

**NPTEL
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**Course
On**

Spur and Helical Gear Cutting

**by
Prof. Asimava Roy Choudhury
Department of Medical Engineering
IIT Kharagpur**

**Lecture 08:
Differential Indexing**

Welcome viewers to the eighth lecture of spur and helical gear cutting.

(Refer Slide Time: 00:24)



So up till now we have discussed simple indexing and identified some problems of simple indexing like there might be some numbers of T for which simple indexing would not really work or you would require some an assortment of index plates in order to achieve simple indexing in such cases. So there can be other solutions to these problems and let us quickly go through.

(Refer Slide Time: 00:56)

Problem of simple indexing

- There has to be a hole circle corresponding to the number of divisions to be made.
- If 33 teeth are to be cut, there should be a hole circle with number of holes = integral multiple of 33
- If instead, we write the rotation amount as
- $\frac{40}{33} = \frac{a}{3} + \frac{b}{11}$ where a, b = integers

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Such examples we had been discussing about 40 / 33 being unachievable that means suppose I do not have a 30° old circle, so you might say that is it really possible that you do not have 33 whole circle this is just by way of an example there might be other numbers of speeds like suppose we have 87 or 265 or something like that which is not there okay. So we are taking a problem where this particular full circle is not there and we say that if that denominator can be broken down into you know its respective factors then we might be able to employ the method of partial fractions and achieve something called compound indexing.

That means this rotation which is required which is one and 7 / 33 of a rotation that will be equal to the algebraic sum okay they might behaving different signs also algebraic sum of these two what is the restriction of A and B first of all A and B are whole numbers and they have to satisfy this particular equation. So in that case I mean they are integers they have to satisfy this.

(Refer Slide Time: 02:26)

Compound indexing

- In compound indexing, the required amount of rotation may be reached by the algebraic sum of two successive rotations along two separate hole circles.

- $\frac{40}{33} = \frac{a}{3} + \frac{b}{11} \rightarrow a=2, b=6 \Rightarrow \frac{2}{3} + \frac{6}{11} = \frac{40}{33}$

- We may select rotation through 10 holes on 15 hole circle and 6 holes on 11 hole circle



So what we can see is that if this be so we can easily let us look at the piece of paper.

(Refer Slide Time: 02:39)

$$\frac{40}{33} = \frac{a}{3} + \frac{b}{11} = \frac{11a+3b}{33}$$

$$11a + 3b = 40 \rightarrow 11 \times 2 + 3 \times 6 = 22 + 18 = 40$$

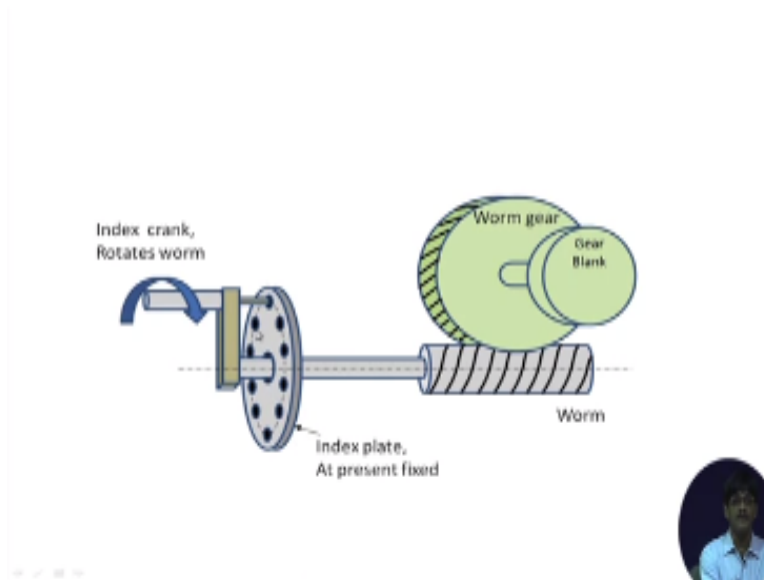
$$\left(\frac{2}{3}\right) + \left(\frac{6}{11}\right) = \frac{40}{33} \quad \frac{20}{30} + \frac{6}{11} \Rightarrow \frac{40}{33}$$

We are having $40 / 33 = a / 3 + b / 11$ and therefore if we take a summation of these two 33 in the denominators $11a + 3b$ okay there are elaborate ways of finding out the you know possible solutions of A and B and finding out the number of whole circles which will be employed in these cases but at this moment we are not going for such an elaborate discussion, compound indexing is not that popular because it essentially combines two different successive rotations we would always be interested to work with a single movement less number of movements to achieve the required rotation would result in less amount of errors.

