

**Introduction to Mechanical Micro Machining**  
**Prof. Ajay M Sidpara**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 08**  
**Scaling Laws (Contd.)**

Good morning everybody and welcome to our course on introduction to micro mechanical machining processes, in the last class we have seen the introduction related to the different aspect of the micro machining process; classification of the micro machining operations with respect to other processes comparison between the different processes like MEMS based process and the precision machining processes.

We have also seen that what things you have to take care when you are working with a micro scale; that means, what are the habits you have to understand that what things will happen when you operate a micro machine in the macro machine. So, development or the creativity of those types of things will create good things about the processes. Now, we are going to the next part of the course that is called scaling laws.

So, this scaling law will tell you about the difference between the system at a bigger scale and what parameter affects those systems when it is bigger scale and when you scale down those systems to a smaller scale; what will happen? That means what things you have to take care at that location. So, let us understand those things in the next lecture.

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**Why scaling laws??**

Demand of miniature and compact products with increasing functionality is increasing.

2004 2005 2006 2009 2010  
pantecholutions

large size and less storage capacity

Small size to large storage capacity

2004 2005 2006 2009 2010  
gtechmini.wordpress

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So, why scaling law is required, now see that there are many components there are 2 example we have given 1 is the memory storage; that means, if you see long time before the size of the pen drive was very large and now if you see at this particular location now what was the problem here the large size and small storage not small, but let us write the less storage capacity.

But if you see here it is a small size and large storage capacity. So, this is exactly reverse, now when you are moving to this dimension without actually compromising with the storage, but rather you are increasing the storage capacity. So, something we have to do with the component or the system we are putting inside it.

Now, second example is the cell phone, now if you will size of the cell phone or mobile phone long time before and now if you will see it is ultra thin ultra light weight and there are lot of operation it is like a palmtop or something you can do everything which you are doing in laptop or the desktop computer. So, these things are very important to understand that how this thing is working. So, what is present demand that we are doing a miniaturization of compact of products and then we are increasing the functionality also. So, when you do both the things that you are minimizing the size also, but increasing the functionality then we have to think about what processes will be used for backing this products effectively and usable.

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**Why scaling laws??**

Demand of miniature and compact products with increasing functionality is increasing.

Individual components should be reduced in size and packed.

Some system cannot be scaled down favorably and some cannot at all.

Scaling laws become important to understand the behavior of structures or systems when miniaturized.

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So, to do these things to find out that what things to be done, then to reach to these location what we have to do we have to individually component should be reduced in sized and packed. So, now here in this particular case now there are memory storage available, so there will be chip available; but the size of the chip will be very small in this case. So, there are small components and the whole thing should be packed in a small size 1 centimetre by 1 centimetre also some time and that means we have to put all the components packed with a reduced size.

So, that is what is important in this case. So, some system cannot be scaled down favourably and some cannot be at all, but now question is that what we have to do in this particular case. So, here luckily everything happened favourably that everything you can scale down you can scale down the size also, you can increase the functionality also; here also happen very fast, but if you see the machine that you cannot scale down the machine like a big milling to a small milling machine because, there are lot of issues available these things are moveable components here, most of the things are static component that means there are no moveable part.

Most of the thing is done by electronic signal. So, that is the advantage of moveable scale down electronic system, but when you scale down mechanical system there are lot of problem, gravity is issue inertia is issue then the momentum the way you are moving components here there those things are creating problem. So, then we have to find out

the which will scale down favourably, which will not scale down favourably and which some system cannot scale down at all; that means, there are some problems which are very serious that you cannot move ahead in their particular direction.

So, what is scaling, the scaling law becomes important to understand the behaviour of the structure or the system when it is miniaturized? So, these are the examples we have taken related to the electronic components, but we will focus on the mechanical part that what will happen when you scale down one particular car, suppose you have scale down a big size car to small size car consider a the size of a car which is size of this particular pen. So, now what you have to do you have to shrink the engine also you have to shrink wheel also, you have to shrink all the electronic connection, you have to shrink all the related component.

