Machinery Fault Diagnosis and Signal Processing
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Lecture – 13
Non-Stationary Signal Analysis

In the previous class we have talked about or introduced you to fast Fourier transform, and there are certain assumptions in fast Fourier transform is what we have going to talk about in this technique in this lecture. And then tell you, what are the other methods to find out the frequency content of a signal.

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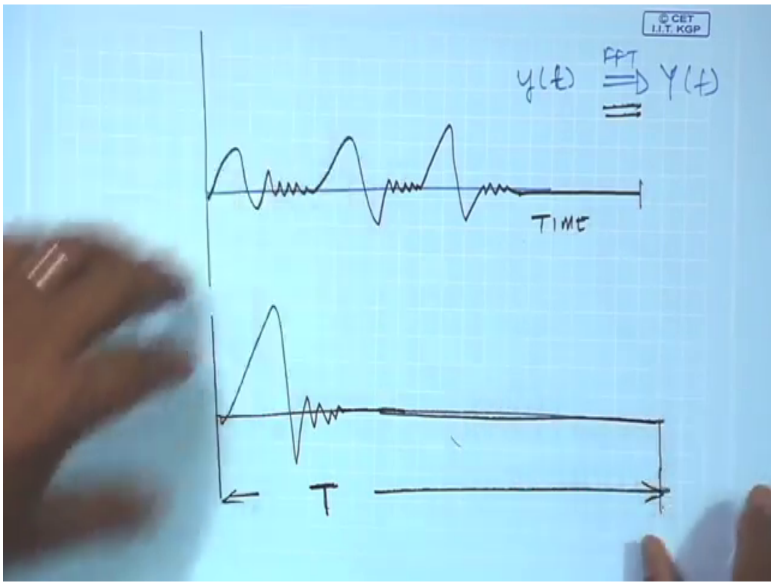

Now, when we do fast Fourier transform, certain time is being used to do this FFT, but the question is what happens in this time my signal is changing; that means, the signals quality has changed for example, I will give you another example illustrate. This I have an impulse and then there is nothing beyond it. And then if I have taken a total time signal of T; obviously, this is not true the signal is not repeating at many at any instance of time.

So, this is a gross violation of the stationarity assumption in Fourier transform. And there are methods which take only very small amount of time. For example, we had seen that the delta F which I measure is nothing but 1 by T or 1 by N times delta T.

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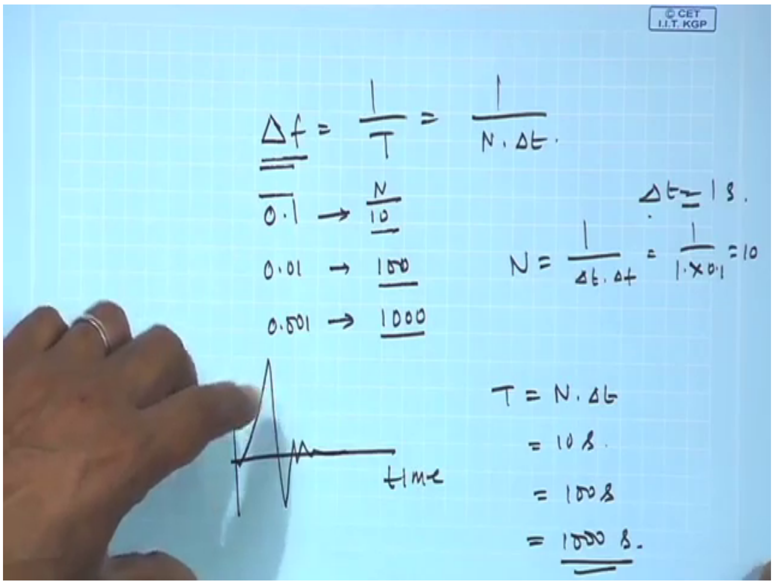

So, if I want high or low resolutions, I need more time of the signals. I want to find out a signal which is having a frequency of 1.1 hertz if delta T is for example, if delta T is one second. Here N is nothing but 1 by delta T delta F is equal to 10.

So, N corresponds to 10. If I want 0.01 N will be 100. With the same sampling 0.001 N will be thousand. So, you see what is T? T is nothing but N times delta T. So, delta T is one second. So, this is 10 seconds in one case, in other cases this 100 seconds. Another case it is thousand seconds. But in machinery, when impacts occur something has broken. There is a large impact, maybe an high impact force occurs only once right.

So, well this is not going to repeat at all time. So, this is a gross violation of the frequency representation on time. For the signal, what I mean to say here.

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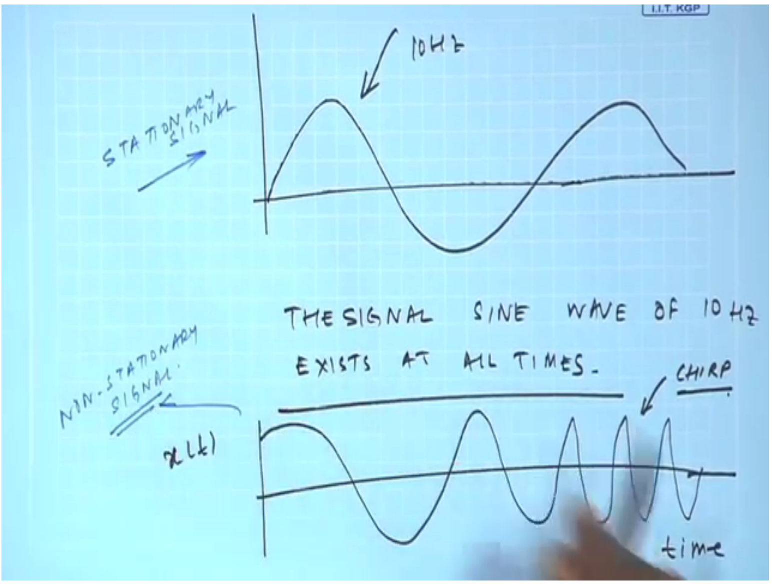

When I have a sine wave of a single frequency; that means, the signal say for example, the sum 10 hertz sine wave, the signal sine wave of 10 hertz exists at all times, which is fine, but suppose I have a signal where the frequency is increasing and decreasing for example. Have a low frequency signal and then suddenly it becomes a high frequency signal. So, this means what in the time domain the signals frequency is changing and this by the way is known as a chirp signal. So, I need to know that this is an example of a stationary signal, whereas, this is an example of a non-stationary signal.

