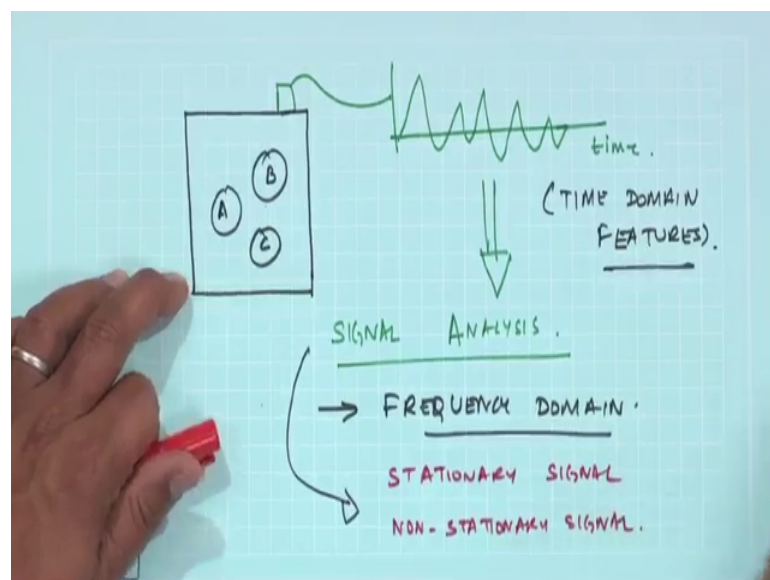


Machinery Fault Diagnosis and Signal Processing
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Lecture – 20
Hilbert Transform in Condition Monitoring

This lecture is on Hilbert transform in condition monitoring. Well as you know signal processing is a very important aspect or element on CBM. And basically depending on the type of signal, we can do analysis, be it in time domain or be it in frequency domain. The idea being that when you analyze the signal, I need to find out the features of that signal, which actually relate to the machine in question or machines condition.

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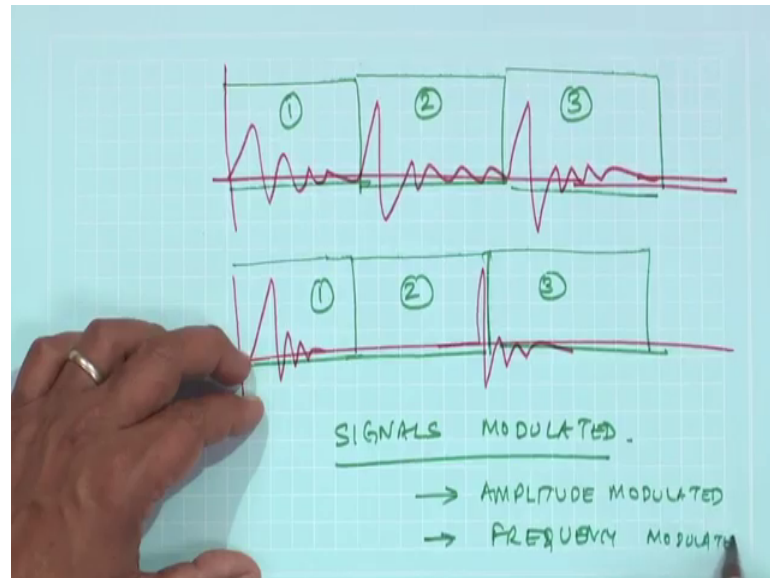


So, you will come across the scenario for example just to recap some of our previous concepts if a machine has different components A, B and C. And if I put a transducer on the machine I will get a signal in the time domain which is an dynamic signal and to do this I have to do some sort of a signal analysis. One very important signal analysis, which we do of course you know apart from then time, we can find out the time domain features.