So, which particular component will work favourably which will not work favourably. So, those things will be understand by scaling law of the different system and the different components.

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**Different types of scale reduction of a system**

Same size  
more components

Small size  
same components

$V_1 = V_2 = V_3$

Components like milling, milc  
Adding sensors for foot and milc monitoring  
we have to shrink or reduce the size of the milc components  
Camera + Additional high speed spindle

294 × 220 × 328 mm<sup>3</sup>  
Bang et al. (2005)

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So, now there are different ways of doing scaling down, the first thing is suppose we will consider this cubic cube the same size component and the same size but the more component; now see the size of the component is same, but instead of this 4 or 5 components we are reading more components and if you continue in this direction what

we are doing that we are adding more and more component, but we are keeping the size same.

Now, consider this particular milling machine, now this is the 1 milling machine size you consider it is a on an average 1 foot by 1 foot or 1 feet. So, this is a milling machine can you get all the oppression by this machine which can be done on the big size milling machine. So, here what we are doing that we are scaling down the component, but that this example is not exactly this one; here what we are telling that suppose you have one milling machine.

So, let us draw a small schematic here and suppose this is the conventional machine and this is the size conventional milling machine. So, now let us note change the size keep the size same, but we add some components which are useful for the efficiency of the component, so we are adding sensors for tool and machine monitoring right.

Now, we know this machine is completely full we do not have space and we are adding more components inside it. So, what we have to do this thing we have to shrink or reduce the size of the machine component, then only we can add something extra; let us go further inside it size remain same let us see the size volume 1 volume 2 volume 3,  $v_1$  equal to  $v_2$  equal to  $v_3$  we are not changing the size.

Now, we are adding sensory now we are adding here other than the we are adding camera plus additional spindle, high speed spindle; in that way we are continuously adding 1 by 1 small component and; that means, that size is not changing the total volume is not changing, but we are adding more and more component in that.

So, now if you see this particular component this is the one component and at this particular location this is the same component at this location. So, we have to reduce the size of the individual components, so that we can add more and more component inside it. So, this is the 1 way the size remains same, but we are adding more and more component. So, another thing is the size is small size and the same components.

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**Different types of scale reduction of a system**

Same size  
more components

Small size  
same components

Small size  
more components

294 × 220 × 328 mm<sup>3</sup>  
Bang et al. (2005)

As the scale of the system reduces, different phenomena / parameters become significant at different scale and others become less effective.

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Now, what does it mean? Now again the same thing is here, now what we are doing that we are not adding more components; but we are just reducing the size of that component, whatever is here that remain 4 remains 4, but if the size is reduce here. So, this is what is happening and this is exactly the example of this part

So, same milling machine is there we have not added anything extra here, but what we have done we have shrink the all the components of the milling machine to a smaller scale. So, this is what is happening in this particular case and there is 1 more categories that is what is happening now day, a small size and more components. So, this is the same thing here but what is happening here, the volume was the constant but we are adding more and more components.

Here we are not adding any components, but we are reducing the size of the component, now if you get the both the things in a single part. So, what we have to add more components here and we have to find small size also. So, this is what is happening in the electronics item that we have seen the earlier example the pen drive and the mobile phone, then we are reducing the size also we are adding more component also. So, can we do the same thing in the machining part. So, that is the questions here.

So, this is what is happening here the size of this particular thing is also very reducing. So, it is here and if you see 1 component so this particular component is also reducing from size to size. So, this all the things will create lot of problem in manufacturing

because just by showing a cubic cube and changing the size it is very easy, but when it comes to the actual operation and how to make this component.

Now, if you consider this is the size of a milling machine. So, here if you see this particular component that means, there is a limitation of the size of the component which you can machine, but that is not a big issue here because anyhow we are going to machine it as micro scale. So, now when you do this thing you consider this is the motor available here, this is the x and y motion and this is the rotation of z around the axis that is called a axis. So, everything you have to encompassed in a 1 feet by 1 feet.