So, we knew also like we did a 54-stationary signal, we should find out what the different methods available for non-stationary signal and that is what we look here.

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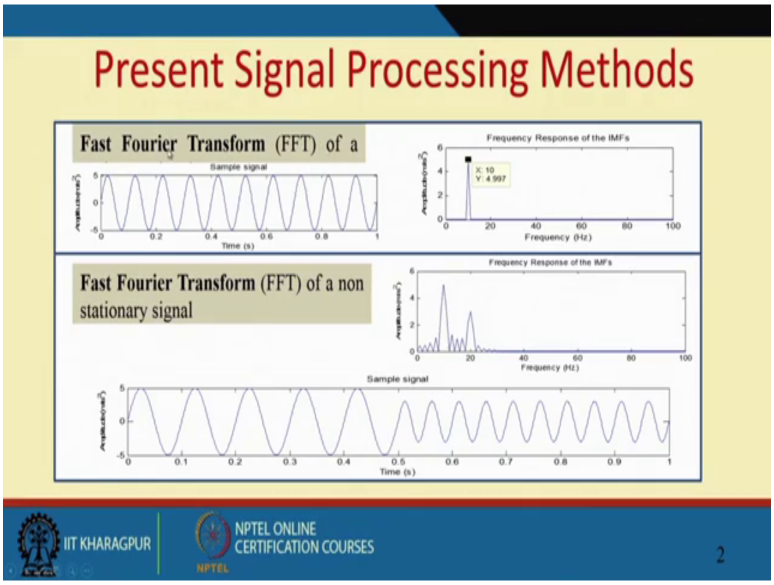

So, some of the signal processing methods, one is this fast Fourier transform if you here, this is a signal of one second durations. Amplitude from minus 5 to 5 here of course, you know this amplitudes meter second square. Should have been actually 10 by 4.997 is close to 5 that is within the limits of the measurement here.

So, I have represented in the Fourier series or the frequency spectrum of the signal is this. But if you look at this single here, the frequency is changing with time of a non-stationary signal if you look at the spectrum here. You may get mislead that sin signal of 5 amplitude at 10 hertz also exist at the same time a single of 20 hertz of close to maybe 3 0.5 volt or 3 volts, it is just that is a gross violation of our assumption. Because if I do an inverse of this signal I will get a sum of sine wave of 10 hertz at 5 amplitudes and sine wave of 20 hertz at 3 volts.

But you see in the first 0.5 seconds, this signal does not exist, and the subsequent 5 seconds the low frequency signal does not exist. So, this is where your Fourier transform would fail. And that is for the analysis of non-stationary signals. Though let me tell you in condition-based monitoring 90 percent of the cases can be solved by Fourier transform. But today that the state of the art is such that people are developing many algorithms for non-stationary signal analysis. For quickly detecting defects in machineries for example, like I has telling you if a defect has occurred like an impact something has broken something has fallen this can only happen for a fraction of a second.

So, is the signal processing algorithm very robust to catch such high frequency or quickly occurring transients.

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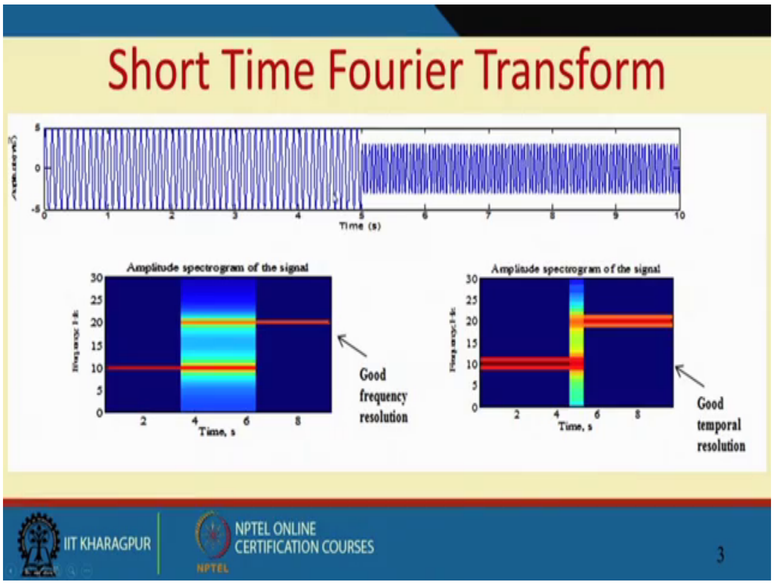

And then we will see the one such method is what is known as short time Fourier transforms. So, in short time Fourier transform what we do we break this time period into small packets of T. And do the FFT and then stand up together. But again, the problem is if I have a good frequency resolutions, I will need to have a by good I mean finer I need to have more time. And if I have more time I will have a poor frequency resolution. So, this policy exist because f is 1 by T or delta F.

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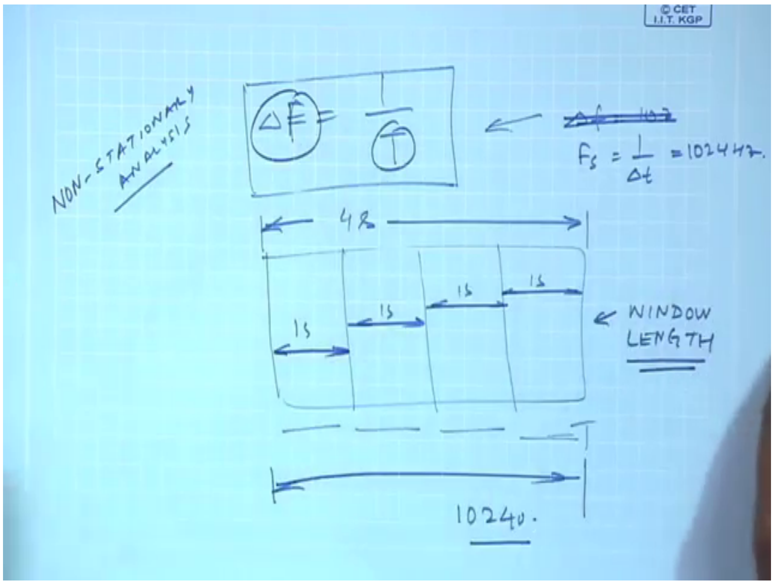

So, this is true. So, I can break up into different windows and do FFT, but then I will have higher resolutions of data F. So, this is what we see here a good temporal resolutions happens finer in the time, but poor in frequency or good in frequency more in time. So, we will miss the characteristics of the signal. So, this is a problem which people face.