So this one means that $11a + 3b$ should be equal to 40 and by the method of trial and error okay you can easily hit upon some solutions this is basically the equation of a straight line a and b have to be integers and so let me suggest suppose $a = 2$, so $11 \times 2 + 3 \times 6$ how much does it give us 3×6 is $18 + 22 = 40$, so this means that if our if AB equal to $2/3$ rotation plus $6 / 11^{\text{th}}$ of a rotation these two separate rotations will give rise to $40 / 33$ of a rotation.

What does this mean? This means that suppose I employ any particular value or say multiple of 3 how much say 30 so on a 30 hole circle so 20 hole on a 30 hole circle + 6 holes on an 11 whole circle this will be the same as $40 / 33$ of a rotation. So that means first of all I rotate. Let us go back to the previous just a moment

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That is it first I take out this pin from the index plate and rotate this by say 20 holes on a 30 hole circle and put the pin down that means I am feeling this particular initial rotation 20 holes out of a 30 hole circle put the pin down and after that if I still rotate the index plate is going to rotate with us why are we doing this because, now I am going to keep track of the rotation by the help of some other fixed device suppose I put a pointer here okay with respect to this pointer now this particular crank a sorry index plate is going to rotate with respect to this say fixed pointer the index crank is now going to rotate.

So first move by 20 holes on a 30 hole circle now with respect to the pointer with the pin inside the index plate in this way it also rotates after this and with respect to this particular pointer the show that you move by six holes on an 11whole circle. So both these old circles have to be present on the same side of the index plate this after this is done the net rotation will be 40 out of 33 so this is called compound indexing.

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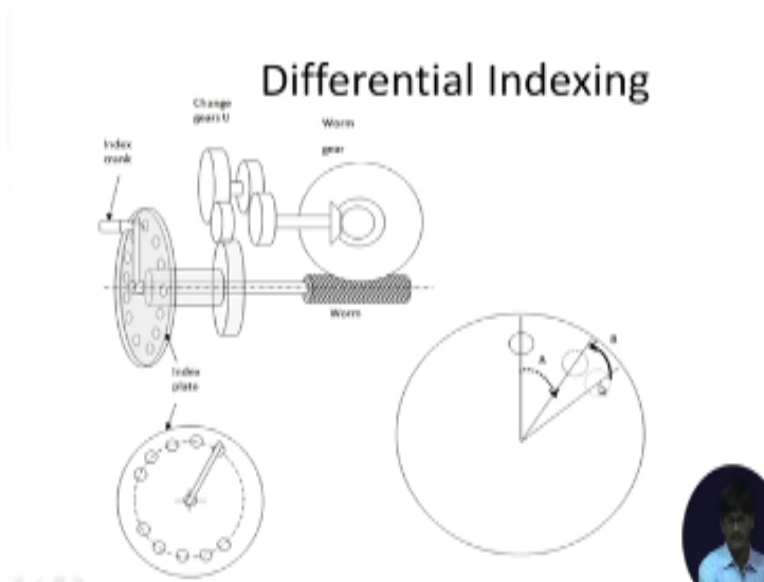
Calculations for the rotation of index crank

- There are 40 teeth on the worm gear while the worm is of single start
- 40 rotations of index crank gives 1 rotation of work piece
- Rotation of work piece by 1 tooth = $1/z$ of a rotation is obtained by $40/z$ rotations of index crank
- There are circles with equispaced holes on the index plate which can help in obtaining fractional rotations
- For getting 30 divisions, $40/30$ rotations = 1 and $2/3$ rotation is required. 1 rotation is completed and from a hole circle with integral multiple of 3, $2/3$ of the holes are covered. Say, 22 holes on a 33 hole circle.
- This is called "Simple Indexing"



Now there is yet another method in which we might be having the use of feedback so indexing with feedback is called differential indexing.

