Machinery Fault Diagnosis and Signal Processing
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Lecture – 12
Frequency Domain Analysis

In this lecture on frequency domain analysis, we will be trying to understand how a signal, which has been acquired in the time domain, can be converted into frequency domain, because as we had told in the last class on time, domain analysis in condition based maintenance. Every machinery component will manifest itself at it is characteristic frequency in the signal. So, a signal will carry it is characteristic frequencies. So, it is our job in frequency domain analysis is to understand what are the methods available to us to find out the frequency of that signal, well in the very first approach we can do is from the time period.

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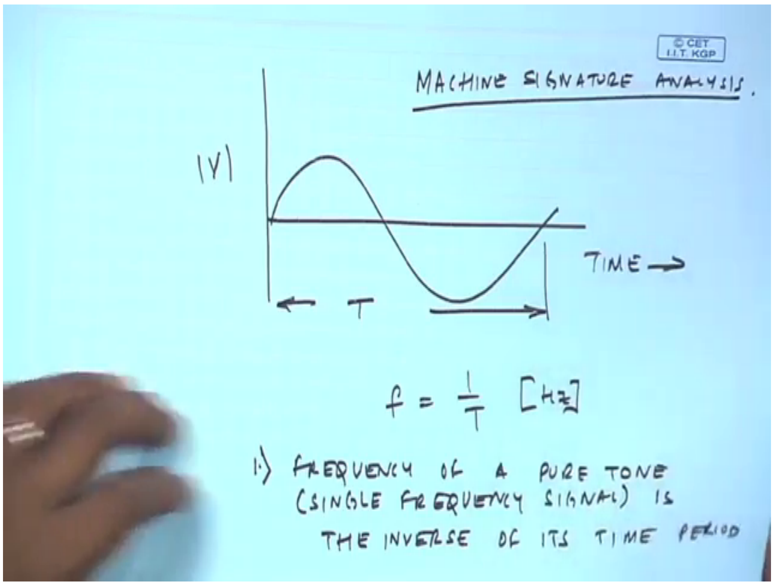

So, you know the frequency of this pure tone signal as it is called in hertz is nothing, but inverse of the time period. So, frequency of pure tone or in other words single frequency, signal is nothing, but it is the inverse of it is time period. Now, if this is time, some voltage, I can do it this way, but like we had seen in the earlier examples or every frequency component.

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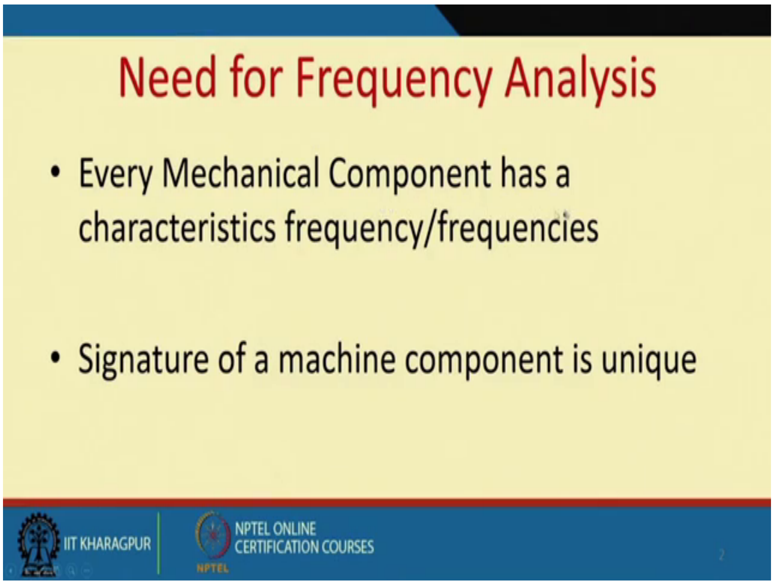

Has a characteristic frequency or frequencies signature of a machine component is unique and we will try to find out this signature. So, it is another one name for, this is the machine signature analysis. This is very simple you know when we have a single wave, but as we know the problem in real world is.

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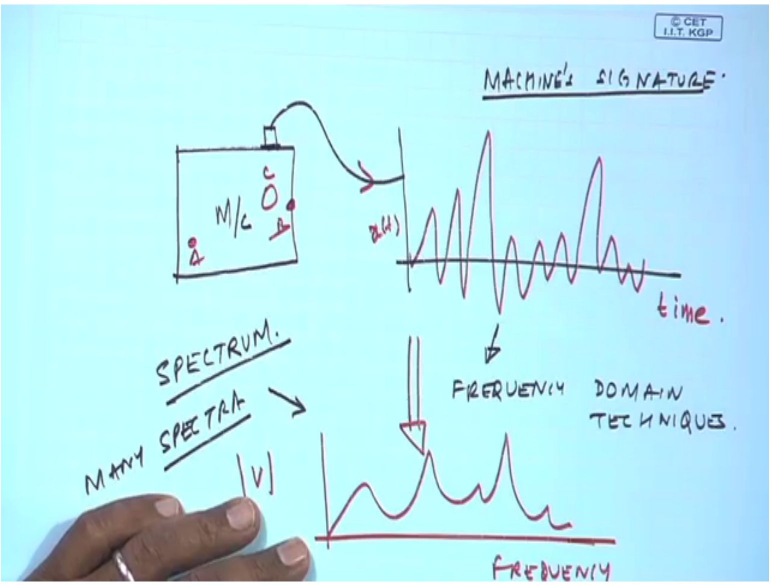

If I have a machine and I have put a transducer on some location did and there could be different components in this machine components A B C etcetera. So, it is this composite signal, which is going to come out in the time domain so; obviously, I am not able to use the inverse time period technique to find out the frequency. So, I need to find out what are the different frequency domain techniques available to me. By the way if I convert this into some frequency domain signature, this is my frequency axis and this is the amplitude as a function of frequency.

So, I get some magnitude. So, this plot is known as a spectrum. So, when I talk about or say vibrational spectrum; that means, the vibration signal, which has been measured, it is frequency domain representation, is known as the vibrational spectrum. So, one spectrum many spectra. So, this vibration spectrum or spectra contributes to what is known as the machines signature alright.

Now, question is how do you get this signature, well one is if I have many frequencies what I could do is, I could physically have filters.