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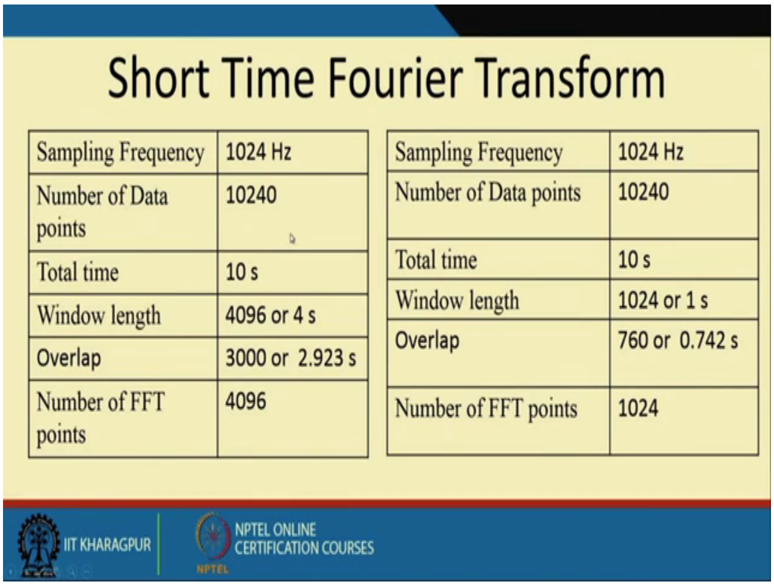

Sampling Frequency	1024 Hz
Number of Data points	10240
Total time	10 s
Window length	4096 or 4 s
Overlap	3000 or 2.923 s
Number of FFT points	4096

Sampling Frequency	1024 Hz
Number of Data points	10240
Total time	10 s
Window length	1024 or 1 s
Overlap	760 or 0.742 s
Number of FFT points	1024

Certain examples of this short time Fourier transform is in the first example I had sampling frequency is the 6 number of data points is total time, window is 10. And this a window length I can break it up into 4 parts, or I can make it up into one window 4 seconds or one window of one second, one second, one second, one second. Or I can have it into 4 sets. I can take it in one go in 4 seconds, or 4 windows of one second each. And each of them, because the total number of data points is delta Fs 1 0 2 sorry, sampling frequency or 1 by delta T is 1024 hertz, total number of data points is 10240.

So, I will I can break it up into 4 second window lengths, or individual one seconds. If I take it 4 seconds I will have 4096 data points, otherwise I will have 1024 points. So, you will see one has to play around with the number of data points or number of times you are taking it all depends on the window length. So, your window length selection plays around with other delta F or T, but people had developed these techniques after to use for non-stationary signal analysis.

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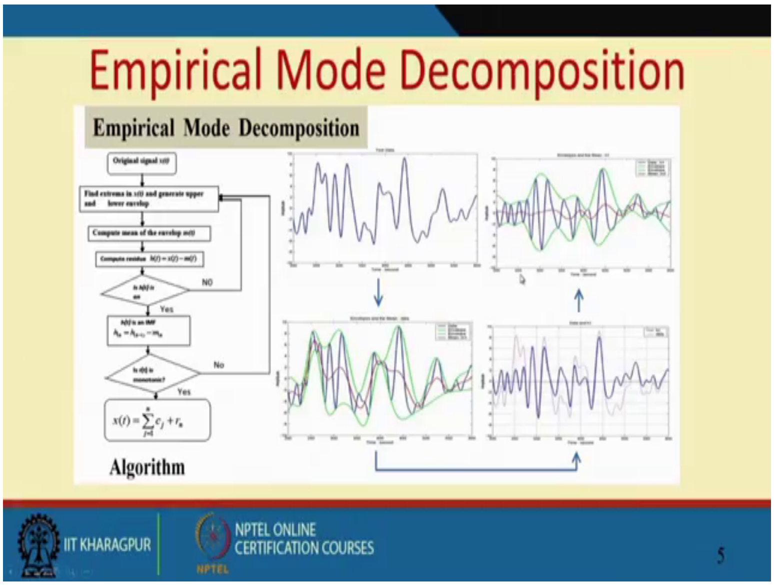

But then there are few other techniques which we at IIT Kharagpur have used to find out the FFT or the frequency content in any transient signals. So, this is known as the empirical mode decomposition. So, if I give any signal there's an I will welcome which has to be followed all we do is you know we do not care about the time I will fit an envelope taking the maximize and minimize. And find out the mean of this envelope, and try to subtract the mean from the original signal. And you will end up with the residue and then we repeat this process till we get a monotonic signal, which is known as an intrinsic mode function.

So, every signal which has been measured for even a short duration can be broken up into such intrinsic mod functions by this algorithm. And one can then find out the frequency content of the intrinsic mode functions. And thus, have a true representation of the frequency content in any signal bit non-stationary bit stationary.

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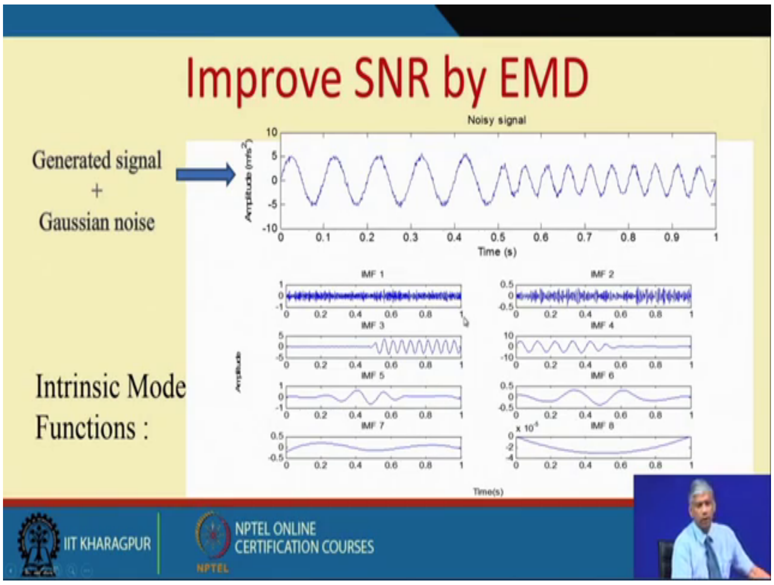

And that is what I am trying to explain to you here through an example. So, this is a sine wave which is again varying in frequency, and then little noise has been added. So, these are the intrinsic mode functions of the signals intrinsic mode functions 1 2 3 4 5 6 and so on, all even for a very small time even in one second of the data. And imagine in machinery condition wanting when transients occurs. They will occur only for a very short duration and you know though to measure a very, very high frequency signal or to have a very fine resolution, I need to have more time period and that is not available to me.