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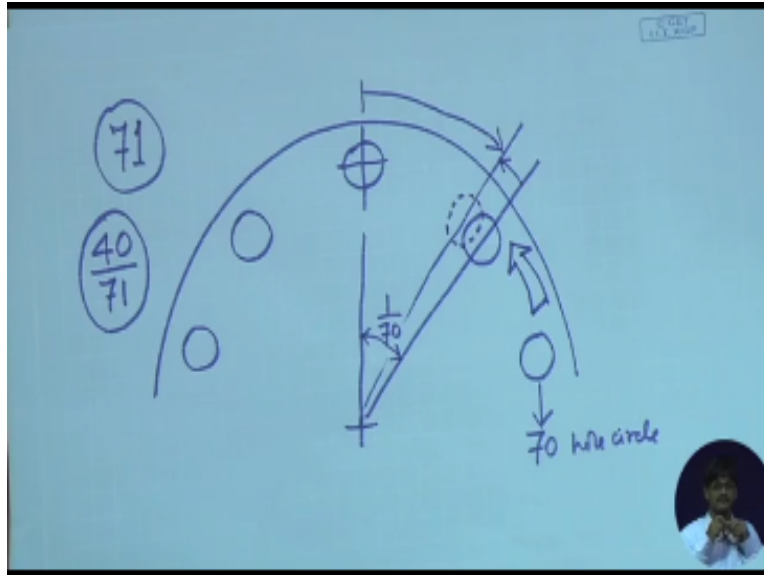


And let us have a quick look at it what does this figure show us this figure shows us once again so let us go through things that we already know just a moment have you missed something know things that we already know this is our index crank this is our index plate index you know this form axis okay this is the worm axis this is our worm gear this is appear of bevel gears which turned the rotation by 90° okay, and we find that there are some gears now existing which rotate the index plate in simple indexing we had not rotated the index rate index plate was remaining stationary with respect to the index plate we were counting off our rotations of that particular index crank.

Now what we do is we have differential motion of the device differential motion means that the ultimate motion is the difference between two motions one motion is the direct motion of the index crank and other motion is the fed back motion of the index plate due to the difference of these two motions we will be achieving our required amount of motion so let us see that in this particular view.

This is the view of the front view of the index plate this is a starting hole okay this is a starting hole from here this is the next hole which is actually existing let me draw a figure so that this is this would be easier to follow.

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If you look at the index plate this is your index plate on the index plate this is they say a starting hole and this is the second hole this is the third one etc, etc. Now you might say which whole circle are you talking about, so let us define the problem previously if you remember we were talking about cutting 70t now the problem is slightly different and more difficult 71teeth have to be cut 71 teeth have to be cut have you been 73 would have you simply missing there would not have been any problem with that.

Now the problem is 71 teeth have to be cut what is the problem the problem is 71 teeth the whole circle is not there 71 whole circle is not there at all so we know the magic number $40 / 71$ rather $40 / 70$ first of a rotation has to be achieved on the crank handle in order to achieve $1 / 71$ of a rotation of the gear $40 / 71$ first of a rotation has to be achieved on the so the crank handle index crank if you do not have 71 whole circle this is near next to impossible. So what do we do? We find out some hole circle which is you know quite near to 71.

So for that suppose we and if I this is the 70 hole circle this is the 70 hole circle not the 71, 71 hole circle is not there and suppose we define this one as our starting point say this is the center 70 hole circle first circle 70 hole circle second one so if I now ask you what is this amount of rotation you will say why this is $1 / 70^{\text{th}}$ of a rotation $1 / 70^{\text{th}}$ of one full rotation yes I accept it now had there been 71holes okay, then the second hole if this is the first hole of that virtual 71 hole circle had it had it been present.