So, size of the motor size of the linear scale size of the travel so everything is reduced to a smaller scale. So, in that case we have to find out a solution by which you can manufacture this component, assembly is the next step but first thing is the how to fabricate this component without compromising the speed feed and depth of cut limitation of the machining operation.

So, what is happening that as the scale of the system reduces different phenomena or parameter become important, different phenomena become significant at different scale and others become less effective; now what happens what does it mean that suppose your system is this now suppose here parameter a b c is important, let us consider here any parameter now if you keep the size constant, but if you add more component this may not be useful here something d e and f parameter become significant, if you further reduce something here x y z important.

As the scale difference or the scale down different parameter play different role at a different scale. So, it may not be the same suppose gravity is important at a bigger size, but if you see the gravity at smaller scale you will see some examples that it does not make any difference right. So, these are the important things the scaling law will tell us that which parameter will affect which way in a different systems.

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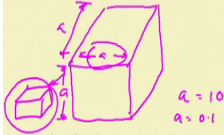
**Scaling laws**

At small scale:

Less effective → Gravitational, inertial forces become less effective

More effective → Van der Waals forces, electrostatic forces, surface tension

The scaling laws are proportionality relations of any parameter associated with an object (or system) with its length scale.



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Now, what are the now scaling laws at small scale, small scale means when you are talking about dimensions of hundreds of micron or something more, things are less effective gravitational forces are less effective; inertial forces that means, whatever the weight is there it is less effective. So, this is so we have to understand how this thing will behave and when what is more effective Vander Waals forces electrostatic forces surface tension those things will affect very large, we will see why this thing will effective more compared to the gravitational and all these forces.

So, scaling laws are proportionality relation of any parameter associated with an object or system when it is a length scale, length scale means suppose we have 1 right. So, we will see that this whole size are let us consider a and this 1 full system workable system; now this is a equal to 10 and a equal to 0.1, so if it is 0.1 all side will shrink to a 1 particular component.

Then what is going to happen when this thing will happen. So, everything is related to the length scale we are not talking anything other than that what will happen, if we change the dimension of any system. So, if your car is 4 meter long let us shrink it to the 1 metre or let us shrink it to the 1 foot also. So, in that case what you have to understand that is everything is related to the length scale not related to any other parameter, if you change the length scale how this different phenomena will emerge as a important parameter at a different scale that we will see by looking at the different examples.



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**Types of scaling law**

1. related to the scaling of physical size of objects  
Geometry  
 $d=10$   
 $d=1$
2. related to the scaling of a phenomenological behavior of an object/machine  
Physical size and material characterization  
20 cm  
defects

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Right what are the types of scaling law? So, 1 is related to scaling of physical size of the object physical size of the object. So, it means geometry, suppose is what we have seen that this is the work piece with a diameter of  $d$  equal to 10 and this is the diameter of  $d$  equal to 1. So, this is called scaling geometrical scaling, so we do not see anything other than that very easy to understand and another thing is related to scaling of a phenomenological behaviour of an object or a machine.

So, what this things are this is the physical size again same thing will kappa but material characteristics are also important. So, how these things will happen now suppose you have 1 block of a any material and this size is consider a 20 centimetre and it is a very big size component; now each and every material has some defects, now if you take a small portion now suppose these are the defects.

So, let us name is a defect right you take out a very small component out of it and now see the defect level, now if the defect level is something here it may not be the same thing here because if the size reduces the number of defects or the number of dislocation porosity or anything it will also reduce. So, behaviour of material will reduce or it will change when you scale down to some limit. So, this is related to material characterization, so how material will behave when you scale down the component not related to the other parts. So, these are the 2 different ways you can actually find out the scaling law or you can understand the scaling law 1 is by geometry another 1 is the

physical size which is also geometry, but addition to that it will also couple the material characterization of the different properties.