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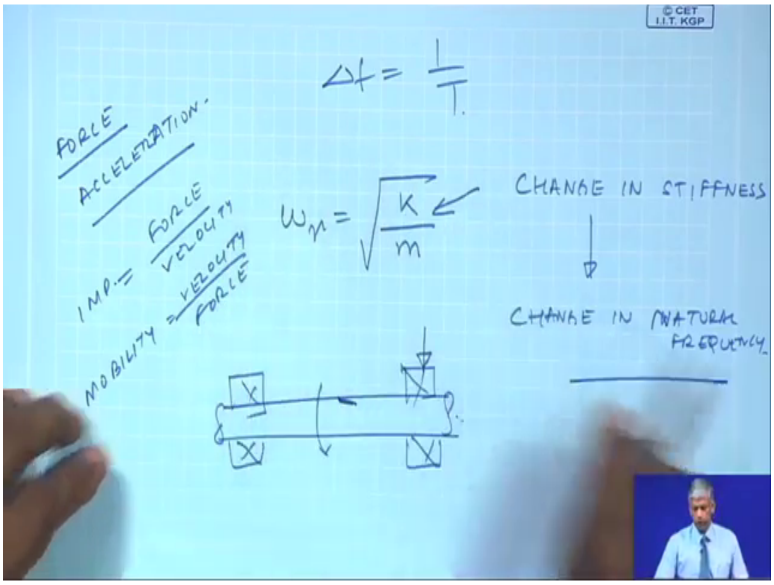

So, more time is not available to me. So, I will not have a fine resolution which did not be done in EMD.

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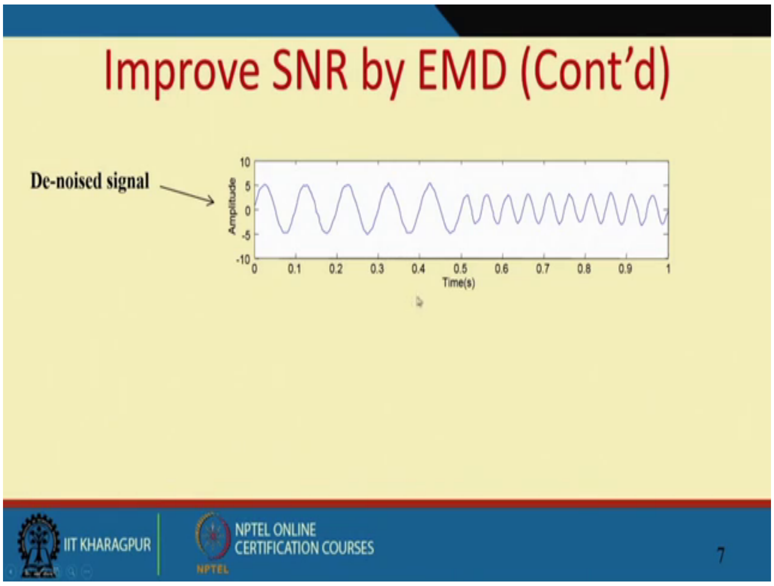

So, this is the d noise signal.

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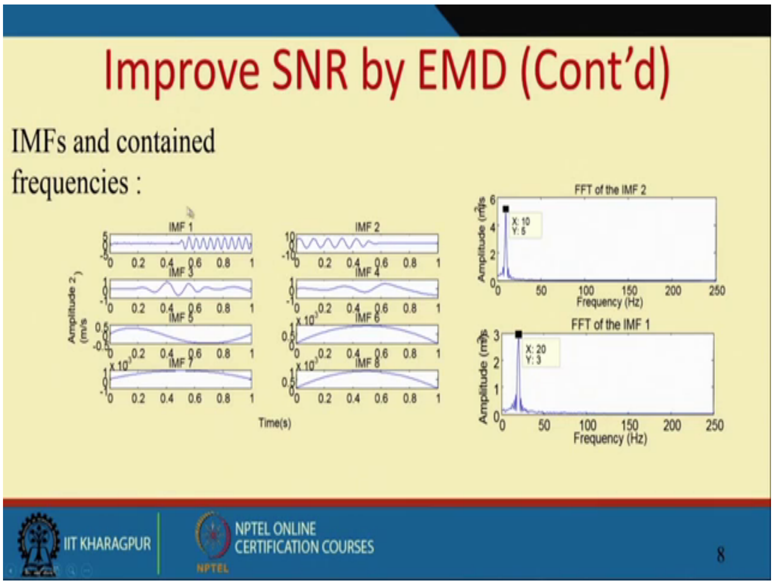

And then even see each of these IMF's contained a frequency. So, the actual representation of the frequency of the signals can be very easily done through EMD.

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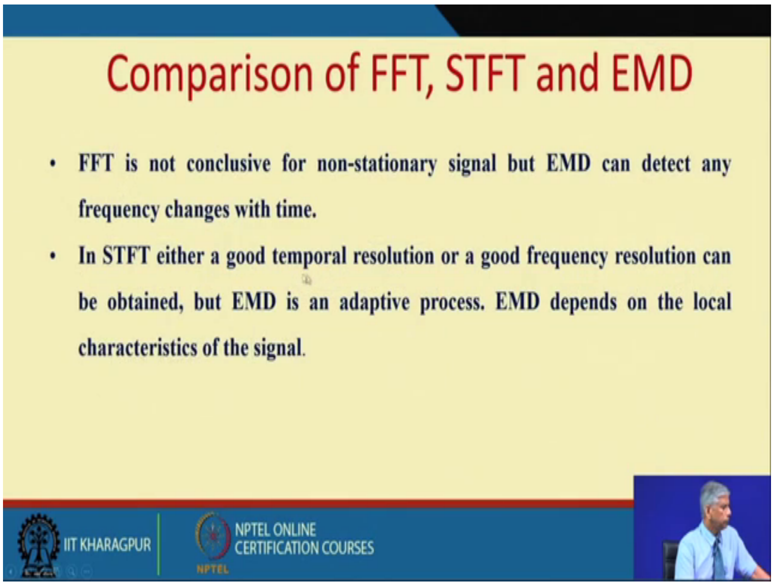

So, if I was to compare fast Fourier transform short time Fourier transform and empirical mode decomposition. So, a 50 is not conclusive for non-stationary signal, but empirical mode decomposition can detect any frequency changes with time. And in STFT either a good temporal resolution or a good frequency resolution can be obtained, but EMD is an adaptive process. And EMD definitely depends on the local characteristics of the signal, in that sense EMD is powerful.

