2 times 10 times 1000 .

So, this will be I am sorry a second we have been given acceleration I have made a mistake here in the sense I am been denoting your displacement as A , but I am writing this as acceleration so I will see this. Make a correction here, this is the acceleration amplitude. So, A omega square is given as this, so to get x dot t I will divide this by 1 by 1000 right. So, this becomes 1 by root 2 times 10 by 1000 this was in meters per second square. So, this is 10 millimeters per second, so this is 0.707 times 10 millimeters per second. So, this becomes 7.07 millimeters per second, so x t rms is the first answer.

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The image shows a hand pointing to handwritten calculations on a grid background. The calculations are as follows:

$$\begin{aligned} |x(t)|_{\text{rms}} &= \frac{A\omega^2}{\omega^2} \\ &= \frac{1}{\sqrt{2}} \times 10 \times \frac{1}{100} \times \frac{1}{1000} \\ &= 0.707 \times 10 \text{ } \mu\text{m} \\ &= 7.07 \text{ } \mu\text{m} \end{aligned}$$

Below the calculations, two formulas are written in red ink:

$$\text{VELOCITY} = \frac{\text{ACCELERATION}}{\omega}$$
$$\text{DISPLACEMENT} = \frac{\text{ACCELERATION}}{\omega^2}$$

A small inset video of a man in a white shirt and red tie is visible in the bottom right corner of the grid.

Now, the second is x t rms the displacement is nothing but $A \omega^2$ by A by ω^2 . So, this is $A \omega^2$ is 10. So and this is 1 by root 2 because it is rms and this is peak times 10 times 1 by 1000 times 1 by 1000. So, this becomes 0.707 times 10 microns or 7.07 microns.

So, from this we see that the by knowing 1 we can get the other. So, acceleration divided by ω is nothing but velocity and displacement is nothing but acceleration divided by ω^2 and we have used this. So, this becomes a very handy tool in the field once we have this handheld calibrated. So, whenever I get some x voltages I can know whether this x voltage corresponds to actually what. So, sometimes what happens in the field the manufacturer sensitivity would have changed and this kind of example of this kind of calibrator helps you do the field calibration.

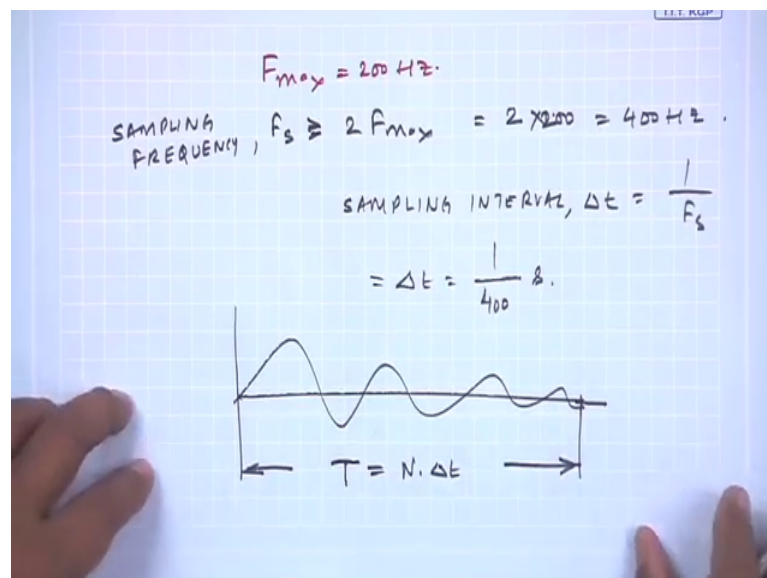
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Q 2. A pure tone signal of 200 Hz is to be digitally acquired by a 16 bit computer aided data acquisition system, what is the minimum amount of onboard RAM memory required to store the digital data in the DAQ system so that a frequency resolution of 1 Hz can be obtained by performing frequency analysis on the acquired time domain signal ?



Now, we will move over to the second problem this is relating to the data equation and a little bit of signal processing a pure tone signal of 200 hertz is to be digitally acquired by a 16 bit computer aided data acquisition system, what is the minimum amount of onboard RAM memory required to store the digital data in the data acquisition system so that the frequency resolution of 1 hertz can be obtained by performing frequency analysis on the acquired time domain signal.

