Introduction to Mechanical Micro Machining
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Lecture – 09
Scaling Laws (Contd.)

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Good morning everybody and welcome to our course, on introduction to mechanical micromachining. In the last class we have started with scaling law effect and we have seen that how different parameters behave or affect a system at different scale, when you scale down one system from very higher scale to the lower scale. And we have seen some of the examples of different parameter like surface to volume ratio and weight to surface tension and let us see some more examples in the same way, what a different parameter settings.

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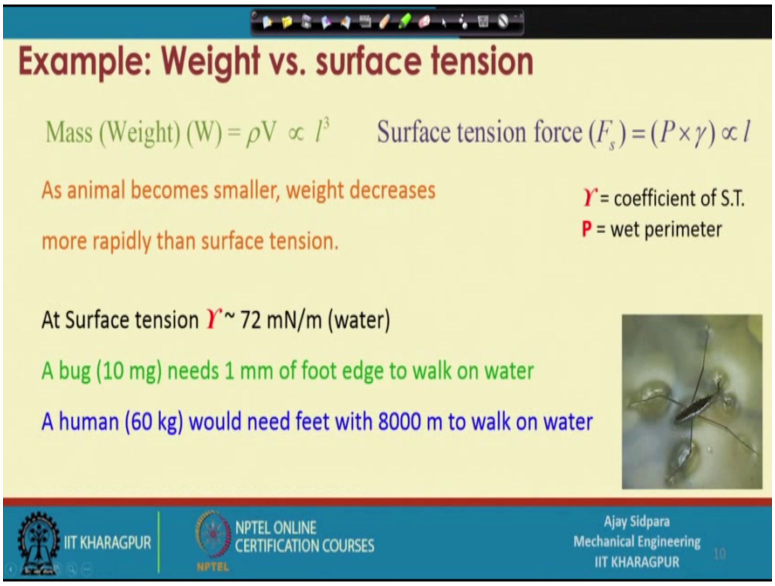

So, now, in the last class we have seen this example that what will happen when the surface tension will be more, compare to the weight in this different scale.

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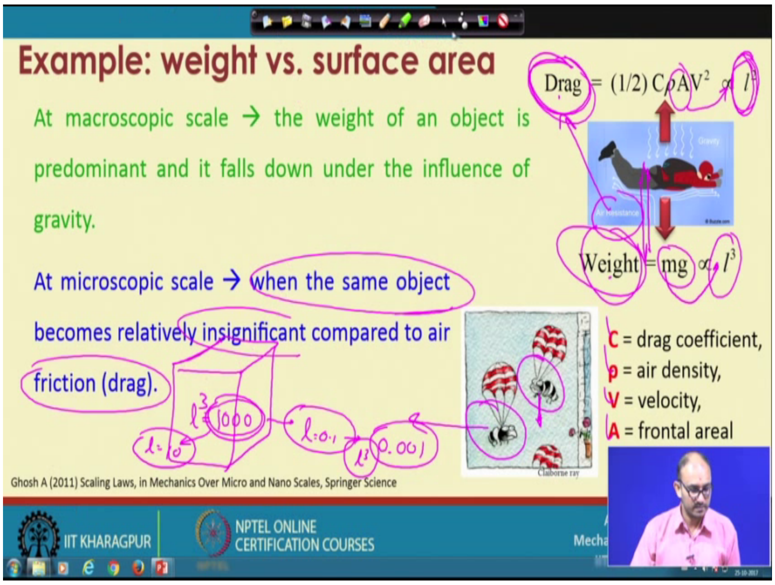

Now, there is one more (Refer Time: 01:06) to weight scale of the surface area at macroscopic. The weight of an object is predominant and it fall down under the influence of the gravity. Now, if you see this particular example there suppose, men or human being is falling down from the top. So, what are the forces acting on that? One is the weight, which will pull it down that is considered mg. So, this mg and it is mgm, is the parameter which will be affected as A l q, because it is a mass of that part and there is a drag force, which is moving, think something in the upward, that is called a resistance. It will allow this person to move up and up that mean; it will reduce the free fall down of the part.