And while we are discussing about the signal processing in time domain, I had shown you what are the different features of the signal, which we can analyze and get. But in the frequency domain to do frequency domain signal analysis, now in frequency domain

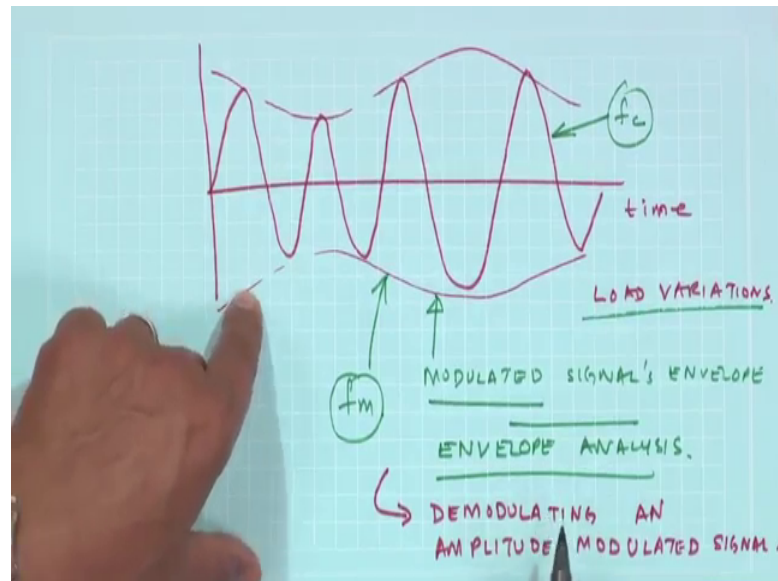
depending the two types of signal which is either stationary signal or another being a non stationary signal. So, just to recap in a stationary signal, we had the case of a signal where the pattern repeats and so on at all times.

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But in non stationary signals I have just once. So, here if I consider this time, here if I consider this time, you will see the signal features are different in this case 1, 2, 3, here 1, 2, 3. So, you will see in the top one, 1, 2, 3 the signal is almost identical, and this could be and if it is repeating and this nature goes for all at all times, this is a stationary signal as opposed to this being a non-stationary signal. But then there are cases where the signals get modulated and I will show you examples of modulated signals. And this could be amplitude modulated or this could be frequency modulated.

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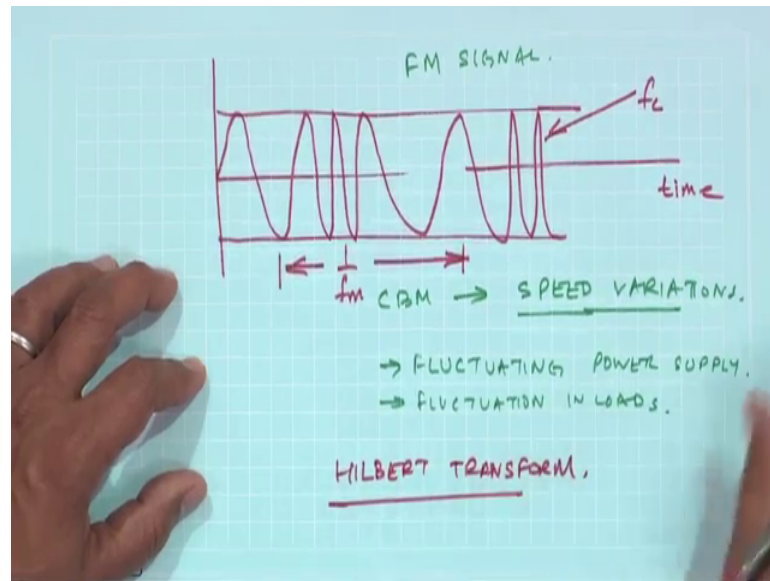


Pictorially to represent amplitude modulated signal or frequency modulated signals, we can visualize it this way I will draw the analog first. So, my signal is actually within this and so on. So, the frequency of the signal is nothing but f_c and it is by amplitude modulate in the signal what I mean in layman terms as this amplitude is increasing decreasing which could be because of load variations.

So, what is happening that I am conveying information as an envelope? So, this is the modulated signals envelope and this frequency of this is nothing but f_m . In CBM such amplitude modulated signals happen in because of load variations. So, once I record the solid red line, I need to find out f_m and f_c , this is one case. And traditionally this is known as what is known as envelope analysis.