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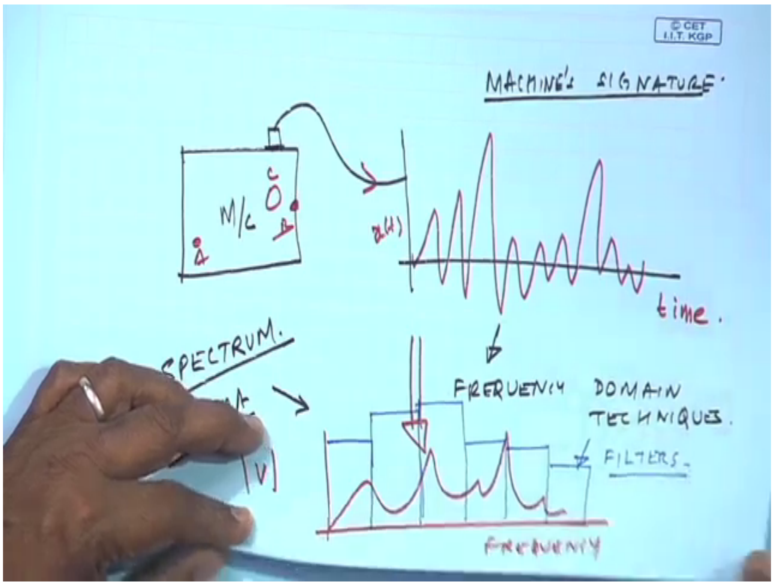

I can sweep the signal through filters. So, I could do.

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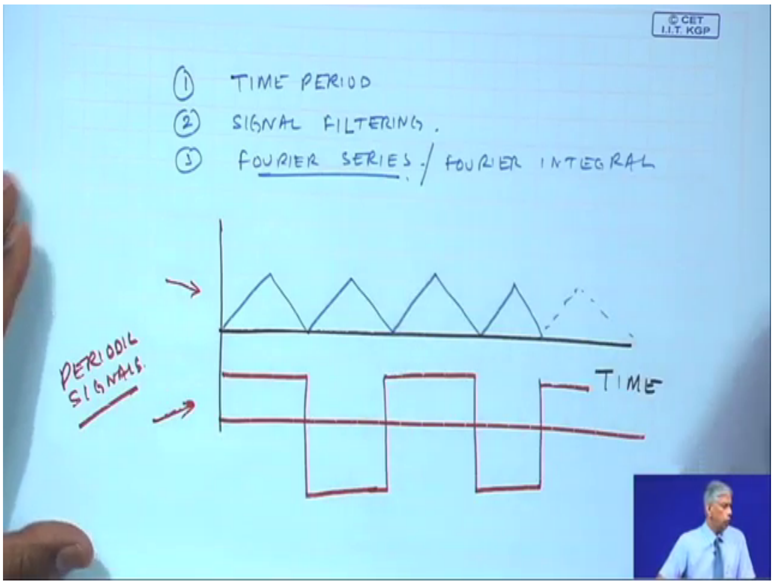

One was what I know is from time period another is through signal filtering. We will talk our filter signal, filter in later on and then another technique which we will see, is by applying what is known as the Fourier series and it is forms or I will write Fourier integral. So, we will see how this has to be; this can be done. You already know about this.

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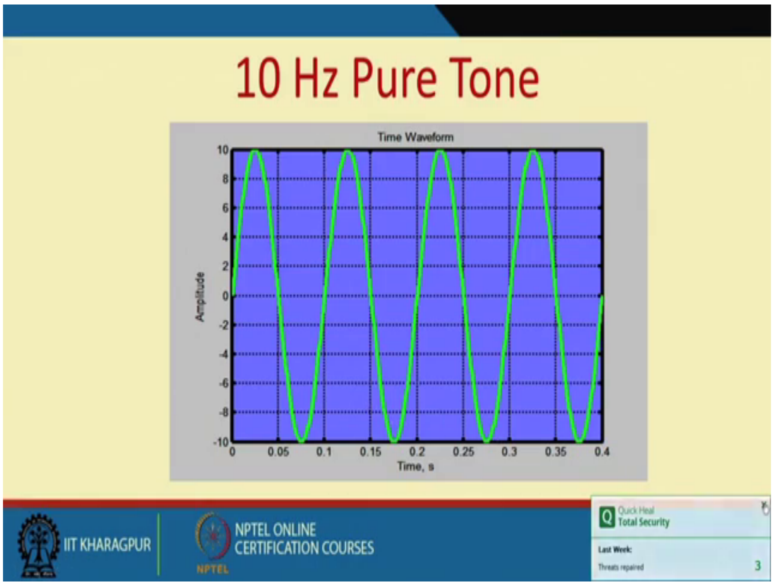

But when we have signal it is inverse time period will give us the spectrum,

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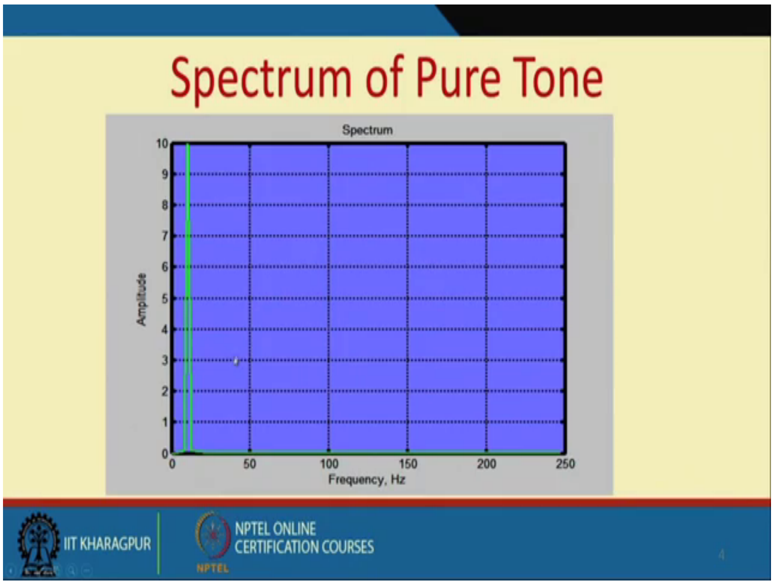

If I have many sin waves in this case two.