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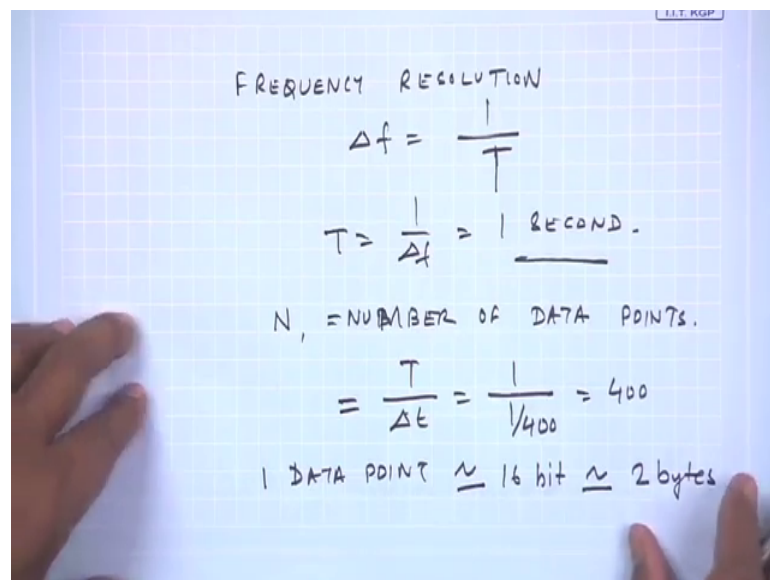


So, the signal frequency the maximum frequency of the signal F_{max} is 200 hertz. So, from my Nyquist sampling theorem the sampling frequency should be greater than equal to twice F_{max} . So, the least would be 2 times 200 that is 400 hertz, so my sampling

frequency this much. So, what is my sampling interval, sampling interval Δt is nothing but inverse of the sampling frequency.

So, in this case Δt is equal to 1 by 4000 of a second. Now some signal is to be acquired, so the question is how many data points I require, so the total time is nothing but N times Δt . So, when is the number of (Refer Time: 15:40) sometimes there is a relationship with the ramp consider N as 1 here or N as 0 here, so the (Refer Time: 15:47) this will be N or N minus 1, so that thing keeping aside that, so I will have this many number of this is my total time, now there is a relationship between frequency resolution and total time, frequency resolution Δf is nothing but 1 by total time.

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FREQUENCY RESOLUTION

$$\Delta f = \frac{1}{T}$$
$$T = \frac{1}{\Delta f} = \underline{1 \text{ SECOND.}}$$

N , = NUMBER OF DATA POINTS.

$$= \frac{T}{\Delta t} = \frac{1}{1/400} = 400$$

1 DATA POINT \approx 16 bit \approx 2 bytes

So, if the time is large my frequency resolution will be smaller and here we have asked that it required Δf is 1 hertz. So, T is become 1 by Δf so T is 1 second. So, basically 1 second of data sampled at 400 hertz is what my data acquisition system needs to acquire in 1 block.

So, 1 block of data is required certain storage space, so N is the number of data points. So, for that what happens to the in our case N is total times by Δt . So, total time is 1 second Δt earlier we are written as 1 by 400, so this becomes 400 data points, now the question is this you know every data point, 1 data point requires 16 bit of storage space which is equal to 2 byte.

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Handwritten calculations on a grid background:

$$\begin{aligned} 400 \text{ DATA POINTS} \\ &= 400 \times 2 = 800 \text{ bytes.} \\ \hline 1024 \text{ bytes} &= 1 \text{ Kilo byte.} \\ \hline 1 \text{ Kb} &= \text{MEMORY SPACE} \\ &\quad \text{(ANSWER).} \end{aligned}$$

So, since I have 400 data points, the memory would be 400 times 2, that is 800 bytes. Well, so now since the memory blocks are available in only 2 to the power something, so the closest number of points would be 1024 bytes, this 1 kilo byte.

So, I would require 1 k b of space in this problem and that is the answer. So, that concludes my second problem, here we basically discussed the relationship between the signal processing parameters in terms of the time length, in terms of the sampling frequency, in terms of the sampling interval and then we calculated what is the minimum number of data points required and of course, another hidden assumption is that any sampling which you do has to obey the Shannon sampling theorem and follow that we got, because in a 16 bit means 2 bytes of storage space we require 800 bit 800 bytes of storage space and then of course, this memory blocks are available in powers of 2, so the minimum amount of memory required is 1024 kilobytes.

