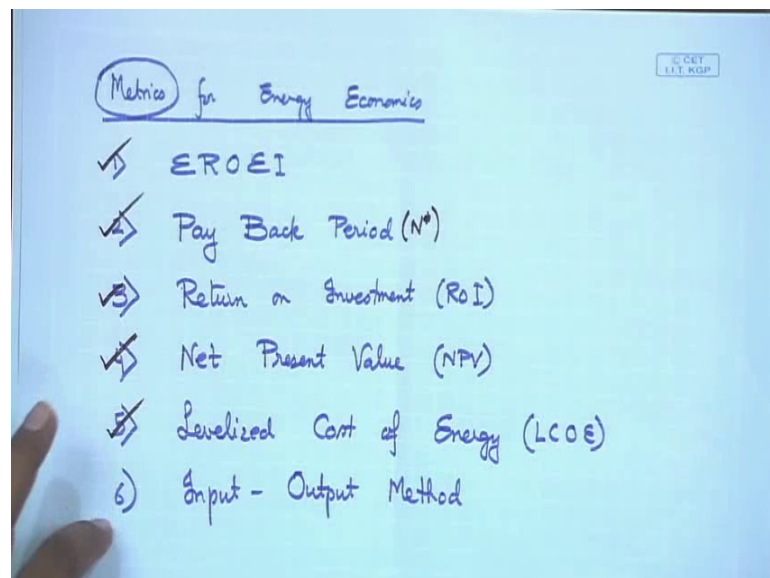


Energy Conservation and Waste Heat Recovery
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Lecture - 66
Energy Economics – V

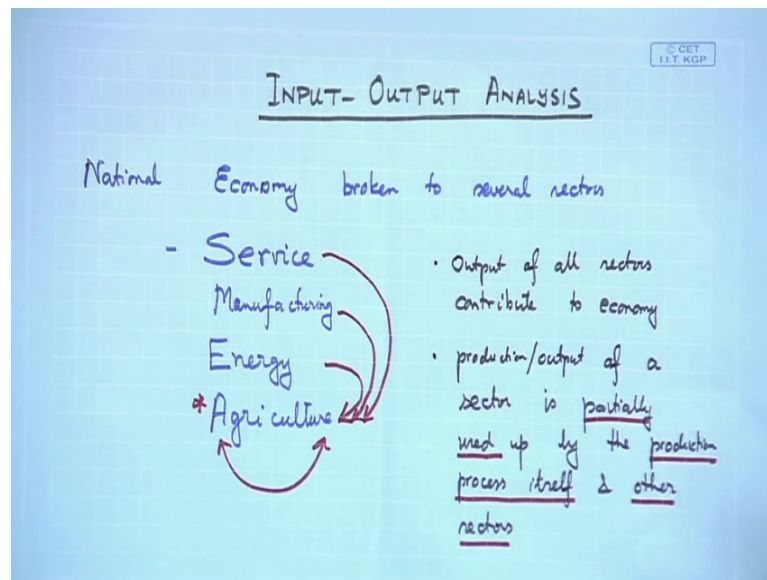
Good morning. Welcome back to Energy Conservation and Waste Heat Recovery. Today, we will continue our discussions on energy economics; if you recall, we were looking at a few metrics on energy or for energy economics.

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So, these are of 6 that we had listed and we have covered 5 till now and in the last lecture we had just started our discussion on input output method of Leontief; who was a Nobel laureate economist and what we said was if you look at input output analysis we started by saying that any national economy is broken down into several sectors and the output of each of these sectors or if you take a certain sector the output of that sector is partially used up by the production process itself and by other sectors also.

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And after that whatever is left is being is available for us for consumption. So, consumption by the population consumption or available for export and so on; so, if you recall we took just as an example; we said that we will consider 4; let us consider 4 sectors service manufacturing energy and agriculture and we focus on agriculture. So, does agriculture depend on energy does output of energy contribute to agricultural output of course, it does because the harvesters that you need to for harvesting that runs on some kind of I know petrol diesel kerosene something some fuel then once the crop is harvested you need to transport it somewhere.