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**Scaling factor**

Scaling of various parameters

Perimeter (P)  $\propto L$

Area (A)  $\propto L^2$

Volume (V)  $\propto L^3$

Mass (Weight) (W) =  $\rho V \propto L^3$

Surface tension (F<sub>s</sub>) = (P × γ)  $\propto L$

Moment of inertia (I) =  $WR^2 \propto L^3$

Resistance (R) =  $\rho(L/A) \propto L^{-1}$

Resistance

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Now, scaling factor scaling factor means here what we are considering we are considering only length scale. So, whatever parameter we are talking we have to convert that parameter into the length scale. So, suppose you are considering a volume, so volume has a length raised to 3. So, it will scale as a length raised to c it is a length of the cube of the length if it is a area then it is a pi d square. So, it is square parameter so the length or the dimension is related to the square.

So, in that way there are different parameters we will see what are this parameters and how we can understand those things. So, let us consider 1 cube it has a 1 length side is a L. So, we are considering all the things as a 1 length. So, perimeter is a l. So, it will behave as a l, l means this is a l. So, let me write it here so it will be clear. So, perimeter is always given by l because perimeter suppose circular is that the perimeter is the pi d. So, it has a 1 length scale only so that is the way we can understand, area has a pi d square. So, it has a d is a square raised to it pi d square by 4 or pi r square. So, it is a square parameter so it will actually scale as a l raised to square now it has a volume.

So, volume has a 3 side l here l here and the lengthwise also so it has a 3 parameter. So, it will scale down scale up as a l raised to cube now the mass. So, the mass we know or weight we know it is rho into v, but rho is not length scale parameter but v is a length

scale parameter because it is volume. So, once it is consider as a volume then what we consider whatever is here that will be same as this. So, mass or the weight also scale as a  $l$  raised to cube, then now surface tension is the coefficient of surface tension  $l$  minute and the weight perimeter area. So, this weight perimeter area that is again going in this location, so it will scale as a  $l$ , but coefficient has no any connection with the length scale.

Now moment of inertia it is a  $m$  into  $r$  square and this  $w$  we are considering from here and  $r$  square is also radius. So, it is considered as a length scale. So,  $l$  cube will come from here so this is the  $l$  cube and  $r$  will be the  $l$  square and these both things will be considered as  $l$  raised to 5. So, this is the way we can consider the different parameters of that resistance is also  $l$  parameter resistance is given by the  $\rho$  into  $l$  by  $a$ . So, this is the resistivity this is the length scale, so it will be  $l$ .

So, because  $l$  is  $l$  nothing is there this will be area. So, it will be square as a  $l$  square so it is a  $l$  by  $l$  square and that is the reason it will be scale as a  $l$  raised to minus 1 right. So, there are different considering only small very less parameter, but you can convert any parameter into length scale by means of their equation that which way they are behaving, based on equation if you find any parameter or any component which is connected with the length, then you can convert into length scale and then you find out whether it is moving in a positive manner or it is moving in a negative manner and there are many.

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**Scaling factor**

Scaling of various parameters

- Perimeter ( $P$ )  $\propto l$
- Area ( $A$ )  $\propto l^2$
- Volume ( $V$ )  $\propto l^3$
- Mass (Weight) ( $W$ ) =  $\rho V \propto l^3$
- Surface tension ( $F_s$ ) =  $(P \times \gamma) \propto l$
- Moment of inertia ( $I$ ) =  $WR^2 \propto l^5$
- Resistance ( $R$ ) =  $\rho(L/A) \propto l^{-1}$
- .....

Knowing the scaling of a physical phenomenon as a power of  $l$ , guides our understanding of how to design small mechanical systems.

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So, now how it will happen that now you have relation of all these physical parameter in terms of the length scale.