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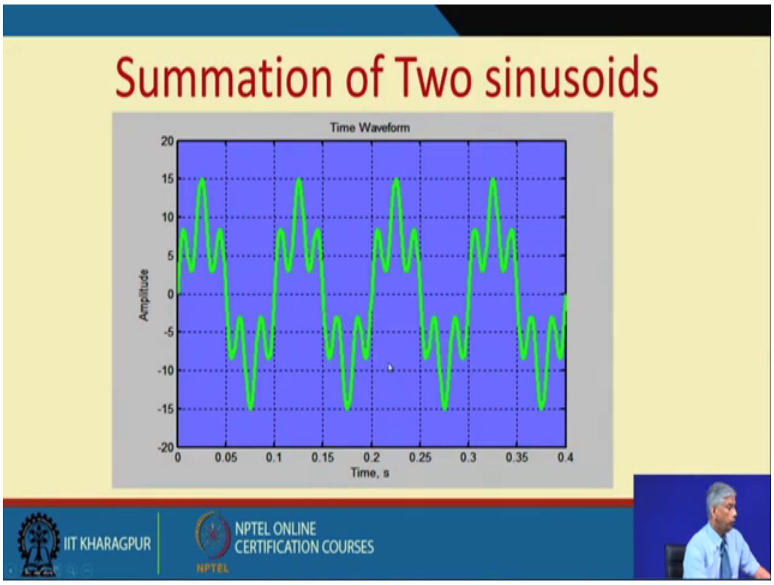

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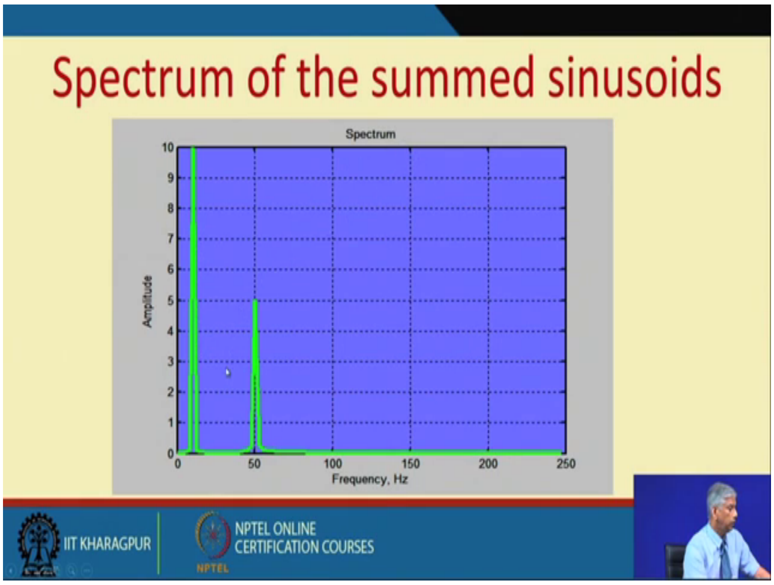

I can see that these two frequencies one of 10 volts 10 hertz and 5 volts 50 hertz gives rise to this kind of a signal.

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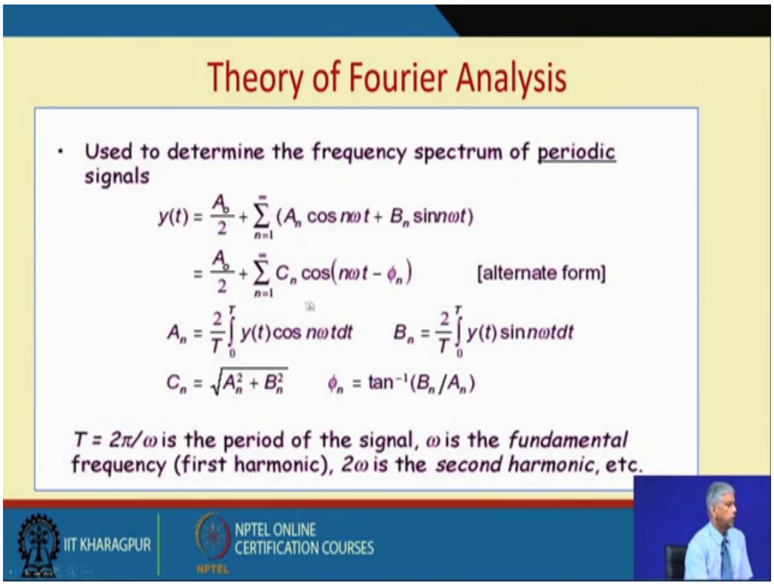

But we must have done this in our Math's class that if I have a periodic signal, I can represent it, periodic signal been y t, I can represent it as sums of sine's and cosines given by this expression, where A n is calculated by this expression 0 to T t is the time period of the signal and. So, on and B n is this and the amplitude C n is given by this and the phase is given by this expression here, where the first fundamental frequency omega is already known as first harmonic and then the second one is known as second harmonic etcetera.

But the idea behind this is the signal has to be periodic. Now what could be periodic signal? For example, signal which is repeating itself. So, these are examples of periodic signal, going till infinite another could be just a square wave. So, these are examples of periodic signals. So, by name, by finding out these coefficients A n and B n, we can plug it back into the equation. So, in other words, a signal in the time domain is represented as sums of sine's and cosines of different frequencies. The first one being the fundamental frequency, the second one being the second harmonic, third harmonic and so on, all the way till the infinite and this A 0 by 2 or A 0 is known as the mean component of the signal which is nothing, but the 0 to T y t dt, but you see here in this expression, here yt has to be mathematically given an expression. So, that this integration can be done, but that is the problem with us.

And for a sin wave

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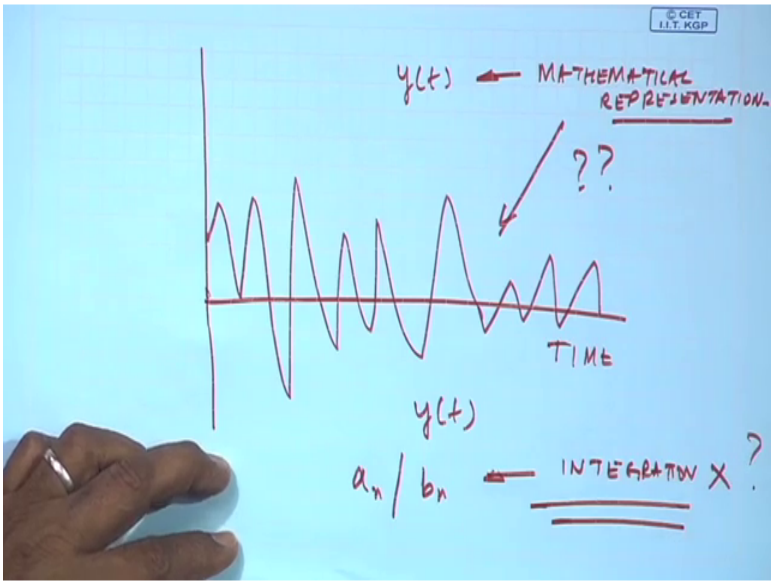