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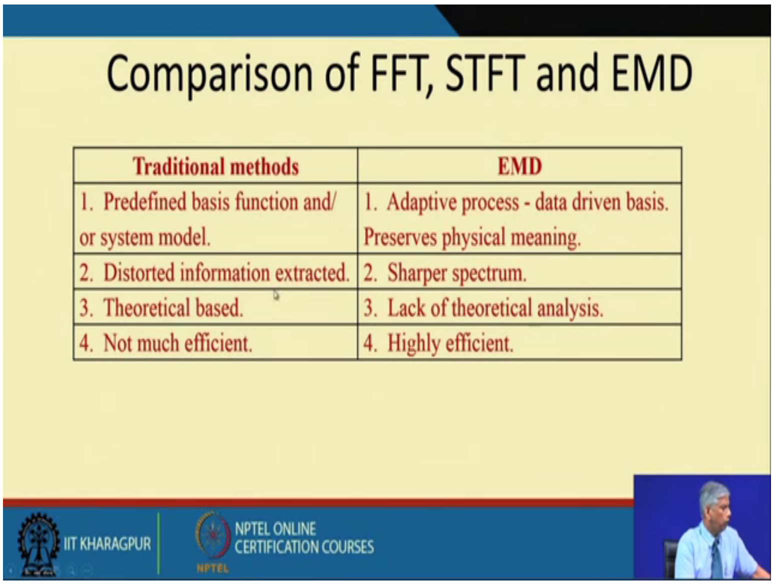

And this is more of an adaptive nature, and it is highly efficient compared to the traditional frequency domain analysis of FFT or STFT.

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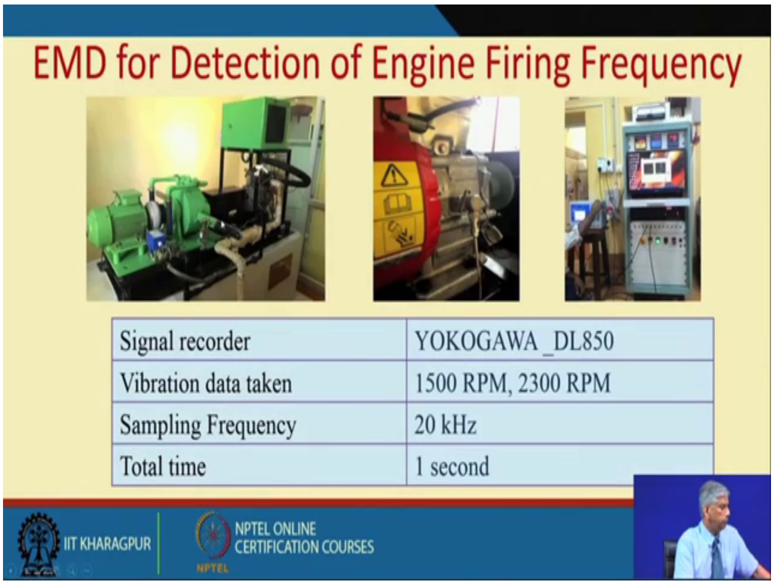

This is an example which we thought ill introduce you from our lab this is an engine single cylinder engine which is being loaded by a dynamo meter, and this is an accelerometer high temperature accelerometer mounted on this engine. And this is the control panel for that engine and the data was taken in a data recorder.

Vibration data at 1500 RPM and 2300 RPM of the engine, was taken the sampling frequency was 20 kilohertz, but if is the total time of the signal was only for one second.

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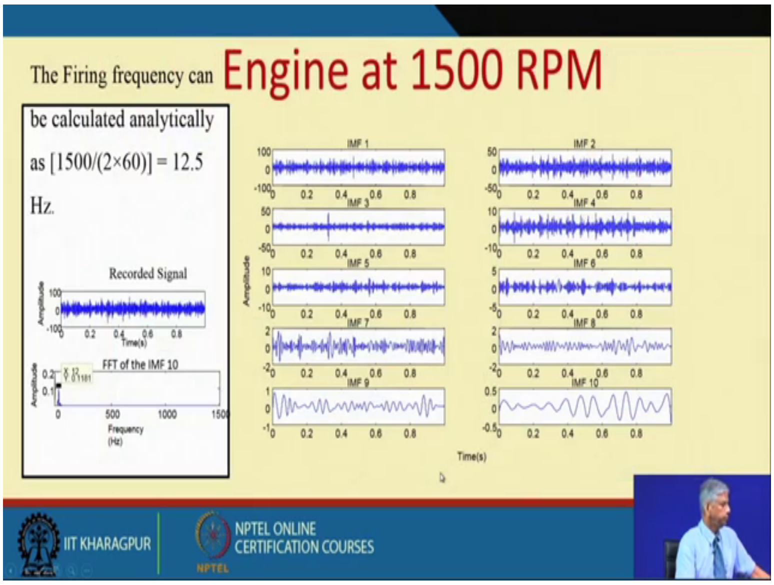

So, the IMF's of the engine was this is the recorded signal, through an EMD process the IMF's were obtain. We can see 1 2 3 4 etcetera. And the tenth IMF you see it gives a frequency close to the firing frequency of the engine. In that way even just in one second of data we can find out the 5 in the frequency of the engine.