So, that is given by the one up C rho A V square then that will proportional to l cube. Now, if you see drag coefficient is the c, air density is velocity, is then frontal area is this part. So, this area is proportional to the l square right. So, here if you see the weight is scaling as a l cube and the drag is scaling as a l square; that means, the things, which is pulling down the person, it is scaling as a l cube and things which is pushing this person in upward direction, it is scaling as a l square. Now, what will happen in this case? Now see, when microscopic scale, when the same object becomes relatively insignificant compared to air resistance.

Now, consider let us take the example of an ant. Now, if you are putting this ant from a 1 floor or the 100 meter or considering 100 of meter, then nothing is going to happen, when it is reaching to the bottom, it will actually go very freely, but same thing will not happen to the man when something is pulling down from a large scale.

Now, what is have different, because if you consider here difference is the weight, if you see weight of an ant is considered as a very small compared to the size. Now, considering the size now, this is the size of a l cube and this one particular thing, all volumes is a one particular part. So, let us consider, this is a 1000. So, it will be scale as a l equal to 10 and now, same thing will happen l equal to 0.1, then everything will scale down very small part here. So, if you put this particular parameter l is l equal to 0.1 and l equal to 10, in this particular part weight and the drag force, you can consider weight will reduce very drastically, because this particular thing. l cube will be 1000 and this thing will be the 0.0 0.001. In this case that is in terms of the l cube.

So, what is going to happen? The weight is reducing very fast, but drag is not reducing that fast. So, comparatively drag is very large, compared to the weight which is pulling down, this particular object from the bottom. So, that is the reason that whens very heavy things will falling down from the bottom, then it will go very quickly, but when the light weight things, which is falling down from the same height or the even more than that, it will not reach with the same velocity or it will not get the same amount of register or same amount of gravitational force, when is moving down.

So, that is the important thing the small object becomes relatively insignificant compared to the rare resistance, which is consider as a drag force. So, this thing is important, all that we will see some of the things, in this spindle design that what is going to happen? When your spindle is over hang for a longer time? Now, coming to strength to weight ratio.

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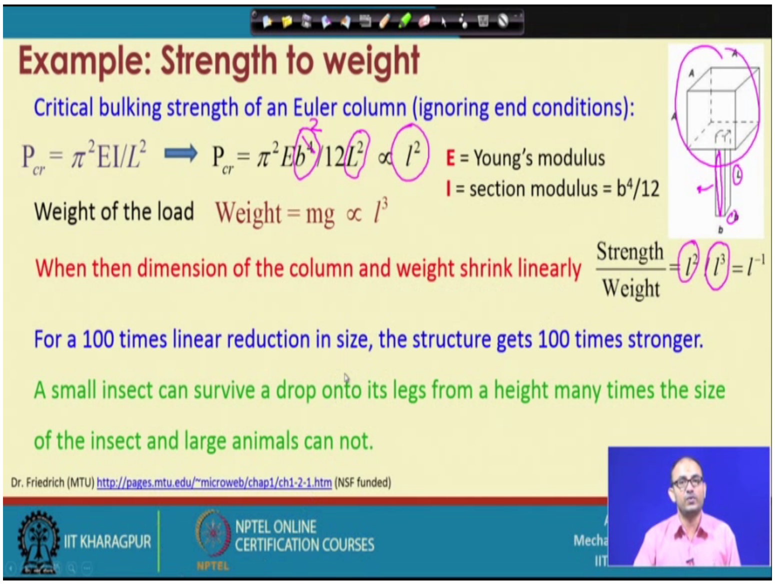

Now, this is also important, now consider the critical bulking strength of a an Euler column ignoring the end condition. Now, this is one particular example; here we have seen. So, this whole weight, it is taken care or it is loaded or it is mounted on the one particular column, it has a b b and L is the length of that particular column.

So, let us see what is going to happen in this case. Now, critical buckling strength is given by PI square E I into divided by L square. What are this parameter? E is the Young's modulus, but it is no connection with the length scale. I has a sectional modulus. So, it is connection, because here both things are b and b. it does not the any different domination, it will be b raised to 4 by 12. So, if you put those things here. So, now, what is going to come here that you keep the PI square as it is, E is not changing here. You put I equal to b square by 12 andL square is also dimension of these particular part.