Square wave I can represent these signals mathematically, but the problem occurs in real world. Signal looks like something like this. So, this becomes tough to find out this expression y t. So, that I am not able to find out this coefficients a n by v n by integration. It is not possible. So, what we do is later on we will see that this integration is replaced by a summation and that is what we will be covering about later on and another thing is that

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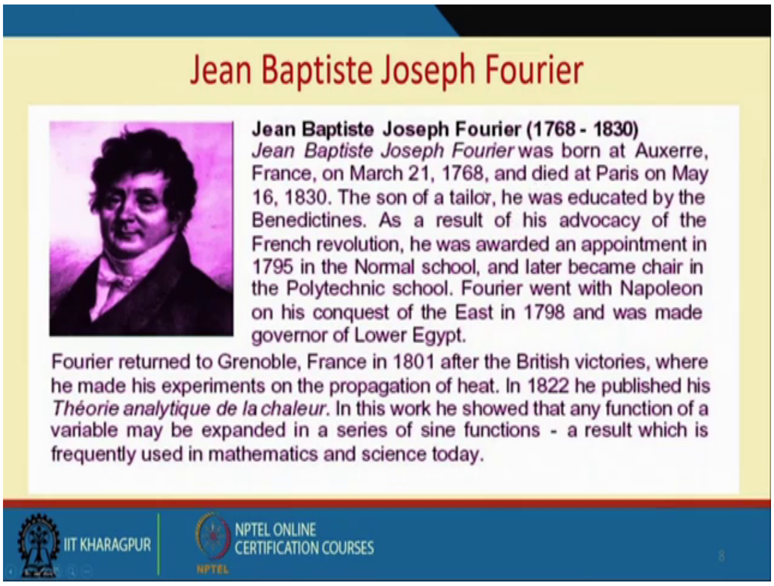

Well just before we get into Fourier transform. I would like each one of you to know about Fourier, they say that this is one of the greatest mathematical boons to engineering is this Fourier series, for that matter anything can be represented by sine's and cosines later on we will see what powerful? How powerful this technique is? Imagine.

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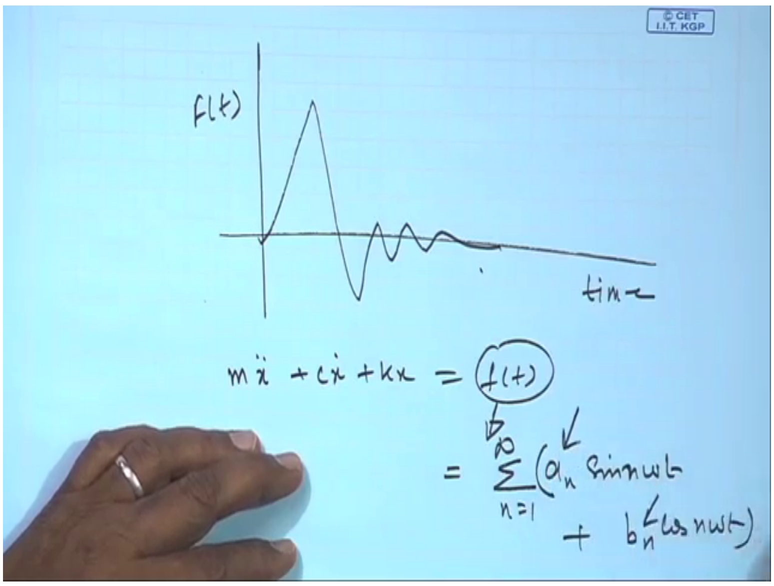

An impact mathematically, f impact functions can be represented as sine’s and sine’s sums of sine’s and cosines and those of you who would have solved an equations mx double dot plus cx dot plus kx is equal to f t. We all have very close form solutions. When f t is a mathematical like an harmonic expression like a sin wave or cosine wave. So, this is. So, powerful, the Fourier series that any force can be given as sums of sine’s. So, this can be all of the caches. We have to find out the An and Bn we will see how Fourier series is going to help us.

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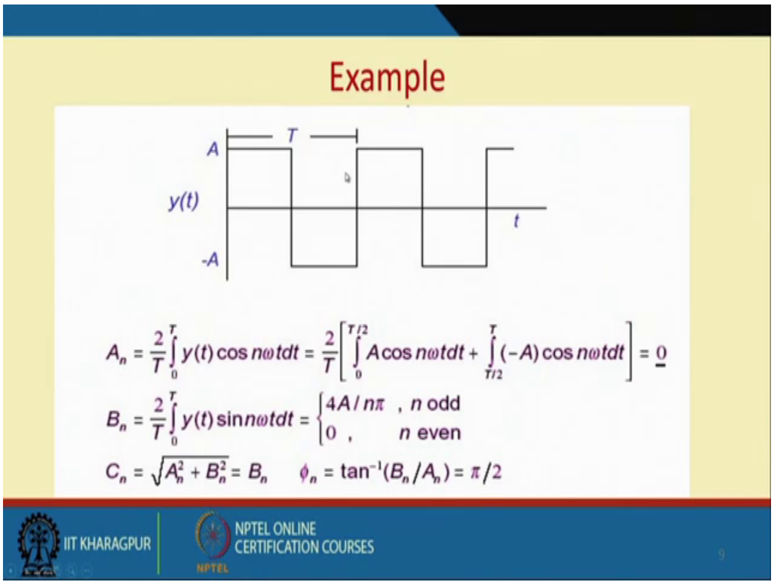

This is the example, wherein I have a square wave from amplitude a to minus a the time period t it goes up till infinite.

So, the coefficients An will be given by this expression and Bn by this expression wherein it is you know this could be boil down to. So, this expression can be split from 0 to t by 2, it is a and t by 2 to t, it is minus a. So, the coefficient A n happens to be 0 and B n becomes this.

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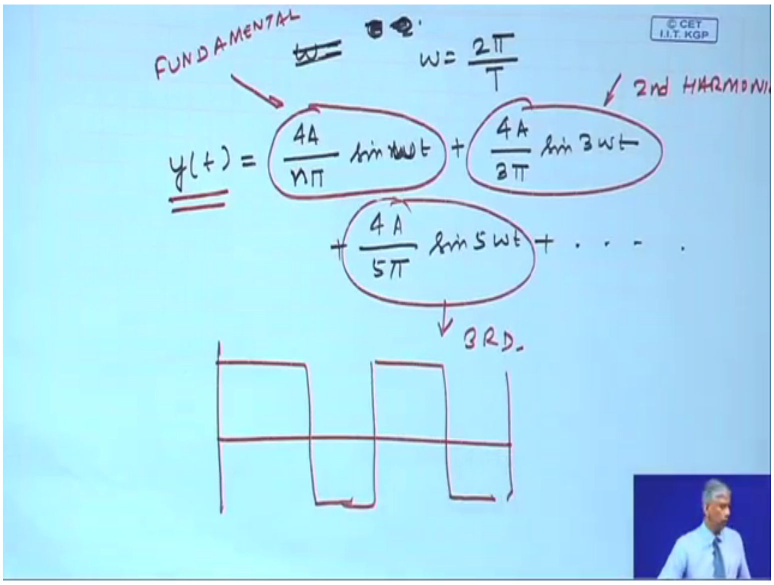

So, cosine square wave can be represented as 4 A by n pi and if you see here it is A n Bn sin n omega T. So, we have ans are 0. So, this will be sin n or omega T plus 4 A, where n is odd 4 a by 3 pi sin 3 omega t plus 4 a by 5 pi sin 5 omega t plus so on till finite terms.