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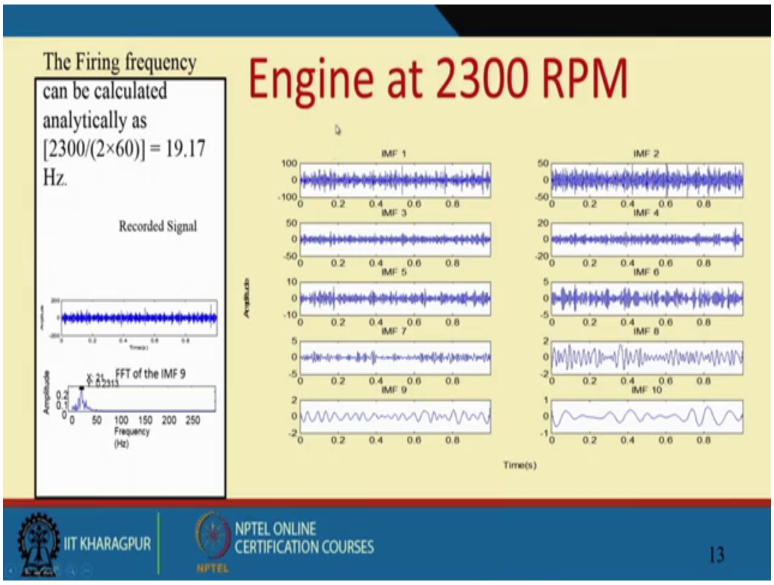

Same was also repeated at 2300 rpm and the engines firing frequency was 19.17 hertz.

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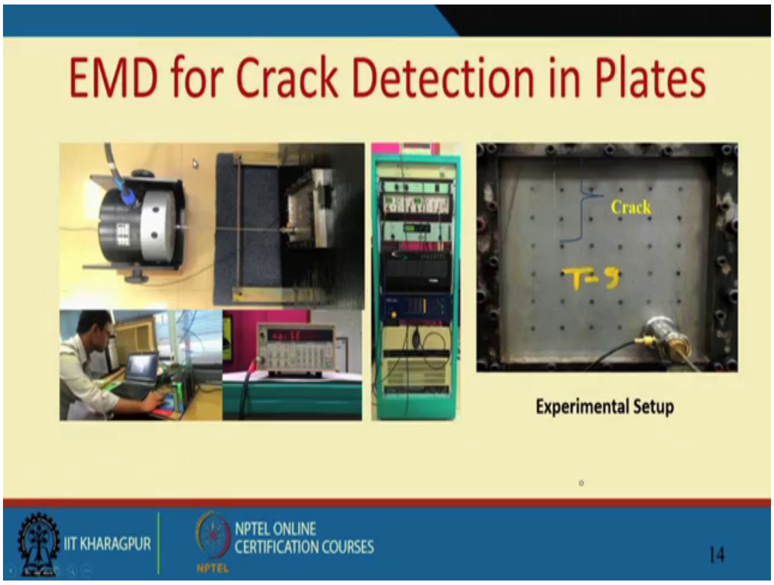

This EMD is also used to find out the cracks in plates. Where in a resistive. So, the reason I want I am showing you this now. So, that you get an feel of what kind of equipment later on will be using in condition monitoring.

Usually, what happens when cracks occur there is the local change in the stiffness and if you will recall any new system? So, is there any change in stiffness, there will be a change in leads to change in natural frequency. So, this is for a plate, but you know this has been tried out in rotor systems where we have you know shafts, which has supported on bearings and if a crack has developed somewhere. So, the natural frequency of the system would change.

Now, in other the characteristic of the system would change. And in this method, this is an impedance head where in simultaneously we can there force and acceleration. You know impedance of a structure is; nothing but force by velocity. And mobility is nothing but velocity by force. So, if I have a signal xt or ft this is my system I give certain force ft. And I got a response x double dot T.

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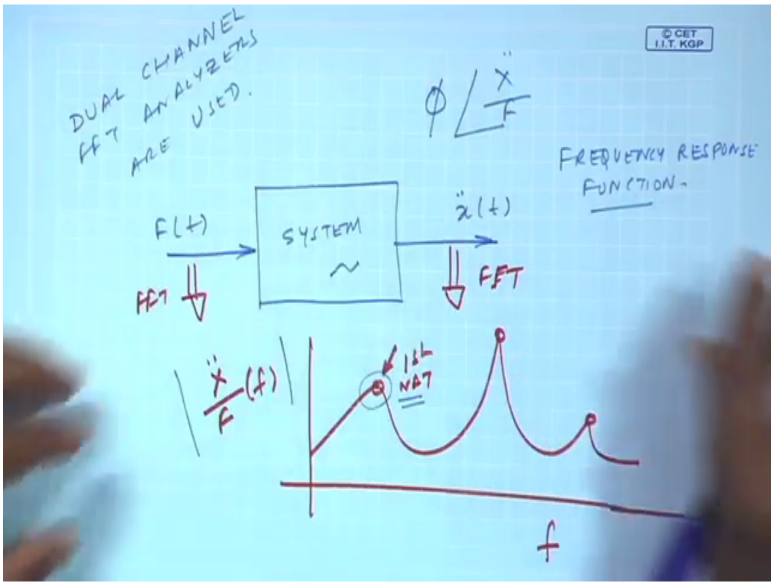

I can always do an FFT of this, FFT and get the plot f nothing but either x dot lot by f or occurred in the frequency domain. So, this corresponds to the first natural frequency and so on. So, there will be a change and this is nothing but the amplitude, and there will be also a phase angle of this and we have seen in vibration and natural frequency is how the phase angle changes by 90 degree.

So, if there is a crack in the system, there will be change in the value of this natural frequency. But question is this has to be done simultaneously, and that is why commercially dual channel FFT analyzers are available are used to do this kind of a competition to find out what is known as the frequency response function of a system. We will talk about this later on when we discuss more about you know signals and systems.

So, to explain this to you we have a random noise. That mean frequency all frequencies are in generated. And this is filtered between the values we want, and this is fed to m power amplifier which is driving an electromagnetic slighter you used to excite this crack plate. And the tip of the exciter we have an impedance head when we are measuring the force and velocity. And this is the FFT and world channel ff which is a multiple channel FFT and these are being used to measure the mobility function of the system.