Many of the commercial software, which are used for CBM, you will see there is an option of envelope analysis this is nothing but demodulating amplitude modulated signal. So, envelope analysis as the name sounds is nothing but demodulating a amplitude modulated signal and traditionally when digital signal processing was not available people were doing demodulation by an analog electrical circuit where this created found out the RMS and averaged it and so on and put a filter to do the envelope analysis. So, that the meaningful information of the signal f_m can be found out, but Hilbert transform is a technique which is implemented to find out modulations such modulations.

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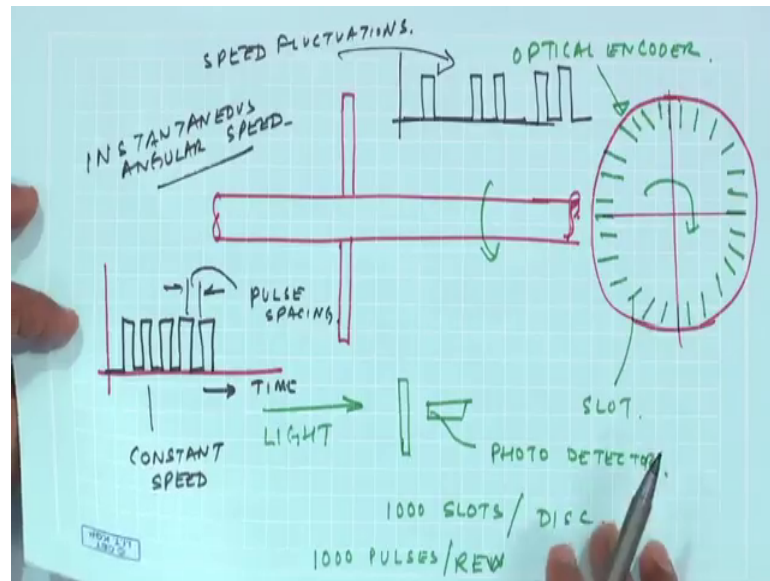


Now, another application you will see is in the frequency-modulated signal. So, this is my frequency modulated signal because in frequency modulation as the name sounds the amplitude remains constant. So, the frequency is decreasing and then suddenly increasing and again decreasing and again increasing and so on.

So, this is an f m signal and f m signal in CBM occurs because of speed variations particularly in machines which are operating which are supposed to operate at constant speed. They normally do not because of fluctuating power supply, sometimes the fluctuating load on the system in loads. We will basically produce speed variations and because of speed variations the amplitude is same, but this frequency content of the signal is going to change.

So, here again I am conveying a high frequency signal with others with a modulated signal. If you see this, this is the wavelength of the modulated signal because from low frequency to another low frequency. So, I can see that. So, this kind of signal variations are there. So, again detection of f m in an modulated signal we will use what is known as Hilbert transform.

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I will give you another example how such f m signals can be generated; imagine, I have a shaft onto this I attach a disc. And if you look at the disc, suppose there are many slots on the disc which I am denoting by these green lines. All the slots are equally spaced and they are physically actually slots and so on. So, this is a slot basically it looks like this. So, in one end I give a light and other end I have what is known as a photo detector. In other words, this is what is known as an optical encoder. And usually these slots are about like you know maybe 1000 slots per disc.

So, what happens if this shaft is rotating at a constant speed, so it will generate 100 pulses per the revolutions? So, how does the pulse look like, this pulse will look like something like this and so on. This is the pulse train, this is in time domain. Now, what happens if there is this is at a constant speed, the pulse spacing is a constant in the time domain. So, I can also find out the speed of this by the number of pulses per given time.

So, because if I know there are 1000 pulses, I know whenever 1000 pulses is there, I will take the time taken and then I can find out the mean speed, but imagine if there is a momentarily change in speed the frequency content will change. So, how would the pulse look like and then suddenly there is a change in the spacing. So, you see momentarily there is speed fluctuations, this is because of speed fluctuations. So, rather than finding out the speed I can always find what is known as instantaneous angular speed.