The second hole would have been somewhere very close and ahead of the second hole, so this I am drawing as the virtual 2nd whole which would have existed had I had the 71 whole circle first

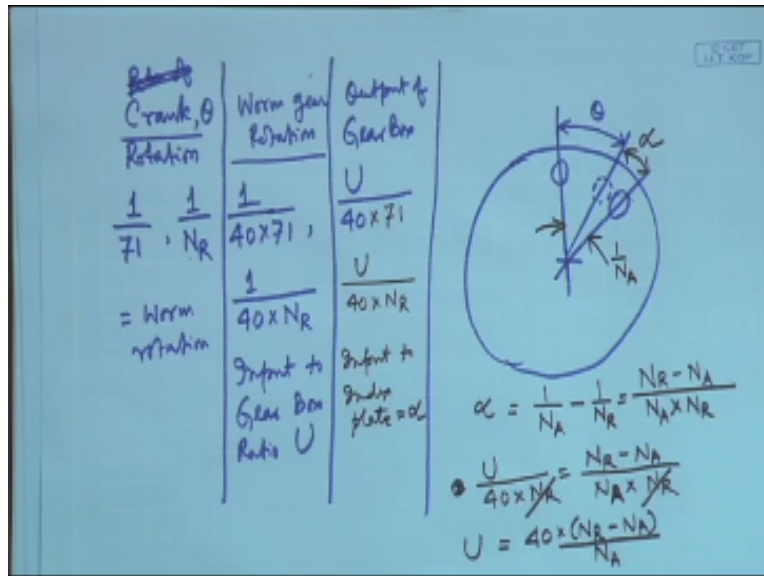
hole second hole of the 71 whole circles it is not there. So my plan is in order to get it here it is virtual things which are virtual they are not tangible you cannot touch them you dream of things and then they disappear something like that but I want it you say I want it here what do you do you bring this one here while you have started your movement from this point as you just reach this point VC video you know crank handle this much amount of rotation in that time you import it to this point.

So you will find a hole and it will get inside it this is differential indexing the whole index plate will be rotating backwards while you have put up your crank and your moving like this you're moving like this in that time this whole plate whole plate will be moving this way. So that this one comes to this point that is it that is what we are going to achieve this is the basic ideas of differential indexing make the plates move so that you achieve what you want you get a hole in the location that you want.

Coming to the figure now let us see how this is physically obtained implemented physically we have we see that there is an addition of an gearbox now all those discussions that we had about gears and gearboxes etc will now become quite relevant what we have done is since I want to rotate this index plate I am tapping motion from the wrong gear putting in a gear box okay some ratio I do not know that have you find out what should be its ratio and I am simply rotating the index plate backwards or forwards whichever be it is.

How much is the amount of rotation required from the index plate this much that is what we have calculated okay how much is the motion of the index crank during that time this one and totally they make up $1/70$ in the general case $1/n$ available. So let us do the calculation okay we do the calculation this way.

(Refer Slide Time: 15:08)



The crank how much does it rotate first hole second hole virtual hole this is the crank rotation θ rotation of crank okay let this is not readable crank rotation θ this much okay how much is it this must be equal to one by the number of teeth we cut this is the virtual hole position and this must be equal to $1/71$ in our case okay, so we write $1/71$ and in the generalized way we write $1/n$ required both are both are correct this is specific is the general case next from this one where are we going if you look at the figure this is going to directly this amount of rotation is just going to the worm.

So we need not worry about the worm rotation but we can worry about the worm gear rotation wrong gear rotation must be related to this is basically equal to we will write equal to worm rotation so wrong gear rotation must be equal to a fraction of this $1/40$, so we have $1/40 \times 71$ and the generalized case is $1/40 \times n$ required okay, specific case general case from the form here we are moving on to the change gears. So we say so this must be input to the change gears we write input to gearbox and gearbox is having ratio unknown to ratio you rewrite and therefore the output of gearbox must be equal to just multiply you with that because we know that this must be equal to output by input.