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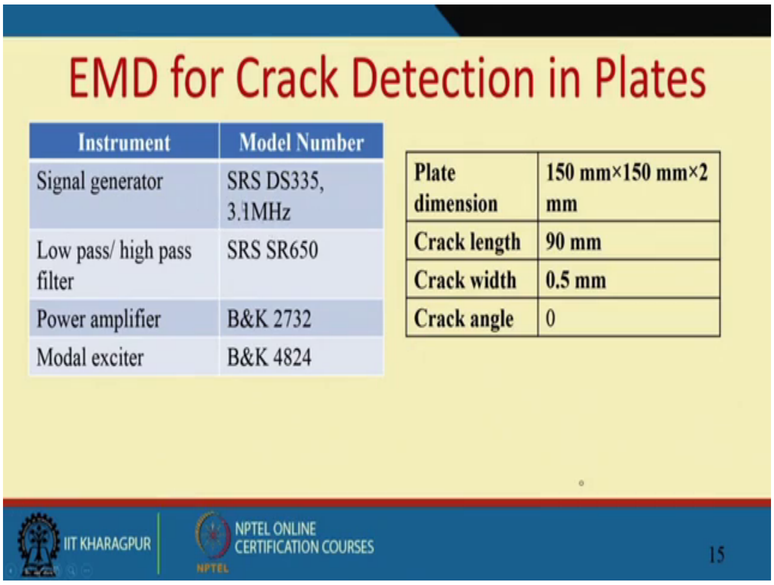

So, this signals now already did an EMD, a plate has dimensions of 2 mm thickness 90 mm is the crack length 0.5 mm is the crack width. And these are some of the instrument which we use to find out the crack.

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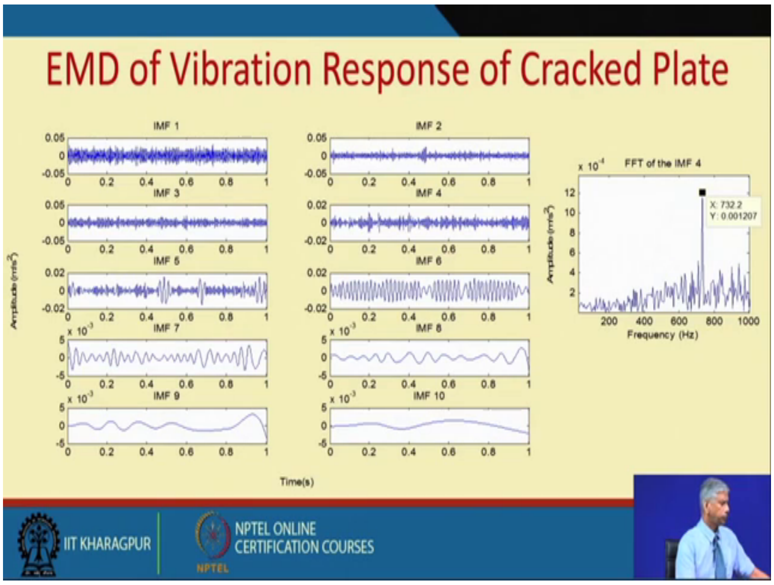

And if you look the EMD of a crack plate the response, I mean here the mobility is not being measured just by the response of a cracked plate IMF one IMF 2 in that same frequency as per the algorithm which I had told earlier. And FFT of the IMF 4 this the natural frequency has 732.2 volts.

So, you see by doing such analysis on signals in FFT, we can find out the natural frequency of the system. At this point I must tell you some other properties of Fourier series.

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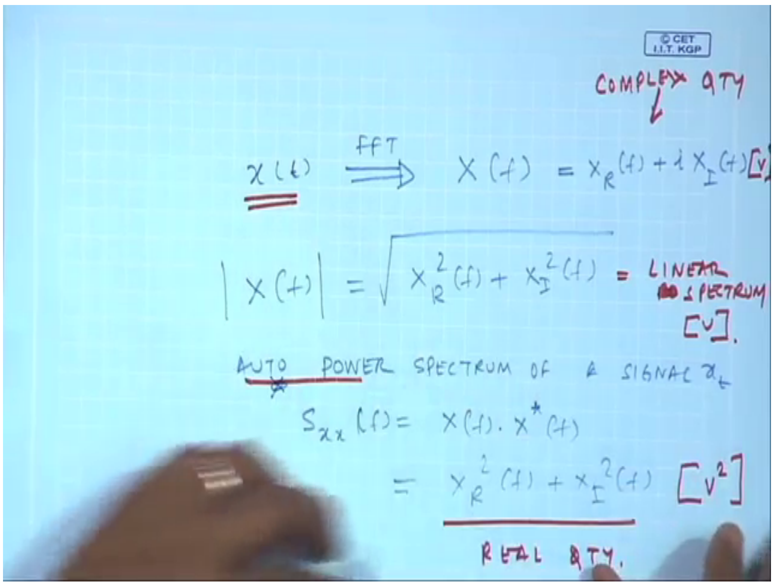

So, if I have a signal xt, if I do an FFT just saying I will get a signal xf, which is a complex number. So, it can be represented as x real plus x imaginary. So, the amplitude or magnitude of this Fourier coefficient is nothing but x real square plus x I square, these are all functions of frequency.

But when I have so, this and there is a term called as power spectrum or auto power spectrum of a signal xt is represented. Whereas, xx f which is nothing but xf times x conjugate f. So, this will boil down xr square plus x I square f. And you see this is though this is a complex quantity, Fourier transform of any signal is a complex quantity that the power spectrum is a real quantity.

So, if it was in volt this will be in volt square, sometimes peoples call this as a linear bar spectrum, linear a linear spectrum. When it is power there is a unit square. Here this unit is voltage. Here the unit is voltage, but this is for a signal. But when we have 2 signals x t and y t this is term called as Sxy f which is nothing but x times y conjugate f.