So, now we have a two dimensions. So, this is also length scale and this is also length scale, but in our b square is on the top and l scale is at down. So, this will become two and this whole equation will behave as a l square in terms of dimension. And we know weight is considered as mg where m is the one of the parameter, which has connection with the length scale and it will behave as a l raise to cube. Now, what is going to happen in this case, when dimension of the column and weights shrinks linearly that we are considering both the thing because this is connection, connected with the weight and this is connect with the dimension. So, if you consider only length scale not considering any other than that you reduce the l, if you reduce the l m g will reduce and when you reduce the l your this whole parameter will change.

Now, what is going to happen that strength weight ratio, now strength is scaling as a l square and weight is scaling as a l raise to cube, the whole thing will scale as a l raise to minus 1. So, what is important thing here. So, I reduce the scale here. So, all the weight will reduce at the l raise to cube. So, if you are 1 kg, you are reducing to the 0.1; that means, it will go very drastically in terms of the weight, but the strength is not reducing such a fast, if it is l raise to square only. So, what is advantage here? So, for a 100 times linear reduction, in the size structure, gets 100 times stronger, because it is l raise to minus 1.

So, what is the advantage here that if you are scaling down, all the system in terms of cantilever beam or something which is taking the weight of the part, then it is very important, that if scale down smaller things will actually keep the weight very large. So, what is the example, we have seen the smaller insect can survive a drop onto it is leg from a height of many heights. If you fall down any even lizard also, it is falling down from the bottom of a 2 3 floor building, still it will walk down very quickly and does not make any difference in they, but large animal cannot, if you do same thing with a elephant, with respect to part then at the bottom it will die.

So, in this case these are the example of a natural (Refer Time: 08:19) same thing will happen with the man made things that is when you design some beam element or something which is hanging on the top of that, and some weight is on the top of it, at that time these particular scaling will important to understand how the strength to weight ratio will affect the design configuration and coming to scaling effect on the spring constant.

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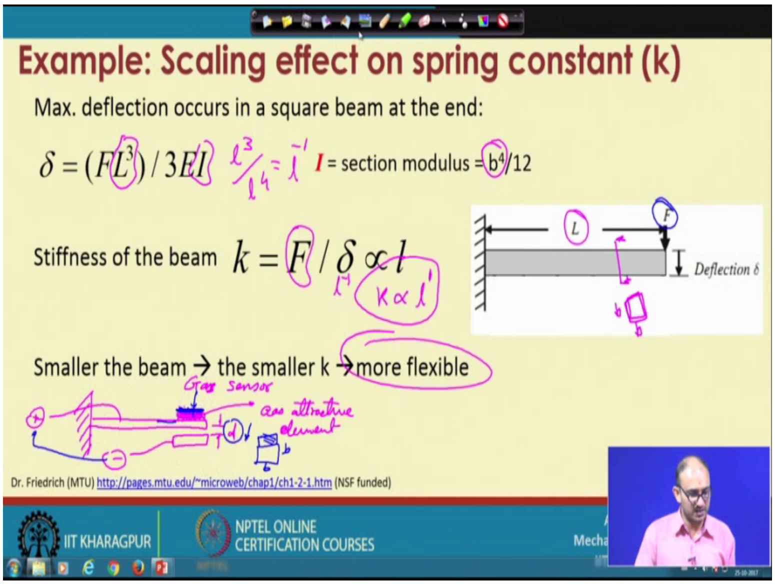

Now, we have, let us see those things. So, now, there is a beam and beam it is a cantilever beam and at the end free end, we have applied a force F and there is, because of their force F we will get some deflection here. So, maximum deflection occurs at the square beam yes. So, this is the square beam. So, this cross section is something like this.

So, what is the equation deflection will be F into l cube divided by E into I. Now, f is the force, which is here, length l is this, l particular length E is the modulus and I is the section modulus and again the section modulus is, this is the b by and consider this is the square. So, this both dimensions are b.