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Q 3. What is the gain in a signal conditioning analog amplifier required so that a 12 bit data acquisition system with a maximum input analog voltage range of 10 V can acquire a signal of 1 micro-volt amplitude ?

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Now, let us look into the next problem. Now what is the gain in a signal conditioning analog amplifier required so that a 12 bit data acquisition system with the maximum input voltage range of 10 volt can acquire a signal of 1 micro volt amplitude. The second problem related to the signal frequency and this one relates to the amplitude resolutions.

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AMPLITUDE RESOLUTION .

$$= \frac{\text{INPUT RANGE}}{2^{\text{bit size (n)}}}$$
$$12\text{-bits} = 2^{12} = \underline{4096}$$
$$= \frac{10}{2^{12}} = \frac{10}{4096} \text{ V}$$

1 μV $1 \mu\text{V} \times \text{GAIN} = \frac{10}{4096}$

So, as you recall amplitude resolution is nothing but the input range by 2 to the power bit size which is denoted as n. So, this is a 12 bit data acquisition system, so the possible values are 12 bit means 2 to the power 12, so this becomes 4096 if I am right here. Now,

so the minimum value you can store, it can sense amplitude resolution which will be nothing but 10 volt divided by 2 to the power 12, 10 by 4096 for the sake of. So, this is the minimum, but I need to, so this is some volt, but I need to sense 1 microvolt of my signal. So, I have to give certain gain to this voltage, so that it reaches this. So, 1 microvolt times again should be equal to 10 to the power 4096.

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$$\begin{aligned}
 10^{-6} \text{ V} \times \text{GAIN} &= \frac{10}{4096} \\
 \text{GAIN} &= \frac{10}{4096 \times 10^{-6}} \\
 &\approx \frac{10 \times 10^7}{4000} \\
 &= \frac{10}{4} \times 10^{7-3} \\
 &= 2.5 \times 10^4 \\
 &= \underline{25000} \text{ (GAIN)}
 \end{aligned}$$

So, 1 microvolt is nothing but 10 to power minus 6 volt times again should be equal to the 10th power by 4096. So, the gain would be 10 divided by 4096 and 10 to power minus 6, for the sake of computations, 10 in 10 to power 7 by if I write this as 4000, so 10 by 4 times 10 to the power 7 minus 3, so 10 by 4, 2.5 times 10 to the power 4, 25000. We require such a high gain, to sense 1 microvolt in such a data equation with 12 bits.

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AMPLITUDE RESOLUTION.

$$\rightarrow \frac{\text{MAX INPUT [10V]}}{2^{\text{BITSIZE}} [2^{12}]}$$

$$\text{MAX INPUT} \leq \underline{10V}$$

SIGNALS OUT OF THERMOCOUPLES.

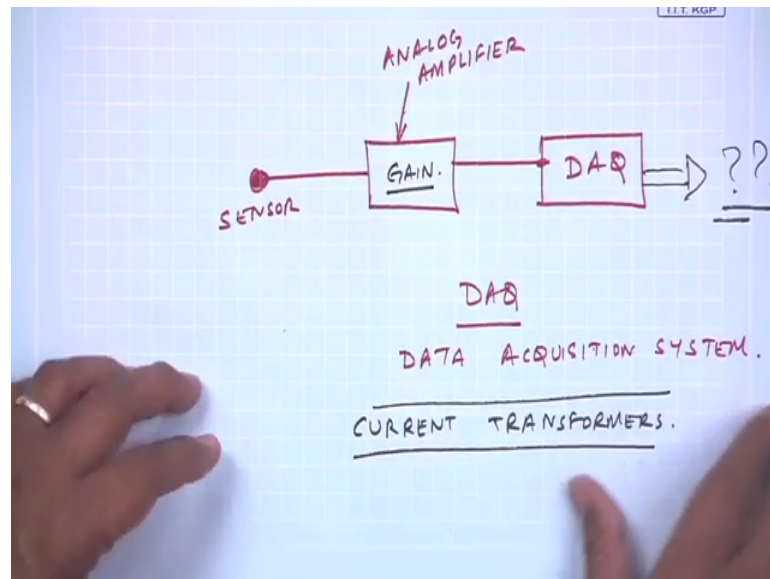
OUTPUT \approx mV.