Now, knowing the scaling of a physical phenomena as a power of  $l$  it will guides us to understanding the how to design a small mechanical system, now if you see that if you are changing the volume parameter and if you find volume as  $l^3$  of the parameters of any system it will be scaled down at a power of 3 and if you consider the parameter is  $l$  of the parameters of any system or any equation, but it will scale down as a power of  $l$  only. So, when you reduce the system with a perimeter it will scale down very slowly, but if you scale down any system where volume is present it will scale down very drastically because, it will reduce it as a power of 3 not the power of 1.

So, now let us see some of the example we living example as well the manmade examples.

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**Surface area to volume**

Generally, smaller things are

- less effected by volume dependent phenomena such as **mass** and **inertia**, and
- more effected by surface area dependent phenomena such as **contact forces** or **heat transfer**.
- **Main driver:** scaling of the surface area to volume ratio

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So, first thing is the surface to volume ratio. So, this is very important to understand because we have many components in the memes part also as well as the other components related to the micro machining operation, where we will understand where this surface to volume ratio is very important to understand.

So, generally we can tell the smaller things are less affected by volume dependent phenomena such as mass and inertia, we have seen a very small creature also ant is also

there and small insects are there. So, those are where very less affected by the mass and inertia and more affected by the area surface area different phenomena such as contact forces and the heat transfer.

So, let us see some of the example that how this thing is doing, so main driving force of all the components whether it is a memes part or it is a micro machining parts surface to volume ratio is very important to understand and it will give you the very scientific reason for understanding of something.

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**Example: Surface to volume ratio**

Surface Area ( $S$ ) =  $6a^2$  and Volume ( $V$ ) =  $a^3$

For  $a = 10 \rightarrow S = 600$  and  $V = 1000 \rightarrow S/V$  ratio = 0.6

For  $a = 0.1 \rightarrow S = 0.06$  and  $V = 0.001 \rightarrow S/V$  ratio = 60

**S/V difference is 100!!!**

**What is the significance?**

- Heat storage  $\rightarrow$  volume
- Heat dissipation  $\rightarrow$  surface area

The small cube has 100 times more heat dissipation per unit volume.

Temp = 100°C,  $\alpha = 0.1$ ,  $h = 100$  → Cools down 100 faster than

So, let us take this example, so here this is 1 cube it has a dimension of a 1 side and now what is the surface area surface area we can understand 6 into a square and the volume is a raised to cube right; now let us consider 1 dimension of the a, so let us consider a as a 10 right, now so 10 maybe micron also millimetre also or it can be metre scale also.

So, now if you consider a as a 10 you put 10 here. So, s becomes 600 and if you put a equal to 10 v become 1000, now let us take the ratio surface to volume. So, if you divide 6 hundred by 1000 you are end up with a .6 correct. Now let us reduce that size, now a is a 0.1. So, how much we have reduce 100, then if it is a 0.1 now s becomes 0.06 you put the point here in this equation you will get this value and then v become 0.001. So, in this particular case now let us take the surface to volume ratio here now it is a 60 right

So, now what is important here, now surface to volume difference is here it is a 0.6 and here is .60, so now difference is a 100. So, where it is making difference what is significance is the heat storage by the volume. So, this is the 1 of the parameters of heat storage heat storage is the function of a surface, heat storage is the function of the parcel no not this 1 minute.

So, heat storage is a function of a volume that how much heat you can store is a function of volume and heat dissipation is the function of the surface area right. So, here is the surface area so now how this thing will happen that suppose you have 1 cube here and here the difference is that suppose you are reducing the size of the cube somewhere here this much you are doing.

So, what we are doing we are reducing the area this volume will reduce very drastically, but the surface area will not scale down that much way because, it is moving it is reducing as a raised to square while it is reducing as a raised to cube. So, what happen that smaller cube has a 100 times more heat dissipation per unit volume, because here what happens here now you can see here the difference between this and this is how much it is 600 from 20 here let us take 40 this much is the difference.