So, you know this is a sin wave. So, what does this physically signify and; that means, the fundamental frequency omega is nothing, but where T is the time period and this is the fundamental frequency and this is the second harmonic and so on third and so on. So, a square wave which is periodic can be represented as sums of sin waves of many frequencies. So, if I look at this spectrum of the square wave this is in time domain.

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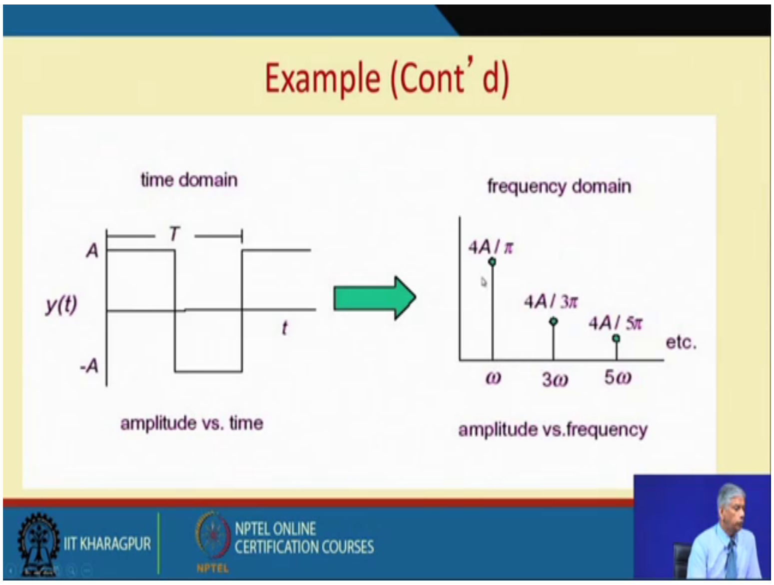

If I look at the spectrum, it will show as, at the fundamental frequency omega. It has an amplitude 4 A by pi, at this second harmonic. It has amplitude 4 A by 3 pi third harmonic by 4 A by 5 pi and so on and all the odd terms are there.

So, if I add these signals, I would get back my original square wave.

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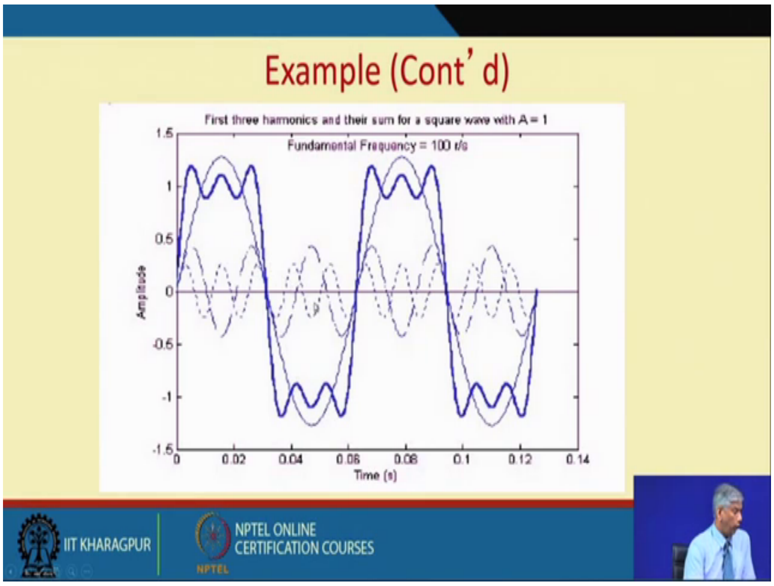

You see, this is the, this one is the first fundamental sin wave, the second or the fundamental, the second harmonic, third harmonic and if I add them up I get this dark blue line and all of you will understand that that if I have infinite such terms, this will closely represent a square wave. So, in other words what are the takeaways from this exercise; that means, any periodic signal can be broken up into it is Fourier series coefficients and these coefficients are nothing, but the amplitudes is nothing but

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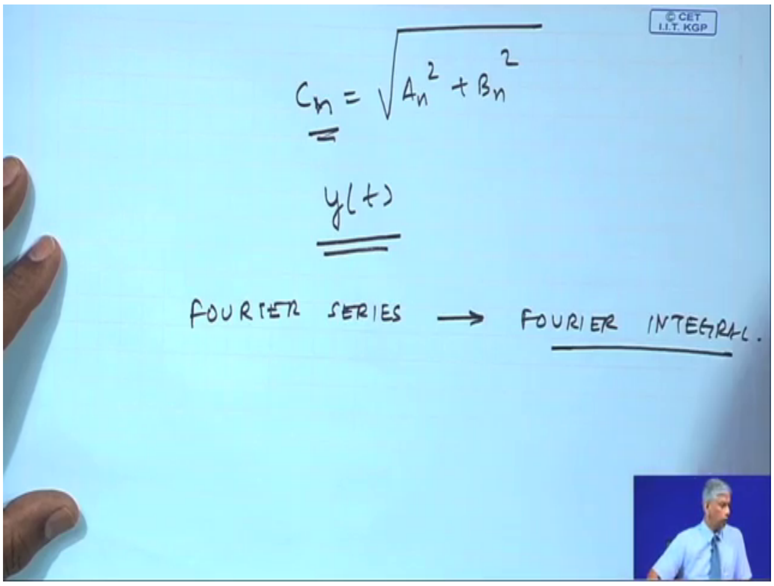

I can find out this amplitudes at any n corresponding to the frequency of that signal.