So, if there is a defect in the machine or there is a change in the instantaneous angular speed, this frequency of this instantaneous angular speed is an indication of the mechanical system, which is causing such a speed difference. And such optical encoders basically produce such frequency modulated signals. And you will see through the maps in Hilbert transform how such analytical signals can be used to find out the frequency enveloped.

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Hilbert Transform

Hilbert transform of a time domain signal $x(t)$ is defined as

$$\bar{x}(t) = H[x(t)] = \int_{-\infty}^{\infty} \frac{x(u)}{\pi(t-u)} du$$

Or in terms of convolution,

$$\bar{x}(t) = x(t) * \left(\frac{1}{\pi t}\right)$$

Where $\bar{x}(t)$ is also a time domain signal.

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So, Hilbert transform of a signal in the time domain is defined by this mathematical function. So, in terms of convolution it is given by this expression where $x(t)$ is my original time domain signal.

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Analytic Signal

What is it?

In signal processing context,

Analytic signals are those signals that have only positive frequency content. So we say they have only 'one sided spectrum'.

This is in contrast to Fourier methods where the spectrum is two sided and we exclude the negative frequency part.

Why should we use Analytic signal?

Advantage of using Analytic signal:

Identification of envelope and instantaneous frequency becomes obvious once we find the analytic signal representation.

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So, what is this analytic signal? So, analytic signals are those signals that have only one positive frequency content. So, we have one sided spectrum. This is in contrast with the Fourier trans methods where we had actually the negative frequencies part, but for CBM I did not discuss about the negative frequency part. But the advantage of using analytical function is or signal is the identification of envelope and instantaneous frequency becomes obvious once you find the analytic signal representation.

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① AM — ENVELOPE
② FM — I.A. & (INST. FREQUENCY)

HILBERT TRANSFORM.

MATLAB.

Because I require for my AM signal the envelope and for my fm signal I require what is known as the IAS instantaneous or instantaneous frequency and you will see in CBM when we talked about dynamic signals they will be a amplitude modulated frequency modulated or some other form of non-stationary signal.

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Analytic Signal Representation

- Analytic signal is represented by

$$x_a(t) = x(t) + j\tilde{x}(t)$$

Where, $x(t)$ is the analytic signal and
 $\tilde{x}(t)$ is the Hilbert transform of $x(t)$.

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So, Hilbert transform helps us identify an envelope or an frequency modulated signal. So, an analytical signal will be made by this where this is the analytical signal and my Hilbert transform if I put it as the imaginary part I will generate an analytic signal, this original signal I add an Hilbert transform as an imaginary part and get an analytic signal. So, the analytic signal can be represented either in the polar form where this is the envelope of the signal and this is the phase.

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Representation of Analytic Signal

$$x_a(t) = x(t) + j\hat{x}(t)$$

We can write this in polar form as



$$x_a(t) = A(t)e^{j\varphi(t)}$$

Where

$$A(t) = \sqrt{(x(t))^2 + (\hat{x}(t))^2}$$
$$\varphi(t) = \text{atan}\left(\frac{\hat{x}(t)}{x(t)}\right)$$

$A(t)$ is the envelope signal and $\varphi(t)$ is the instantaneous phase signal.



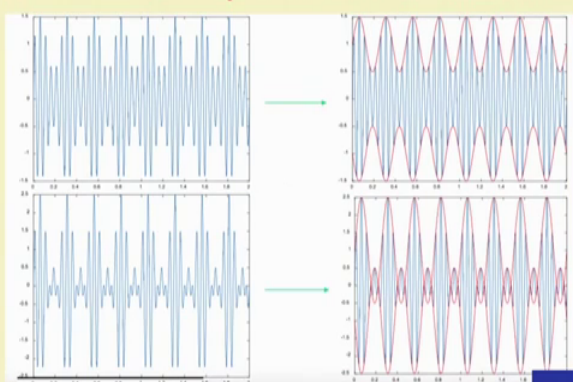
Instantaneous frequency

$$f(t) = \frac{1}{2\pi} \frac{d\varphi(t)}{dt}$$


So, it is the envelope of the signal and $\varphi(t)$ is instantaneous angular phase signal and instantaneous frequency is nothing but $\frac{1}{2\pi} \frac{d\varphi(t)}{dt}$ of the instantaneous phase signal. So, this is what we are obtaining Hilbert transform. So, any signal I can use it in the analytic form using an Hilbert transform and get the envelope or get the instantaneous phase, and then get the instantaneous frequency, and this has a lot of applications in CBM.