So, there also you need unit transportation and where you need energy and of course, probably for irrigation etcetera for pump for pumping water you also need it manufacturing definitely all the agricultural tools and machinery are directly are manufactured in the manufacturing sector. So, that is a direct input of or the output of manufacturing is an input to agriculture sector and service when we talk about service let us we said that the farmer may have to take loans from the bank to buy crops or seeds. So, therefore, the banking sector also come into picture and that is how the service industry contributes to agriculture and finally, is there a self dependence because you know the harvest from previous year that is where we get the seeds for the next harvest.

So, the seeds from earlier harvest is used in the next harvest. So, therefore, that is the self dependence we are talking about. So, agriculture output feeds into itself.

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Simple Example

Consider:

Agriculture (A)	c) 1 unit of Energy (E)
Manufacturing (M)	: 0.1 unit of A
Energy (E)	: 0.5 units of Mfg (M)
	: 0.1 unit of E

a) 1 unit of Agri(A):

0.1 unit of A
0.2 unit of M
0.3 unit of E

b) 1 unit of Mfg.(M):

0.1 unit of A
0.3 unit of M
0.5 units of E

So, what we will do today is we will just take a sample problem example problem and see what all we can do. So, let us take a very simple example where we will only consider 3 goods or 3 sectors consider let us consider agriculture one manufacturing and energy. So, from the previous example we have not considered service or banking at this point we will just take these 3 and then go ahead.

So, it is given that out of these 3, it is given to you that to produce let us say one unit of agriculture needs what does it need it needs 0.1 unit of agriculture. So, I am going to denote this as A, M and E. So, 0.1 units of agriculture 0.2 unit of manufacturing and 0.3 unit of energy; so, we saw this; why it would need we just discuss that. So, b; I would say one unit of manufacturing or M; I am saying that that needs 0.1 unit of agriculture where the agriculture is also food and you need to feed the workers in any manufacturing plant. So, that is one and you can also say that maybe something runs on you know biodiesel or something.

So, that that is also an output of agriculture 0.3 units of manufacturing and 0.5 units of energy definitely, you run any machine you need energy and of course, to manufacture anything you need these machine tools and manufacturing infrastructure which again has to be built in the manufacturing sector and let us go up here and I would write like to add here c; I would say one unit of energy that requires 0.1 unit of agriculture 0.5 units of manufacturing and 0.1 unit of energy clear. So, this is what is given to me; all right.

So, what I will do next is these are the outputs 1 unit, 1 unit, 1 unit and these are the inputs. So, that is the input output method comes from there, all right.

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$$A = \begin{matrix} & \begin{matrix} A & M & E \end{matrix} \\ \begin{matrix} A \\ M \\ E \end{matrix} & \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.1 \end{bmatrix} \end{matrix}$$

3x3 matrix
(Transaction Matrix)

$$X = \begin{bmatrix} A \\ M \\ E \end{bmatrix}$$

Production Matrix

$$AX = \text{Amount consumed by production}$$

↓
3x1 matrix

$$= \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.5 \\ 0.3 & 0.5 & 0.1 \end{bmatrix} \begin{bmatrix} A \\ M \\ E \end{bmatrix}$$

So, what we will do is now try to explain this dependence in the form of a matrix; how am I going to do that I will write a matrix where I would denote it in this manner. So, what I will do is over here, I have agriculture I have manufacturing and I have energy and here also, I have agriculture, I have manufacturing, I have energy. So, what happens what have we said that one unit of agriculture requires 0.1 unit of agriculture. So, I am going to put 0.1 here then what else it requires 0.2 units of manufacturing.

So, what I will do is I will put 0.2 here and then it requires 0.3 units of energy. So, I would put 0.3. So, likewise let me put the others as well 0.1, 0.3, 0.5 and here again 0.1 0.5 and 0.1. So, this is one matrix, all right and what I will also write is I will write another matrix as X which is the production matrix and I am going to write it as A M and E, all right. So, then what is the. So, before that let me write this is a 3 by 3 matrix and this one is also called transaction matrix typically denoted by A.