So, couple of takeaways from this numerical example, one is how could I have made the amplitude resolution small enough, so that it was able to sense I could have, because if you see this is the input, maximum input right 2 to the power the bit size, so to make this quantity smaller I could have reduced maximum input by reducing the input range and here, this corresponded to 10 volt in our case and here this corresponds 2 the power 12.

So, if you will see here, the way I can reduce it is reduce the numerator. So, I can reduce the maximum input to something much less than 10. So, in fact, there are data equation systems when we are dealing with very small voltages there is a method by which you could reduce the input range of such inputs to less than 10 volt by just by a potentiometer they can reduce the input range.

Another is, now suppose I made it 1 if you go back to this numerical example, instead of the input range being 10 if I made it 1, your gain would have reduced by 20 factor of 10, another way to reduce the amplitude resolution, not making more sensitive to low amplitude is increase the bit size. Now 2 to the power twelve I could have made it 2 to the power 16, 2 to the power 32, 2 to the power 64. So, you see by increasing the bit size the gain in this device would be reduce to sense such low amplitude signals. In fact, when we deal with particularly signals out of thermocouple signals, out of thermal couples the output voltage is of the order of few millivolts.

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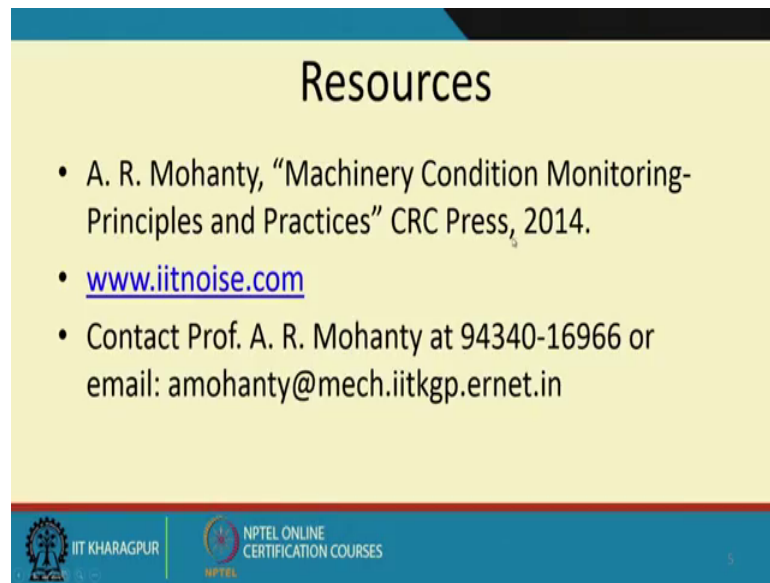


In such systems, we can always between the sensors, I can put an analogue amplifier and of course, then I have my DAQ system. Well you know DAQ the way I am writing is its, in brief for data acquisition system and of course, then you will have the right voltage out of this, so depending on the amplifier I can put some gain.

Even you know sometimes I can put a net unit also, to reduce the voltage of this high voltage signals or sometimes you know when we are measuring signals from high voltage transformers we can put what is known as the current transformer coils, where we can reduce the voltage or increase the voltage accordingly with such transformers provided it is an signal.

So, this numerical establishes how a particular gain could be found out for that data aggression system is able to sense the signal, because if I cannot sense the signal I will not get anything meaningful from my signal. So, I would otherwise get in garbage. If it is not able to hit the minimum noise floor of this device, so that is what we want has to be careful about in such systems.

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The slide is titled "Resources" in a large, bold, black font. Below the title, there is a bulleted list of three items. The first item is a book reference by A. R. Mohanty. The second item is a website URL. The third item provides contact information for Prof. A. R. Mohanty, including a phone number and an email address. At the bottom of the slide, there are two logos: the IIT Kharagpur logo on the left and the NPTEL Online Certification Courses logo on the right. The slide has a yellow background with a blue header and footer.

Resources

- A. R. Mohanty, "Machinery Condition Monitoring- Principles and Practices" CRC Press, 2014.
- www.iitnoise.com
- Contact Prof. A. R. Mohanty at 94340-16966 or email: amohanty@mech.iitkgp.ernet.in

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So, in summary what we discussed in this class was on the data acquisition problems really relating to data equation, basically the frequency resolution issues and the amplitude resolution issues and more of these examples you will see in my book.

Thank you.