So, as you will see this is different, if this was a sin wave, I would only have one term. The sin wave is just a pure tone signal. So, if I have a trapezoidal wave or a triangular wave or a saw tooth wave I will have different forms of this expression to represent them in Fourier series and if you look into any handbook on mathematics you will see the Fourier series expansion of periodic such waveforms and this is also available in any books, but the problem with traditional Fourier analysis is; obviously, difficult to implement numerically for a measured signal one having no obvious mathematical form like I has told you, I need to have expression yt known to me, but this is not known to me and this is limited to periodic signals.

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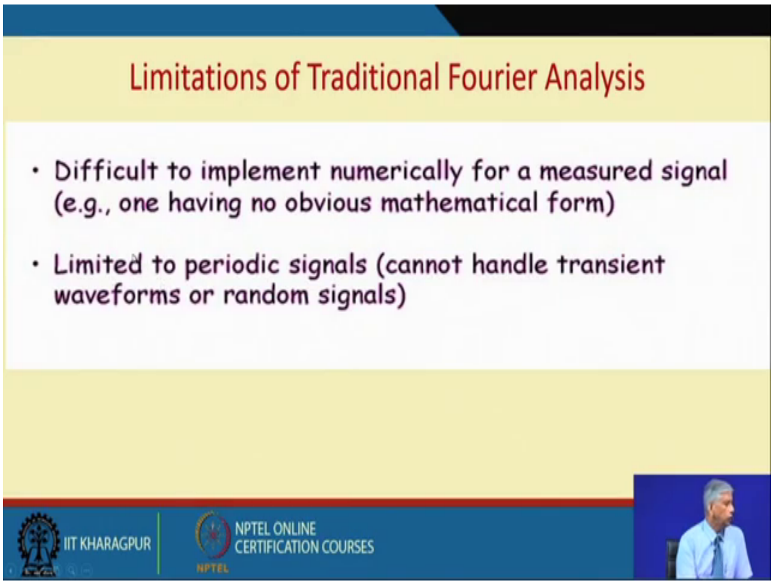

So, this Fourier series, thus gives rise to what is known as a Fourier integral, which can be done for non periodic signals like; transients, random signals etcetera.

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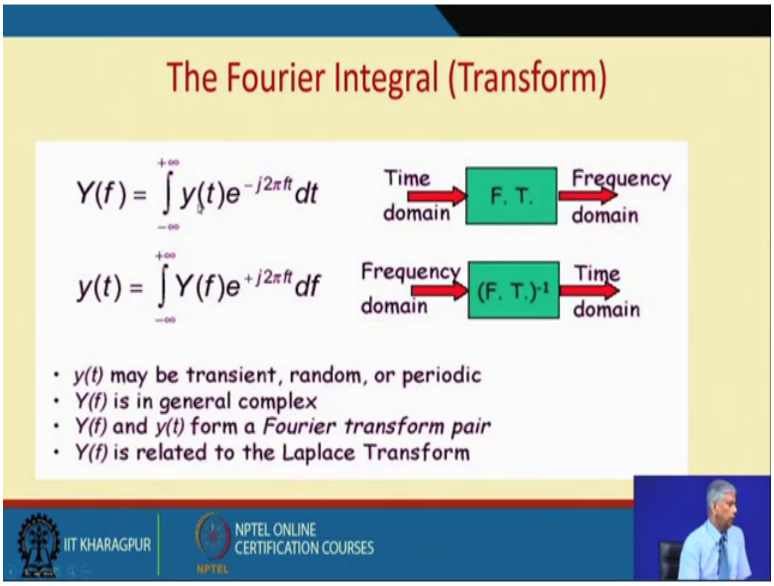

So, any signal yt can be represented, if I do this multiplication by e to the power minus g 2 pi f t dt from the time domain, I can go to the frequency domain or from the frequency domain by an inverse transform, look at the signature signs. Here I can get back my Fourier the signal. So, this is very important that I have a signal

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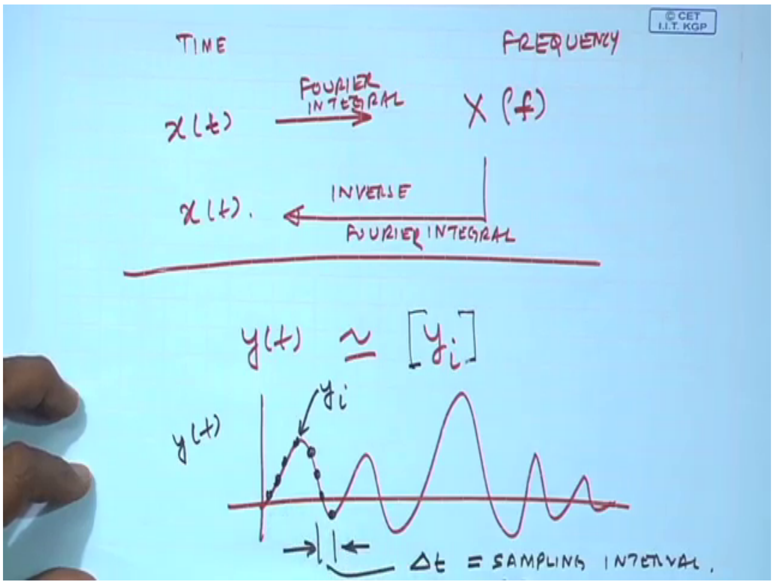

xt, I will do the Fourier integral, I will get the signal x f. This is in time domain and there is in frequency domain. So, this is the frequency and this is in time.

Now, the same signal, I can do an inverse Fourier integral and get back my original time domain signal. So, this is very powerful in terms of application in machinery condition modeling for because for any signal i can do this and we will see how this is implemented, but again you see this yt may be transient random or periodic yf is a complex number.

So, this becomes a complex quantity and this pair is known as the Fourier transform pair now mathematically to represent yt all i have to do is i will represent as yi; that means, if I have a signal any signal, I will pick it up a different points and each one of them this was my yt each one of them is yi and this is related to the distance, this is known as the sampling interval.