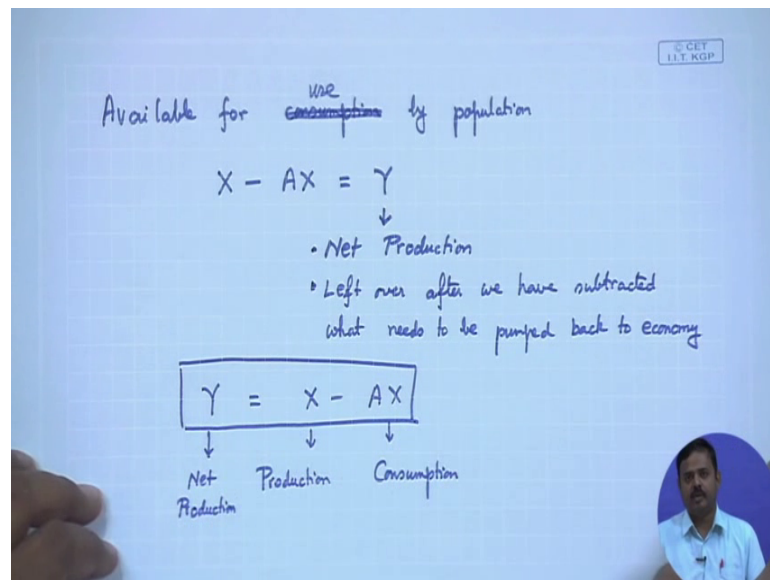
So, once again do not please do not get confused between this A and the A for agriculture unfortunately that is how we have to denote, but these 2 are different the A matrix is different from a agriculture, all right and what is this? This one is called a production matrix. So, this is the overall production this matrix or whatever the elements of this matrix denote the overall production. So, therefore, if I take A times X this matrix what

does it denote, it denotes the amount consumed by production. So, the output of each sector of the economy that is consumed by itself; so, it is the output of economy that is consumed by itself.

So, what is that going to be let me write it as 0.1, 0.1, 0.1, 0.2, 0.3, 0.5, 0.3, 0.5, 0.1 and this times the production of the economy am and E clear all right. So, this is what is being consumed out of the out of the production of X; A times X is consumed and remember this is also going to be turn out to be A 3 by 1 matrix or vector if you call it all right k.

So, therefore, what happens?

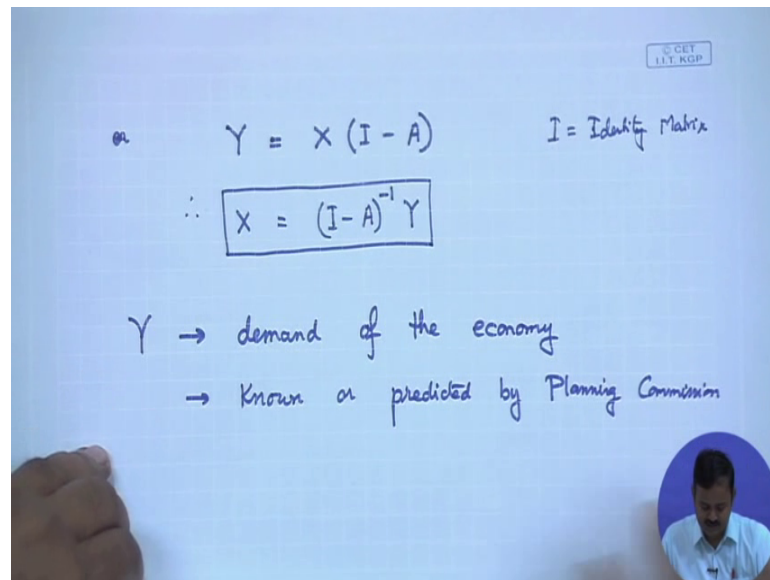
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So, what is available for consumption by population and maybe export whatever that would be; X minus AX and let us denote it as Y where Y is going to be the net production. So, that is the production that is available to us for consumption as population after taking into account whatever is needed what the fraction of the total production that needs to be consumed to produce that is consumed by the economy itself. So, therefore, the net production is this. So, again we will we can write it as leftover after we have subtracted what needs to be pumped back to economy recall this is something like what we said whatever is produced is needed to support the existing infrastructure this is similar to that all right.