So, here that is why it is a b raise to 4 divided by 12 and stiffness of the beam, given by the force divided by deflection and if this is the equation. Now, what we are coming here. So, now, if you put this equation, let us take this equation first. Now, here though, this is the parameter, which consider as a dimension of parameter. It comes directly, connected with the length l and I is also dimensional parameter, it has a connection with the cross section of the b. So, here it will be the l cube divided by l raise to 4.

So, that will; that means, it will be l raise to minus 1. So, now, consider this same thing, again a force is here, but delta that deflection is at the denominator. So, this will be the l raise to minus 1, when it will come back on the top; that means, k proportional to the l raise to 1. So, this is the formula, the smaller the beam. So, now, let us consider now it is linearly scaling with that part, the smaller the beam smaller the k value and more is the flexible. So, that is the advantage of going down into the dimensional case. So, your beam become more and more flexible, because this beam, if you consider the mems or the micro electromechanical system, there are many applications where you use the cantilever beam as the sensing element.

Now, if you take one example, let us take one example. So, this is the cantilever beam and this is the consider, it is a consider as a capacitor. So, this is connected with the positive. This is connect, is a negative and this is the gate between the two plates and now you are adding one particular component gas sensing component. Now, consider this examples is a gas sensing con sensor gas sensor, that we want to detect the concentration of a particular gas in a environment.

So, what we do that, we there are different elements available, which will attract only particular type of gases. So, let us take one particular element which is this, is the element a gas attractive elements. Now, we are putting that thing and now what will happen this beam is very small. So, it is more flexible. So, what is going to happen with this, that when some we were putting this old things, old system in environment and it will start catching the gas molecule of a one particular component.

So, this gas molecular will start sticking to the top surface. So, these are the gas molecules, it will start sticking to this part and because of that what happen that it is considering as a one of the parameter as a f parameter. So, when it is happening at that time, what will happen that your f is reduce increasing, you are loading this part, because of that and then your d will decrease as that d will decrease, you will get a different capacitance here and because of that you will sense that what is the concentration of that, because this particular beam if you see, here it has a dimension of a b by b and this is the top area always.

So, you know what is the area, which is expose to the gas. So, you know the area, you know how much a deflection. So, you can find out the what is the concentration of the gas or that particular element in a particular environment. So, these are this is one of the example, there are many example like that which you can use as a sensing element or gas element or actuating element.

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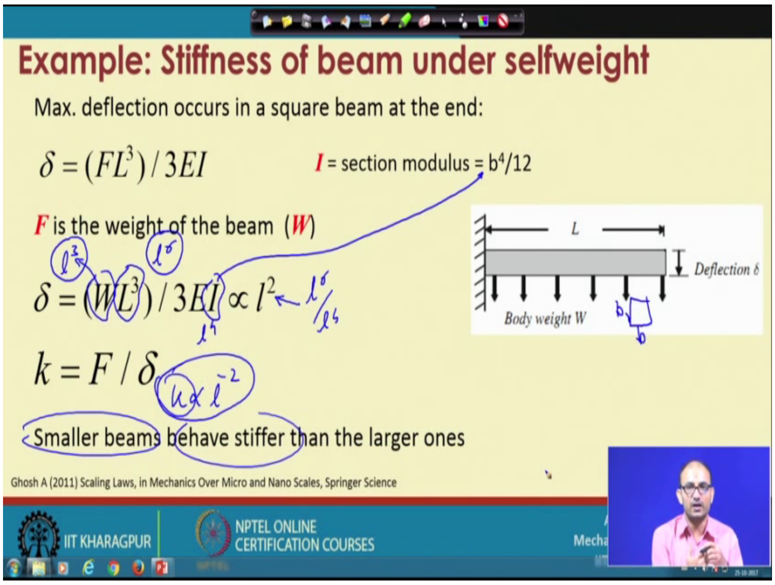

So, stiffness to beam under selfweight let us take this examples also, now here what we are doing that, we are not putting any weight here, let us see that what is the weight of the beam which will safe sustained; that means, up to which length or which weight you have to make a beam. So, that it will not fall down on the, it is selfweight.