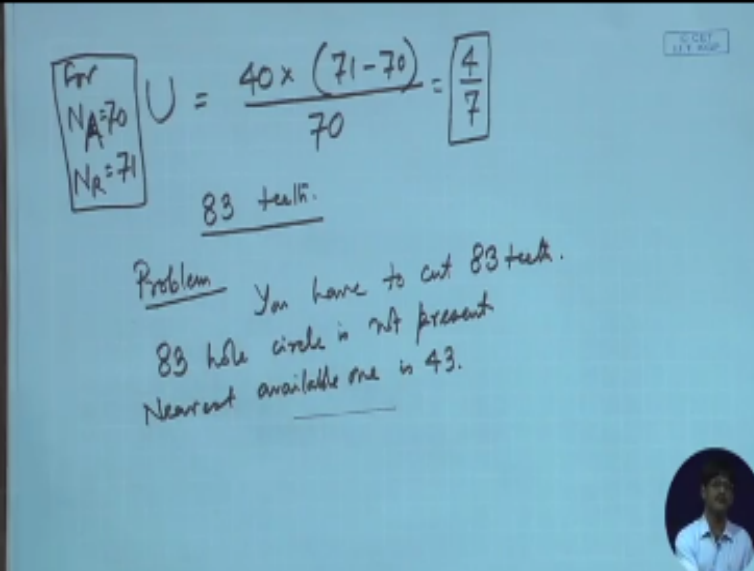
So this one into input must be equal to the output so we write u y 40 now just a minute in to 71 general case $u/40 \times N_R$ and this is input to index plate in a 1 is to 1 ratio, so see this same value is input to the index plate and what is the index plate rotation this amount so this must be equal to this say let us call this $\alpha = \alpha$ so we make this particular equation, now this thing is equal to

alpha and what is α , α must be the difference between n available $1 / n$ available - $1 / n$ required so let us put that here $\alpha = 1 / n$ available this one is 1 by n available and this one was 1 by n required - $1 / n$ required equal to $n_a \times n_r - n_a$ okay.

So this must be equal to this value equal to $U / 40$ into N_R and therefore we have from this one let us copy it to our different sheet of paper or write it here itself cancel this out and you get the relation $U = 40 \times n_r - n_a / n_a$ that is it, so I can state so the value of u has been you know exactly calculated and therefore we can state if we come back to the figure once again. if I set the change gear ratio this being the input side and this being the output side if I set the change gear ratio to a value $40 / n_a \times n_r - n_a$ in that case I will exactly achieve a backward rotation of not backward always anyway a resultant rotation of the index plate.

So that I will be achieving exactly $1 / n_r$ rotations on the gear blank this is the most general statement that we can make okay that is this change gear ratio if it is set to $40 \times n_r - n_a / n_a$ in that case it will result in rotation of the index plates. So that a rotation of $1 / n_a$ on the index crank will result in $1 / N_R$ rotation of the gear blank and that is what exactly we want.

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For $N_A = 70$
 $N_R = 71$

$$U = \frac{40 \times (71 - 70)}{70} = \frac{4}{7}$$

83 teeth.

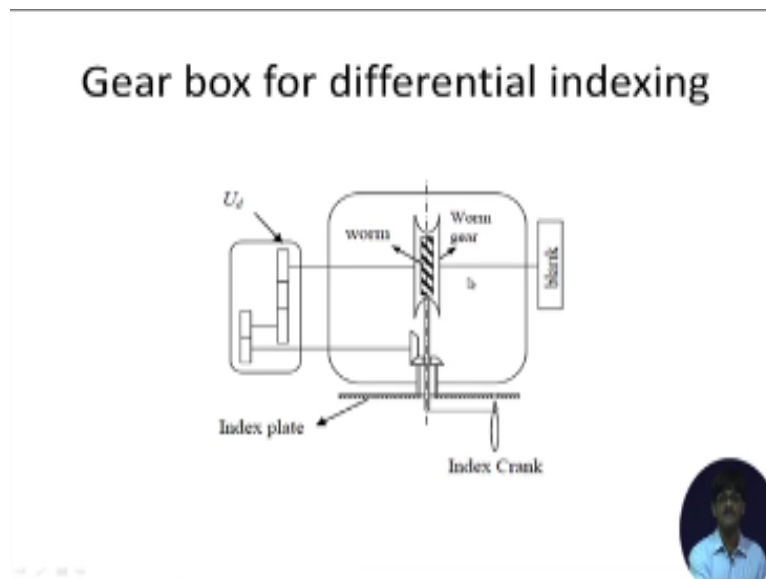
Problem You have to cut 83 teeth.
83 hole circle is not present
Nearest available one is 43.