So, therefore, Y is X minus ax and I can also write this as production this is consumption let me remove the word consumption here and I would several for use by population because consumption we are using for what is this ax term and what is this? This is my net production as I said.

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$Y = X(I - A)$ $I = \text{Identity Matrix}$

$\therefore X = (I - A)^{-1} Y$

$Y \rightarrow \text{demand of the economy}$
 $\rightarrow \text{known or predicted by Planning Commission}$

So, this is a very powerful expression; what we would do with this one is we will also write it in this form that Y is X times I minus A where I as we know is identity matrix. So, I is the identity matrix clear and same sizes A it has to be. So, therefore, what is X going to be X is going to be I minus A inverse of Y, clear.

So, I think we all know how to invert A matrix if it is like 2 by 2, 3 by 3 we can do it ourselves otherwise we have we have to write a small computer code or subroutine to calculate invert of inverse in inverse of a matrix or to invert a matrix, but this is an extremely important expression why because let us say in an economy say the Indian economy we know the different sectors and we know that what is required by the population for consumption by the population what is required for export and so on.

So, this Y matrix many a times is known to us what do I need to support my population and also if I need to export after that I know that what is the demand. So, therefore, Y is like the demand of the economy, but I just cannot produce exactly equal to what is demand because part of it is going to be consumed by the sectors themselves. So, this sometimes is known or predicted by planning commission for example, let us say that

today my agricultural output of the of the country is X, but we want to increase it over the next 10 years I want to increase it by 20 percent. So, it becomes one 0.2 X.

So, if one if the agriculture component in this Y matrix goes up by twenty percent what happens to the X matrix. So, what does it mean? In terms of production how much extra do I have to produce and that extra production is not just going to be agriculture, but it is going to be several other sectors on which agriculture depends on right all right. So, therefore, if we know this demand matrix and if we know this transaction matrix which kind of gives us the interdependence of each sector we can find out what is going to be what is the production needed. So, that we can the final output can meet the demand Y.

So, what I will do next is I will just leave it here for the time being, we will come back to this example again. So, I will leave it here and we will take up a more generic case. So, let us go back now and say that we will just generalize what we discussed with a specific example.

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Generalized Formulation

Sector 1: Total Production is X_1

$$X_1 = X_{11} + X_{12} + X_{13} + \dots + X_{1n} + Y_1$$

$$X_2 = X_{21} + X_{22} + X_{23} + \dots + X_{2n} + Y_2$$

$$\vdots$$

$$X_n = X_{n1} + X_{n2} + X_{n3} + \dots + X_{nn} + Y_n$$

X_{ij} = consumption of output of sector 'i' by sector 'j'
 Y_i = final demand of products of sector 'i'
 X_i = total production of sector 'i'

So, generalized formulation; all right; so, let us say sector 1 will consider the total production is X 1; now there are several sectors like this X 2, X 3, X 4, X 5 of that sector one sector 2 sector 3 sector 4 like that. So, I can write that out of this total production of X 1, part of it is consumed by the sector one itself, then part of it is consumed by sector 2 sector 3 and finally, sector n and then whatever is left is going to fulfill the demand of the economy or of the nation.

So, similarly I can write X_2 is going to be X_{21} plus X_{22} plus X_{23} plus X_{2n} plus Y_2 all right and similarly X_n all right. So, what do I mean by this let me write down here X_{ij} is consumption of output of sector i by sector j clear what is Y_i final demand of products of sector i and what is X_i which we know total production of sector i .

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Define $a_{ij} = \frac{X_{ij}}{X_j}$
 = input from sector 'i' required for unit production of sector 'j'

$$\begin{aligned} a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + \dots + a_{1n}X_n + Y_1 &= X_1 \\ \vdots & \\ a_{n1}X_1 + a_{n2}X_2 + a_{n3}X_3 + \dots + a_{nn}X_n + Y_n &= X_n \end{aligned}$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & & & a_{nn} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix} + \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$$

So, what we will do next is we will define something called a_{ij} this is a small element A_{ij} which is nothing, but X_{ij} over X_j , clear.