So, now we are reduce removing that particular f from here again the same equation deflection is given by F l cube by E I again the same thing here. Now F will be reduce by the weight of the beam, because we have removed this weight load external loading, but we consider this beam weight itself behave as a again the beam weight is given by W and we know that W is; that means, we have consider the earlier case that this beam is a square beam the dimensions both things are b b and that is written here also.

So, this is the w now you are in terms of in place of f you are putting w here and if you are putting w here then what will going to happen here now W as a l cube we have seen that it is rho into v and already l cube is here. So, it will be l raise to 6 and at the denominator I is then I is connected with the b raise to 4. So, at the bottom it is l raise to 4.

So, it will be l raise to six by l raise four and that is the reason you are coming as a l raise to square right. So, and we have the stiffness that is force divided by deflection and this this will scale as like that. So, smaller beam behaves stiffer than the large one. So, now, consider this k is here. So, your delta is here delta will be at denominator. So, k is proportional to the k is l raise to minus two. So, if you reduce the dimension; that means, smaller beam behave more stiffer; that means, if you reduce the dimension k we will go up. So, if we there is no any external loading now considered it is as this it is just a beam all then you are you want to see the what is the total length or the total dimension of the beam you have to make in such a way that it will not fall down from that part.

So, again you have to consider stiffness. So, its scaling down as a l raise to minus 2. So, smaller is the beam more stiffer is the part. So, that is the way you can design the beam for the different system also now strength of a cantilever beam

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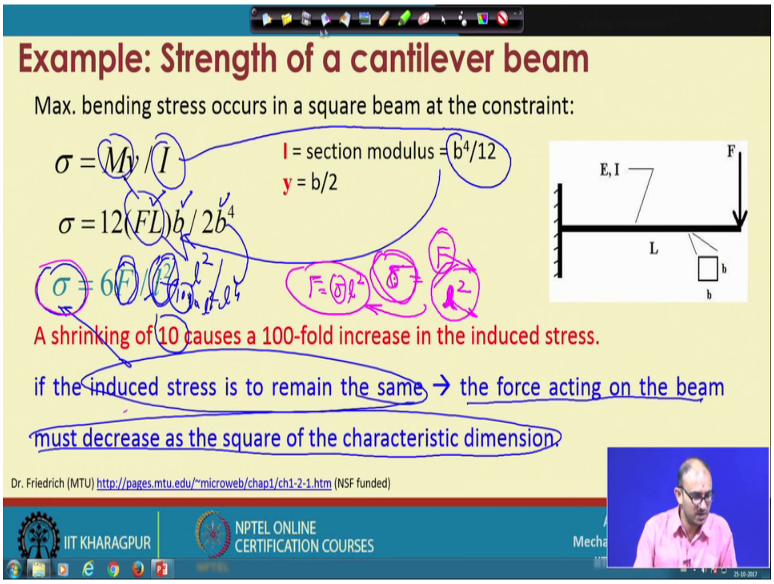

Now let us consider strength because why we are taking example of cantilever because it cantilever is a mechanical system it can be used for the different design element when it is in assembly in different part. So, maximum bending stress occur in the square beam at the constraint same again we have taken them x here that the square beam dimension cross section b by b we are now putting we are one putting one load at the free end and dimension total length of the beam is l and we are uses the modulus and the section modulus.

So, now, it is a sigma equal to m y divided by I now sigma this is the maximum bending m is the movement and y is the b by two it is the distance from the outer surface to the neutral area and I is the section modulus and now let us put I equal to b raise to four by twelve. So, we are putting that and m we are putting f into l. So, that is the load given here divided into l area. So, this is multiplied by this way. So, this m becomes f l and I we are using from this equation then we are putting it here in this case. So, what are the parameters which has length scale. So, this is the one length scale this is another length scale and this l as one more l square and this is also l square.