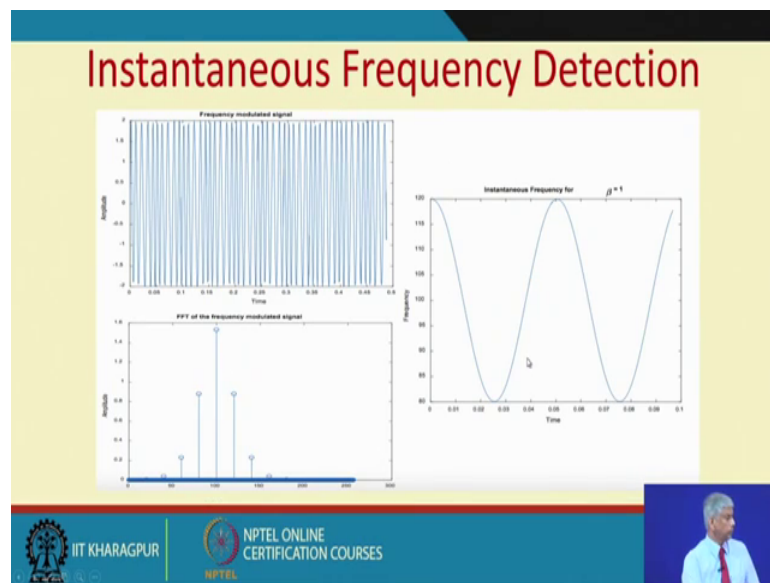
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Envelope Detection



For example, this is my envelope this is an amplitude modulated signal. So, you see here the red function here is actually the envelope, and this is an amplitude modulated signal. So, this can be very easily detected through an Hilbert transform, this is another signal and you will see its envelope even here.

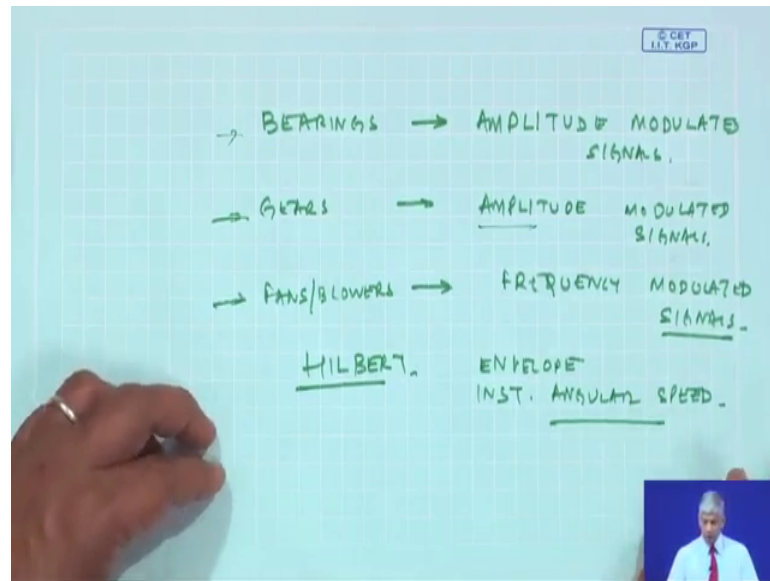
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As oppose to you see this, this is an frequency modulated signals. You know the amplitudes are constant unlike the amplitude modulated signal where there is a considerable change in the amplitude modulations in frequency modulated signals I can see the instantaneous frequency and then you can see the instantaneous frequency of the frequency modulated signals.