So, what does this mean this is input from sector i required to produce unit or required for i would say required for unit production of sector j . So, remember in our example we were sake take tell saying that one unit of agricultural output requires 0.1 units of agricultural input or 0.1 units of agriculture, 0.2 units of manufacturing, 0.3 units of energy. So, this is what it means these are all these a_{ij} terms. Therefore, what can I write with this definition I can therefore, write this set of expressions or these equations in a slightly different manner I can therefore, write as $a_{11} X_1$ plus $a_{12} X_2$ plus $a_{13} X_3$, $a_{1n} X_n$ plus Y_1 is equal to X_1 it is a same thing.

Similarly, I will just skip and write $a_{n1} X_1$ plus $a_{n2} X_2$ plus X_n plus Y_n is equal to capital X_n . So, this now can come in the same manner this also can be written therefore, as a_{11} , a_{12} , a_{1n} , a_{21} , a_{n1} , a_{nn} and like this, right, times what X_1 , X_2 , X_n plus Y_1 one Y_2 Y_n is equal to what is equal to the same thing the X matrix the production matrix X_1 , X_2 , X_n . So, look at this friends and see what does what can we write it as this is the

same thing as or a matrix times X matrix plus Y is equal to X consumption plus demand is total production, right.

This is exactly the same formula that we got using that simple using that simple 3 by 3 matrix and this is a more generalized form in an n by n matrix a will be an n by n matrix X will be an n by one and Y will be n by 1 vectors right . So, therefore, this is what we did was; so, the final demand of Y.

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So for a final demand of Y, if we know A, we can calculate how much we have to produce

- A is typically known for a given economy

$$X = (I - A)^{-1} Y$$

~~if we wish to increase demand~~

- if demand for a sector 'i' increases by say 10%, the above equation will help us calculate how much the production of each sector needs to increase to meet the additional demand

So, for a final demand of Y, if we know a we can calculate how much we have to produce, right. now this a is typically known for a given economy even I think in the Indian government if we can access the data of the planning commission this a matrix is known, all right.

So, many a times, the other thing I would like to say is many a times these are these a1, a2, etcetera; they are many a times expressed in monetary values in rupees. So, for example, what they will say is if we; if I have to produce a product worth one rupee from sector i or sector j what is the worth of products in rupees that I need to pump in from sector i. So, for example, what I would say the input from sector I in rupees required to produce goods worth one rupee of sector j if we do that then these are all in monetary values clear this can also be expressed in energy values etcetera so, but monetary is the most common if you look at the planning commission reports etcetera we would mostly

see a in terms of I mean a is cast. So, that these outputs and product the both demand matrix and the output matrix are in monetary units.

So, now let us see I say that Y if you recall I said Y can be I am sorry; X can be calculated in this form and I said if we know Y and if you know a we can calculate x. So, we know the production let us say; if we wish to increase or if in or increase or if that sorry; I should cast it in a different manner I would say if demand for a certain sector for sector I increases by say 10 percent the above equation will help us calculate how much the production of each sector needs to increase to meet the additional demand.

So, therefore, if any element in Y increases by say 10 percent what will happen the entire X matrix will change? So, it is not just a production of sector I that needs to change, but production of all the other sectors or at least the sectors on which the output of sector I depends will all change and this method the input output method that we just discussed we will let us calculate that we will let us calculate how much X has X as a whole the entire X matrix all the individual elements how much that has to change so that the 10 percent increase of Y_i can be met clear. So, you see how beautiful this method is it is; so simple yet; so, powerful, right. So, that is why this is such a well known and widely spread widely used method and it is relevant even today and used by most countries.

So, what we will do is we will end this lecture with this discussion and in the next lecture what we will do is we will wrap up our input output analysis we will go back to the previous example the simple 3 by 3 example that we started with and complete that.

Thank you very much.