So, now what is happening here that here we are ending with a sigma is equal to 6 f. So, this is going down and down and this l as one parameter b as one parameter. So, it will be l square and divided by l raise to 4. So, this is coming as a l square and this b is coming as l raise to 4 right. So, now, if you consider sigma equal to 6 f divided by l square, now we are considering everything in terms of the linear scale. So, this is the l raise to 2 scale. So, your force will coupled as a 6 f into a. So, a shrinking of a 10 cause is the hundred fold increase in the induced stress. Now if you consider a f is a constant we are not looking at the f right now, but you consider dimension only.

So, if you shrink the dimension of 10 times; that means, l equal to 1 it will come to the l raise to 10 times l raise to 2. So, that is a l becomes 100. So, at that is the reason that your, if you reduce the dimension then your force will because this whole thing is in denominator. So, your shrink or the total induced stress will be very high in this particular case. Now what we want to do? So, let us take in another way then let us keep the induced stress to remain same. So, let us fix this particular thing now if you want to keep this thing then we have one another variable that is called f right.

So, if you are keeping the stress will you constant then what will happen the force acting on the beam must decrease at the square of the characteristic dimension why it is happening like that. So, now consider here only. So, it is a sigma equal to let us on ignore 6 right now it is l raise to square right.

Now we want to keep this thing as a constant, and we do not want that this particular thing will pass through the bending state or it should not buckle like a anything. So, if you want to do this thing then this if you reduce the dimension one. Now you are reducing the dimension by keeping this particular constant what is a variable. Variable is f. So, to keep the sigma constant whenever you are changing the dimension one, the reduction the force should be also as a proportional to the l square right.

So, because now here f is equal to sigma into l square from this equation, keeping sigma constant f should proportional to the l square. So, if the stress remain constant the force acting on the beam must decrease because this is on the opposite side one is in denominator one is in numerator. So, in that case if you want to keep this constant if you reducing the l, you have to reduce the f also then only that proportionality will be maintained. So, here in this case we have can take this example in two way then what is the maximum shrinking or the maximum induced stress here in this case. So, that when you are scaling down on other side that we let us keep the constant for induced stress then let us work with the force the how much force we can apply on to the surface.

So, this is where how it will work in the gases

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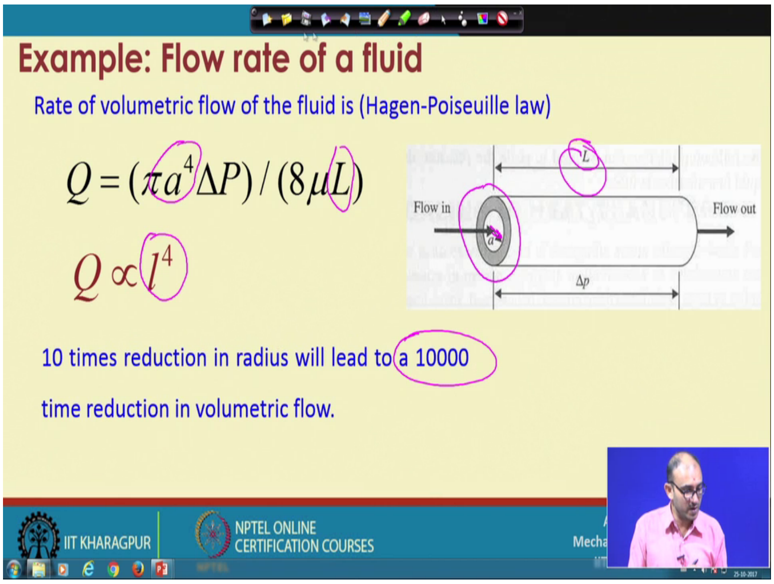

Now let us take some example of the fluid mechanics. Now if you see the rate of volumetric flow of the fluid is given by one of examples. So, this is the equation, where Q equal to pi a raise to 4 delta P divided by 8 mu into l. So, this is the one of the tube, it flow is coming from this direction going out from that length is l, and the diameter is a and the total pressure drop. You can phase measure or that water is coming is a delta P. So, now, here what is the important part, the a is one of the parameter, this is the parameter at dimensional parameters this a; that is right. Now we are discussing about the diameter only we are not talking about the length. So, this Q is a proportional with a l raise to 4 whatever is the flow diameter per unit length. let us consider this one has a per unit length.