And if I have a set of numbers in a series in a the constant sampling interval. I can digitally implement this Fourier transform and this is known as

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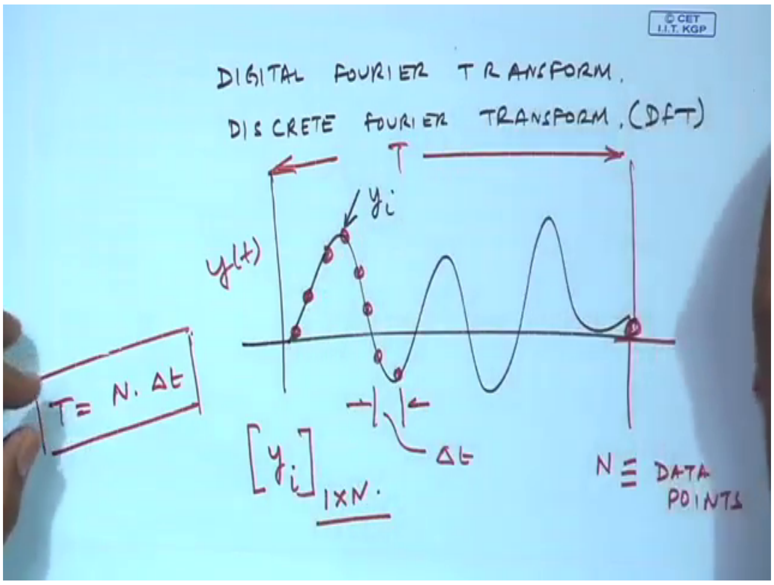

Digital Fourier transform or sometimes known as discrete Fourier transform. So, any signal DfT any signal, which is available to me I will, we will discuss about the sampling points, later on I can select through a mechanism yi for you know maybe n data points. So, this yi will be represent by an array series of number and this is known as the delta t. So, total time taken t is nothing, but n times delta t.

So, if I have a signal of total time t which is nothing, but number of data points times, the sampling interval, I will sample it and implement it, this integral which I discussed here, this can be digitally replaced by a summations and a series I will have, if I have n data points I will have n square complex operations done with the sequence to find out the Fourier coefficients of that signal and.

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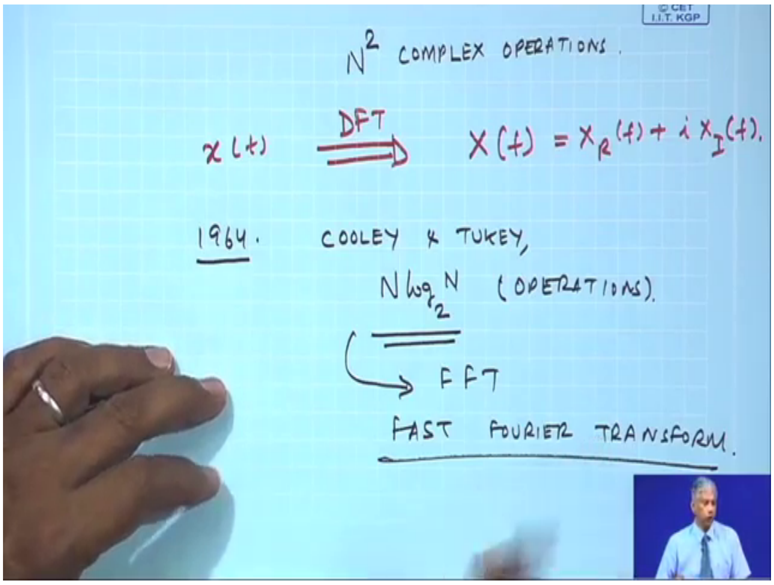

So, if I do a signal xt, if I do a digital Fourier transform, I will get a signal x in the frequency domain, which will have a real component and an imaginary component, but this d square as you will see requires n square complex operations. So, in 1964, one algorithm was developed by Cooley and Tukey, where this operations was only n log to the base 2 n number of operations. So, when the number of data points was log this required insignificant amount of computation time and this then came to be known as FFT or the fast Fourier transform.

So, nowadays you know any signal you get, you can do a fast Fourier transform. There are algorithms available based on n log and to the base 2. So, that we can do the FFT operations.

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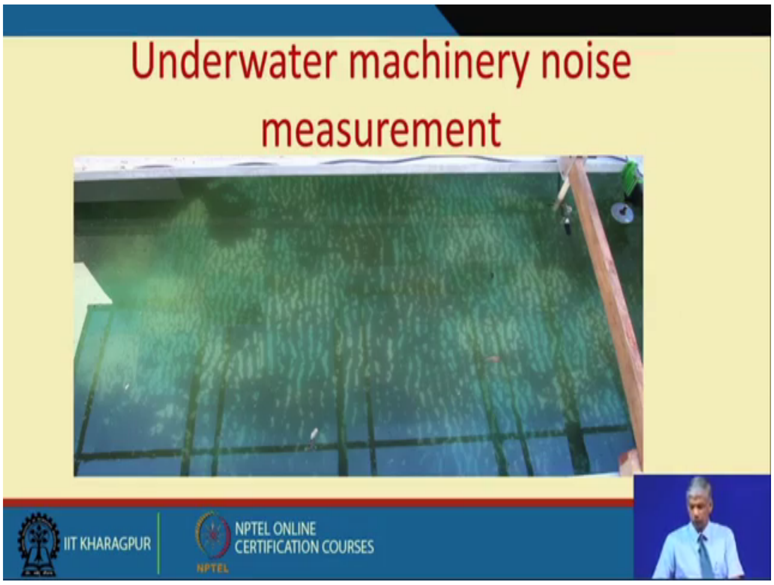

We will talk about the relationship between time domain and frequency domain in the next class, but this should suffice to say that any signal I have, I can.

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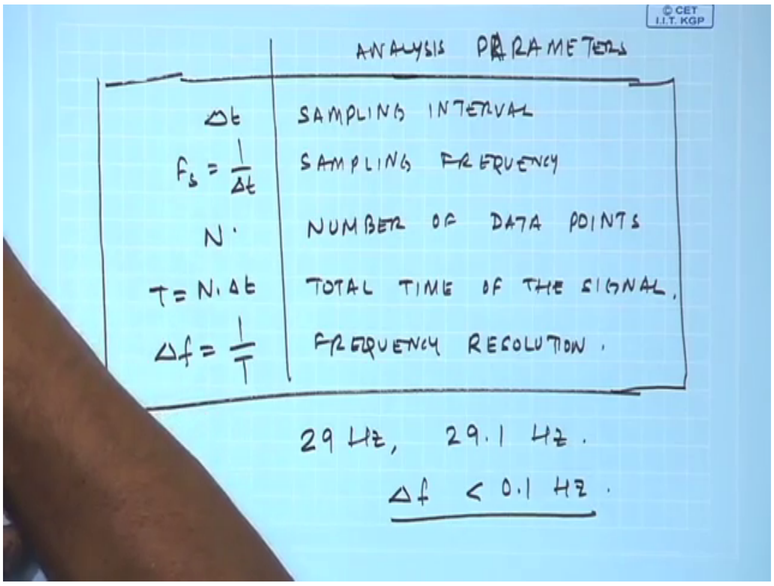

take the parameters delta t is nothing, but the sampling interval f s is 1 by delta t is known as a sampling frequency and in the number of data points t is equal to n times delta t is nothing, but total time of the signal delta f is equal to one by t is the frequency resolution.