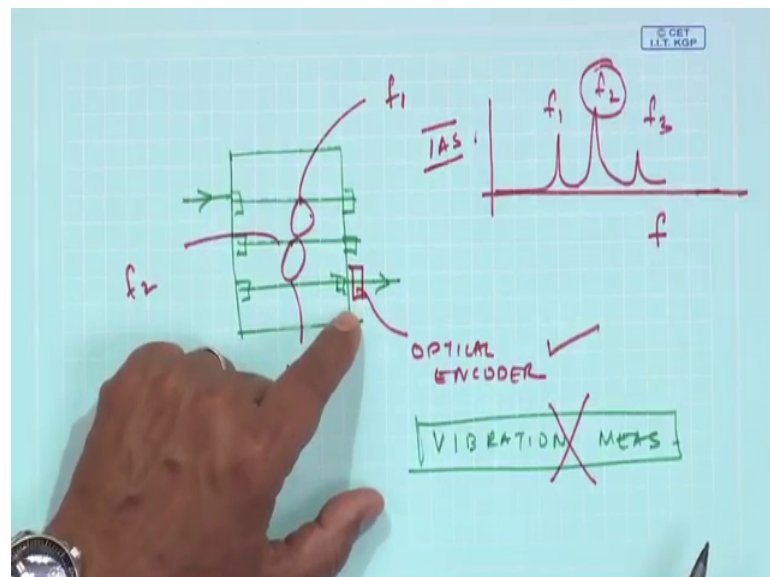
So, this is how the envelope analysis is going to help us look into these signals. Now, I will give you an example in so there is a software called matlab which we will discuss in some of the subsequent classes through matlab, I can get the Hilbert transfer of any signal and just do the envelope detection. But envelope detection or amplitude modulated signals in CBM works in many areas and some of them could be in bearings.

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So, in bearings - we have amplitude modulated signals; in gears - we have amplitude modulated signals; fans or blowers where there is considerable speeds fluctuation, we have frequency modulated. So, we have to use such Hilbert transform to find out either the envelope or instantaneous angular speed or frequency.

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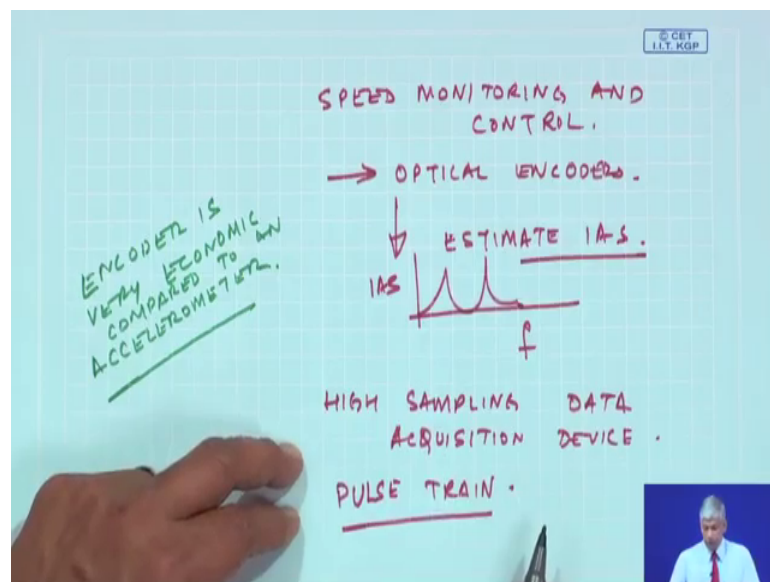


Now, imagine I have a gearbox. So, if you put an optical encoder here, and imagine if all of these this has a frequency f_1 , this has a frequency f_2 , this has the frequency of three and there is a gear meshing frequency here and so on. So, if I correct the optical encoder

pulse and do the frequency domain of the IAS, I will see a speed variation in particular variation and this could be only few orders. So, if this is f_1 , this is f_2 , f_3 , so I can see that whatever frequencies these variations are there I can find out that what is perhaps the defect is in the shaft which is f_2 or shaft which is f_1 .

So, this is how it is going to help you see in this kind of an study we never did any vibration measurements. Vibration measurements is not required, is not required variation measurement is not required rather we will just measure an optical encoder. So, you see the convenience of doing CBM through Hilbert transform if I have a pulse train out of an IAS out of an optical encoder.