So, now what is going to happen, if you reduce the dimension 10 times only then what will happen the dimension there was dimension one; that means, a is the dimension if you reduce the diameter or the radius 10 time, then what is going to happen that your reduction in the volumetric flow will be the 10,000 times, because it is behaving as a l raise to 4.

Let us keep the length, length is also one of the parameter of a length scale, but let us not change the length right now, but let us see the how much flow we can pass out through this. So, reduction in this particular length scale; that means, it is the diameter what is going to happen that volumetric flow will reduce with a 10,000 times. So, that is a problematic situation which we have to encounter in the microfluidics.

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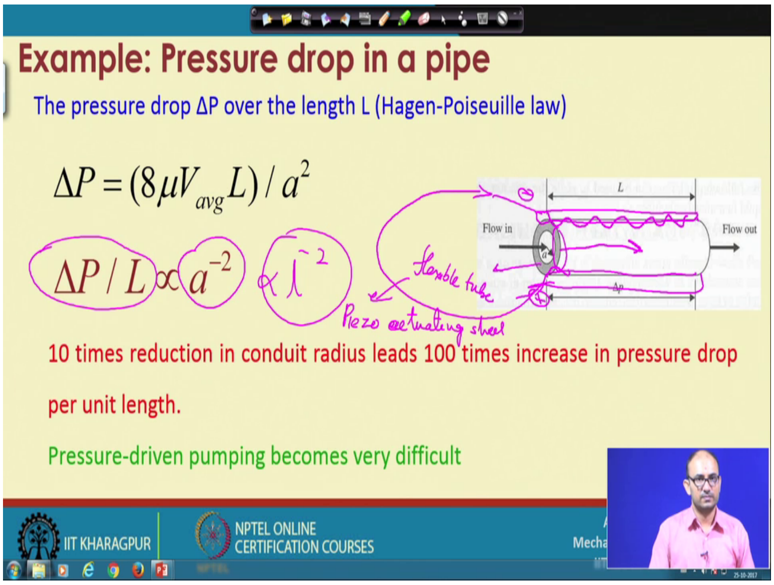

Coming to the pressure drop, again the same formula we can use it, same example. now we are want to see that how much is the pressure drop between these two part. So, pressure drop you can calculate from that earlier equation. So, that is divided earlier we how found the P delta P. So, from delta P you can find that, and final equation is coming as a 8 mu v average l divided by a square. Now again l square a square is coming as a denominator here right. So, now, consider pressure drop per unit length. Now we are considering length, but length is not a one of the part of the length scale, but we are considering as a unit length. So, now, what is going to happen in this case, that it will be l a square. So, it will be l raise to minus 2 whatever is coming. So, here what is important part.

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So, 10 time reduction. Now, let us consider this a 10 times reduction in the radius will lead to 100 time increase in the pressure drop per unit length.

So, if you reduce, the diameter your pressure drop will be very high pressure drop means the difference pressure at the inlet and the outlet pressure. So, what this is the problem is pressure driven pumping becomes very difficult. Here, if you are working with the microfluidic. So, if you are not just pushed the liquid from one location to the another location. So, that you can get the, you need some type of extra element or the external forces, which will give, which will drive, the force drive, the liquid inside it.

So, many examples available in this particular case, where people are putting some type of a piezo actuator. Here on the surface and this tube is a flexible tube, and when this particular thing will these are the piezo actuator piezo actuating sheet. So, what will happen that these particular thing will create a one type of waves. Here, it will create a one type of pressing here. So, if you are providing one signal here, then it will contract it little bit and something like that, it is something like a squeezing a toothpaste.

So, it will create a one type of waves here. So, because of that waves the whatever liquid is there, liquid will be pressure as it will move in this direction. So, that is the one of the way, you can pressurize that thing externally or you have to reduce, you have to work with the diameter variable diameter of the air so that at the later stage your diameter will be very high. So, movement will be very easy in this case. So, let me finish this lecture here, we will continue with the same topic in the next class.

Thank you very much.