So, we need to get a feel of this quantities the sampling and depending on a, this is this field, because you will see always that there is an inverse relationship between time and frequency. If time is more frequency resolution is less, if time is less frequency resolution is more. So, one who does FFT to understand the frequencies in a signal, you want us to be careful about this quantities.

For example, I have a signal 29 hertz, coming out of a machine in another signal 29.1 hertz. So, unless I have delta f less than 0.1 hertz, I will not be able to distinguish these two frequencies and that is very important for anybody who is doing condition based maintenance or monitoring to exactly. Identify the signals you know are the frequencies in a signal, one has to FFT, I will show you a signal of an earthquake signal and this is an earthquake, which occurred in 2015 I believe.

So, my students in the laboratory were doing measurements with an underwater hydrophones, here you can see a hydrophone here and suddenly that earthquake occurred in Nepal and we were in Kharagpur, you know from close to more than you know 1500 kilometers or 2000 kilometers and this earthquake waves are seismic waves coming in underground and you know my students were recording it. So, they did not realize that it was an earthquake, till the later on switch on the tv but

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If you see here, we were, this is where this earthquake occurred on 25th April, sometimes in the early around, in the late morning and we are here, it closed to Calcutta. Kharagpur is close to Calcutta about 120 kilometers.

And this earthquake happened and they were recording the signals.

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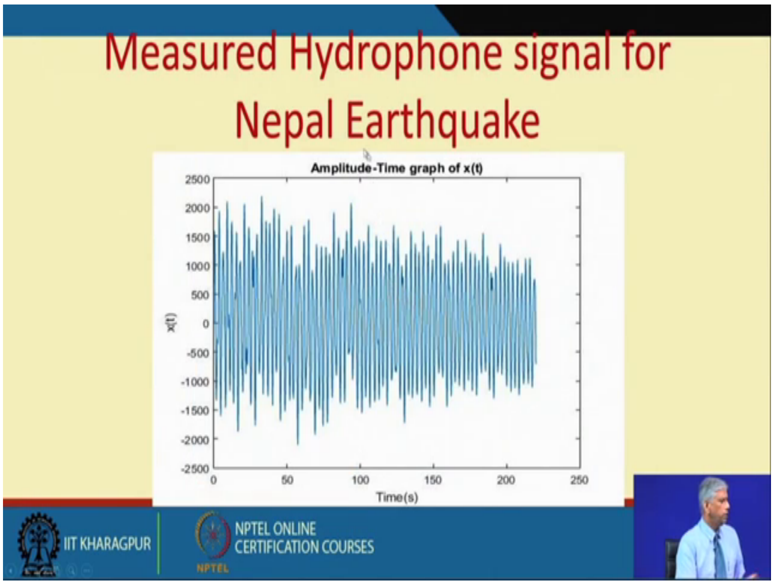

So, immediately the hydrophone captured this signal, because this is the time domain signal, time in 250 seconds and they found this signal, which is because of an earthquake and then they did the spectrum.

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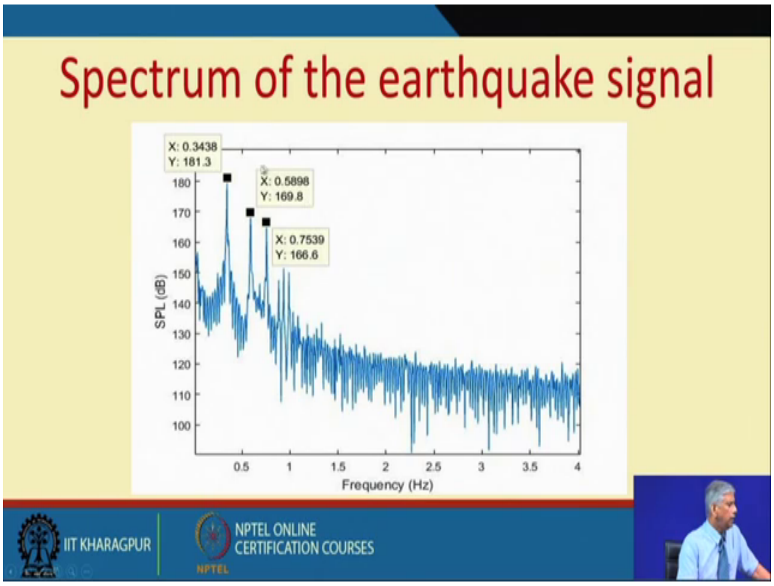

Of the earthquake signal, this is an frequency and you will see, this is the sound pressure level in decibel, in underwater, but you see a frequency of 0.3438, a very low frequency signal is predominant. This is characteristic of earthquake signals.

Now, something like in less than 1 hertz, you know this is a point 0.0589 hertz 0.75 hertz. So, this has been done by FFT of an actual captured time domain signal. So, with this I would like to say that this could have been an actual machinery. So, every machinery or every object gives such signals and one has to do an FFT to understand the frequency contained in the signal. So, in the subsequent classes, we will see what are the other techniques of doing FFT and so on

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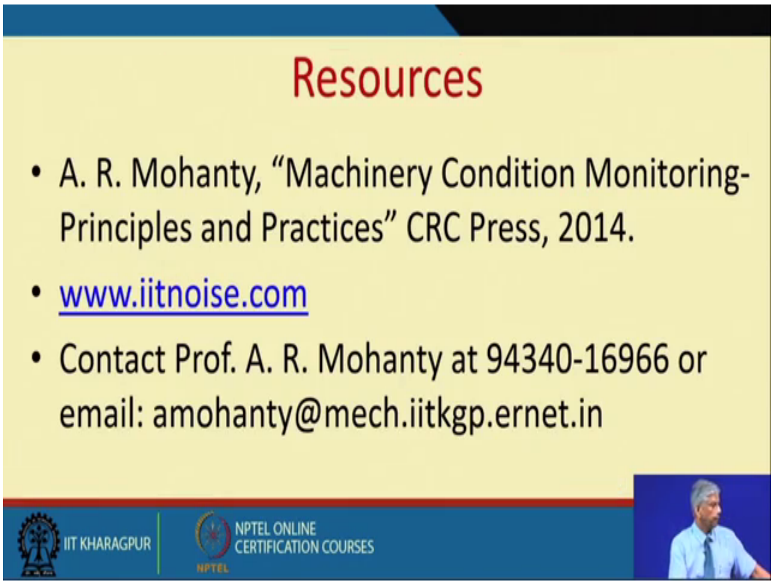

Thank you.