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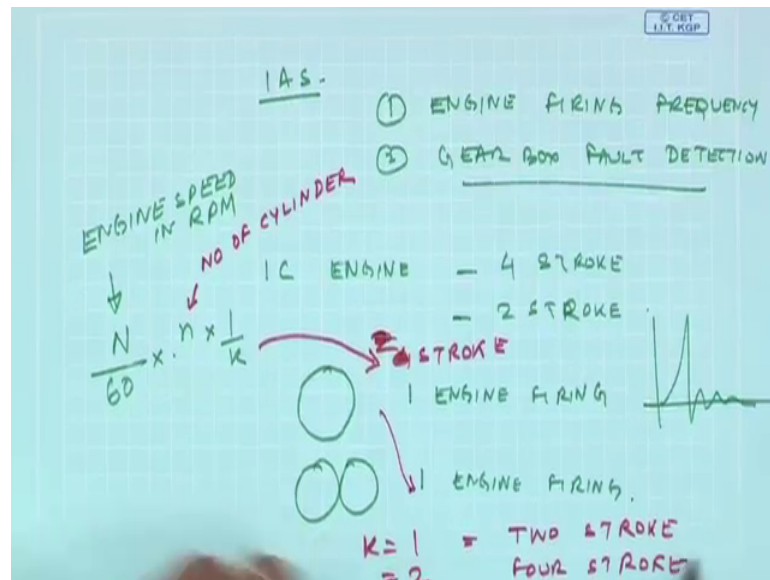


And today you see all of these machines which we be it an CNC machine, be it gearbox, be it precision machine for many of the speed monitoring and control they have optical encoders. So, taking pulses from optical encoder I can do what is estimate the estimate I a s instrument. And see at what frequencies this IAS is varying, but only the catch is I require a high sampling data acquisition device to calculate the pulse train to capture the pulse train, but again the beauty of this is you know you could be just having a pulse signal.

Of course, you know now with new technologies of modulation and WIFI transmission they suppose right at that encoder which you know an encoder is a very relatively very cheap device compared to conventional the variational measurement by an

accelerometer. So, encoder is very economic compared to accelerometer. So, I can all do is just measure the pulse train, and then use Hilbert transform and get the frequency content of this signal.

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In our laboratory, we have done the IAS estimations for finding out engine firing frequencies for gearbox fault detection. I will just for the students who are attending this course I will give you a definition of engine firing frequency. For example, in an I c engine whether it is four stroke or two stroke, so if it is two stroke, so in every rotation I have one engine firing, but as in four stroke for every two rotation I have one engine firing when engine fires basically there is an combustion. So, this combustion creates a pressure wave. So, this pressure wave is responsible for the dynamics of the engine and this is the responsible for the vibrations and an engine.

So, in one case if n is the rpm, so n by 60 is in hertz divided by if the single cylinder engine when n multiplied by 1 by k where n is the engine speed in rpm, n is the number of cylinders and k is equal to 1. If it is a four stroke sorry two stroke, and it is equal to 1, just to be clear k is equal to 1, when it is two stroke; and it is equal to 2 when it is four stroke.

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1200 RPM, 4 CYLINDERS, 4 STROKE



$$f_{\text{ENGINE}} = \frac{1200}{60} \times 4 \times \frac{1}{2}$$
$$= \frac{2400}{60} = \underline{40 \text{ Hz}}$$

So, if an engine is running at 1200 rpm this is four cylinders if it is four stroke the engine firing frequency firing engine is nothing, but 1200 by 60 times 4 times 1 by 2. So, 2400 by 60 that is 40 hertz. Now, if we have modulation around the engine frequencies firing frequencies because of a speed fluctuation I can sense that by putting an optical encoder at the output shaft getting a Hilbert transform and doing an signal analysis.

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Resources

- A. R. Mohanty, "Machinery Condition Monitoring-Principles and Practices" CRC Press, 2014.
- www.iitnoise.com
- Contact Prof. A. R. Mohanty at 94340-16966 or email: amohanty@mech.iitkgp.ernet.in

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So, many of these examples you will see on IAS in my book, and details you can find at my website.

Thank you.