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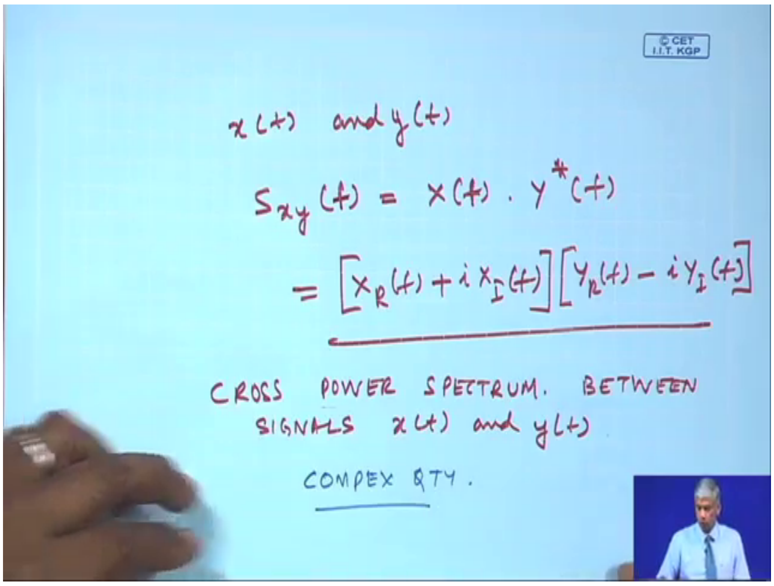

So, this will be a complex quantity. So, this is x real plus i x i f y real minus i y imaginary.

So, this is what is known as the cross-power spectrum between signals xt and yt. So, this is a complex quantity cross power spectrum is a complex quantity. So, this auto power spectrum and cross power spectrum are used to find out the frequency response of a system like we saw the case for a plate. So, how do you do that?

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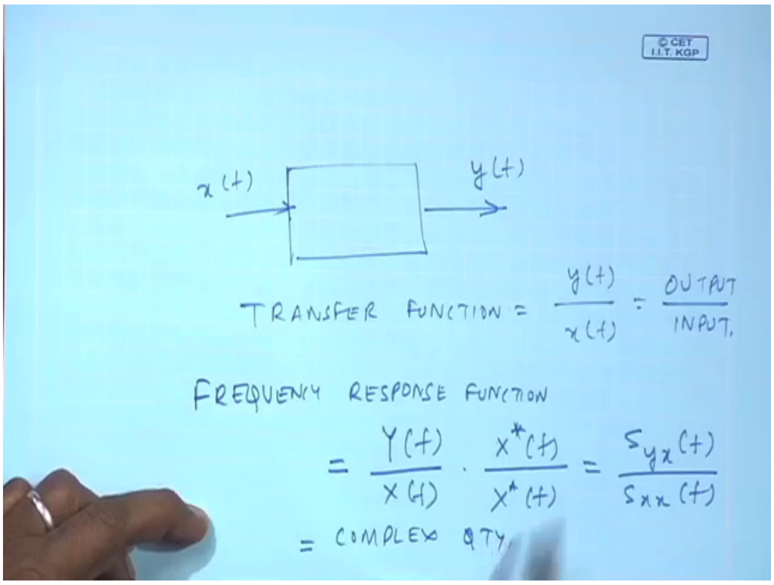

So, if I have a system I will get some yt and xt. So, the system transfer function is nothing by yt by xt or output by input. So, I can find out the frequency response function as y x by x f. So, if I multiply them by x conjugate f or x conjugate f. So, I will get term s y x f by s x xf.

So, this is just the cross power spectrum between y and x and the auto power spectrum affects. So, this is again a complex quantity.

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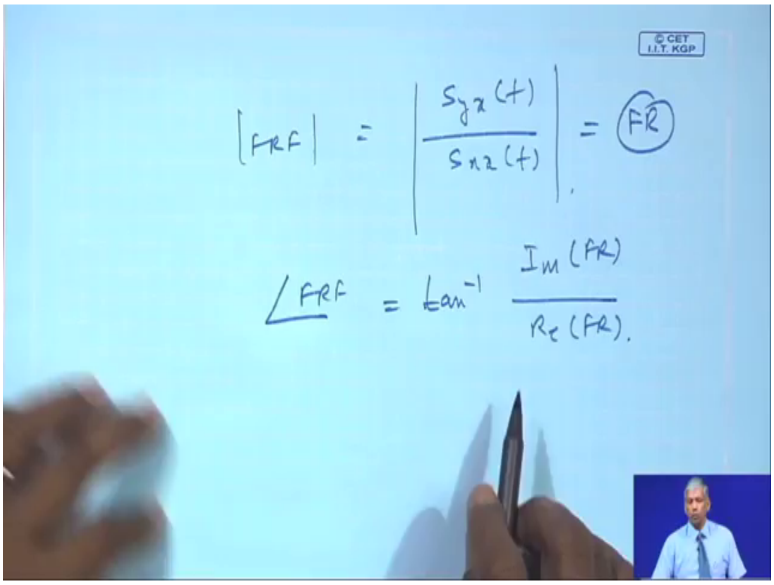

And the FRF magnitude is nothing but the magnitude of this term as y xx. And the phase of FRF is nothing but tan inverse if this is equal to f or tan inverse imaginary part of FR where real part of FR. So, commercial FFT analyzers can give you frequency response functions depending on the cross spectrum of 2 signals and the out of power spectrum. And cross spectrum also tells us what is the time delay between 2 signals and later on we will see it is application in finding out faults in systems and it is applications.

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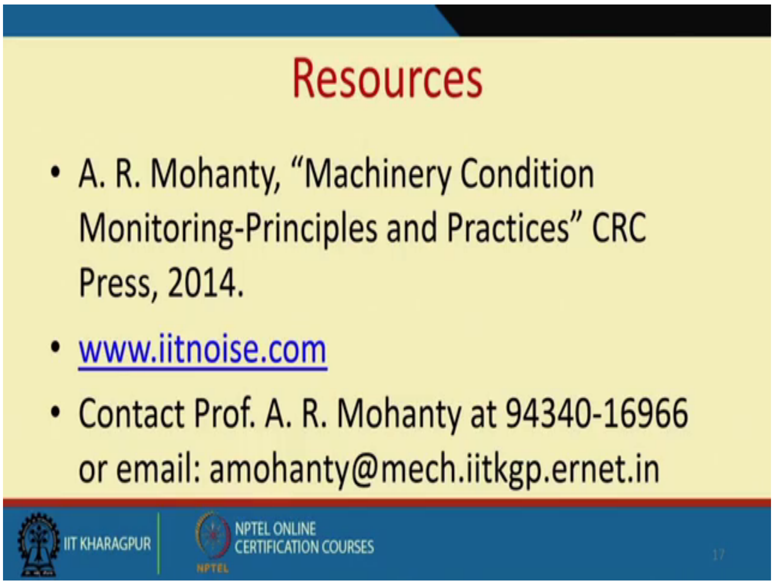

So, more resources on FFT and it is fundamentals can be found in the signal processing chapter in my book.

Thank you.