But here difference is a 1 2 3 4 5 and 6 zero. So, this much is the difference when you scale down 1 system 1 particular system about a 100 of a scale. So, here reduction in a volume is very large, but the reduction in the surface area is less. So, heat this a here storage is very less, but your dissipation is very high. So, if you heat 1 component with the same temperature suppose starting temperature is a 100 degree of a equal to 0.1 object and a equal to 10 object; now what will happen then you put both the things in the same environment and then see the which 1 will cool down fast, then what will happen this particular thing will cool down 100 times faster than this 1.

So, that is important even heating is also that thing it is not cooling down, if you want to heat it will heat down hundred times faster than that. So, in this way you have to understand then, what will happen when you do machining in a micro scale, now suppose you have considered the you have scale down the full machine to a micro scale that we have seen 1 example that all machine is in a 1 foot by 1 foot.

Now, you are doing physical contact; that means, tool is in contact with the surface it will do friction it will generate heat also and it will many heat will be carried out by the

chip and chip is fallen down on the base. So, when it is fallen down on the base at that time if this thing is playing important role what is going to happen that that particular localized area will be heated very fast because the area is small, if the area is large but the volume is small.

So, when there is a heat at that time there is a 1 particular parameter it is called coefficient of thermal expansion that particular material will expand let it be at micro scale, but it will also degrade the total loop or that is called the stiffness loop of the machine and ultimately you are end up with the wrong size of the component. So, this is 1 of the example by which it will create a important role in this particular part.

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**Example: Weight vs. surface tension**

When a glass full of water is overturned the water gets spilled

Water confined to a capillary tube does not come out even when the tube is upturned.

Why ??

Mass (Weight) ( $W$ ) =  $\rho V \propto l^3$

Surface tension ( $F_s$ ) =  $(P \times \gamma) \propto l$

The force due to surface tension becomes predominant at very small scales.

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Now, another example is the weight versus surface tension, now you consider the when you are you fill a 1 glass with a water and overturn the glass all the thing will spill over because nothing will be stick there in that glass, but if you see a same thing in a micro scale now if you put a water to a capillary tube it is very tricky that even if you put it down, when means upside down you will not spill out very easily in this, so because it will happen because of the size scale.

So, what is happening here in this case because, here the surface tension and the weight will create a important role here; because we know that whatever the weight is a actually scaling as a 1 raised to cube, but the surface tension that means, whatever is this particular part is here that cohesiveness of the particular water molecule that will scale as

a  $l^2$  only. So, when you are scaling down from here suppose here it is a 1 centimetre by 1 centimetre of water and that will come down as 10 micron cube, then it will be very small in this case. So, here the weight whatever is weight here that the weight will be reduced as  $l^3$ , but the surface tension will reduce as  $l$  only. So, that is the important thing here that what is happening when it is. So, here it is a very difficult to move, you have to pressurize something from the top and then only it will move from the downwards right.

The surfaces due to surface tension become predominant at a very small scale that we will see in some example of the fluid mechanics also.

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Now, another example with a weight versus surface tension we know the mass is a function of  $\rho \cdot V$ ,  $V$  will be scale as  $l^3$  and this will be important in this case.

Now, surface tension is this particular parameter, now if you will see in that some insect will stay on to the water surface. So, here the surface tension will scale as a perimeter area into surface coefficient and it will scale as  $l$  an animal becomes smaller, weight decreases more rapidly than the surface tension right.

So, surface suppose consider a example of water has surface tension around 72 milli Newton per meter and if you see at that time at this location the bug with a 10 milligram



of a weight, it needs only 1 millimetre of a foot edge to walk on to the water, but if you consider men or human it has weight around 60 kg, then what we need if you scale down with respect to that what we need that, we need a feet of a 8000 meter to walk on to the water, so that is the way it will behave differently. So, let me finish this lecture here we will continue with some more examples in to the scaling effect.

Thank you very much.