Now for the specific calculation let us apply this there for you for $n_r = 70$, sorry I am making mistakes for $n/a = 70$ $n_R = 71$ this is our requirement $u = 40 \times 71 - 70 \times 70 = 4 / 7$ that is it since we very simple $4 / 7$ if you if you can put such gears herewith the ratio $4 / 7$ in between the worm gear shaft and the index plate shaft you will be achieving differential indexing in this case. So we will be definitely doing some problems during the problem session and let me define them so that you can think out the idea and we can compare our results during our questionnaire session.

For example suppose you have to cut 83 teeth and the 83 hole circle sorry let me let me frame the problem this with problem you have to cut 83 teeth 83 hole circle is not here not present not available not present okay and the nearest available one nearest available one is 43 why 43 I could have given it 80 here the idea is if 43 is present you can do the calculations with 86 in mind okay with 86 in mind and after that you will find that 1 2 will cancel out so that ultimately you can work with 43.

So this is also another thing that we will be learning exactly the nearest whole circle might not actually be present but with its virtual presence you can do the calculations and after that you might find that a fraction of it if it is present that we do the trick so this one please work this out we will carry out the calculations during our questionnaire session okay. So after this so I will be giving you more problems of this type when we even discuss further.

(Refer Slide Time: 25:57)




Now for a quick look at what we have discussed so that our doubts if any will be clear, I have tried to those people who have had troubles in following the 3-dimensional picture I have tried to give the explanation of the thing you know flat two-dimensional drawing let us have a quick look at it this is our index crank when you are rotating the index crank this is the shaft of the index crank which goes right through to the to the worm and the worm gear is shown here okay like this and it has its shaft on one side it is carrying the blank on the other side it is scanning the change gearbox and the change gearbox output is given to the index plate through a pair of bevel gears and the index plate rotates due to that. So this is a typical figure which has the details of a gearbox for differential indexing.

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Calculations for differential indexing

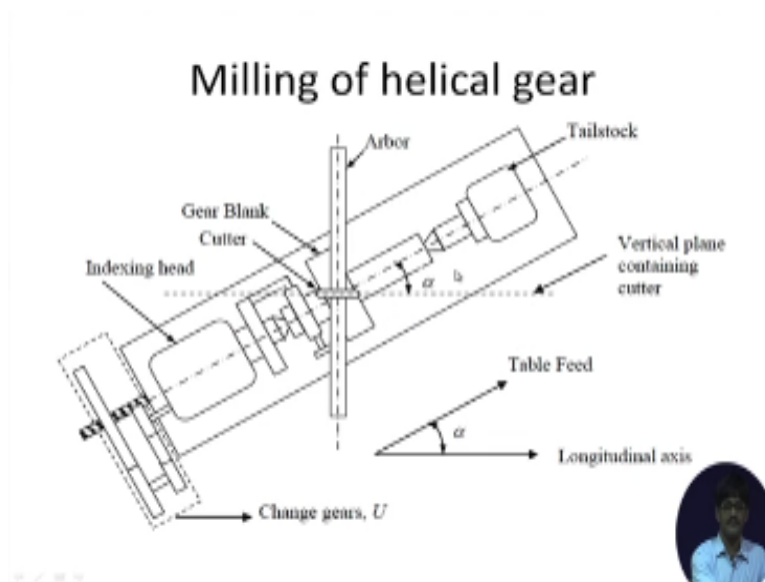
- How much to rotate ? $40/N_A$
- What is the gear ratio ?

$$U = \frac{40}{N_A} \times (N_A \sim N_R)$$



This is the calculation that we have worked out, so I am not doing it once again.

(Refer Slide Time: 27:13)



Just a moment there might be a mistake in not a mistake the one difference in sign is there I have written in a difference in our to generalize the case, now question is the operator who is carrying out the differential indexing you might say that okay so far so good you have put a particular gear ratio and we are expecting that cutting will be done quite you know properly with that but how much do I rotate the index crank if 71 teeth are to be cut and 70 hole circle is available how much is the rotation that I give?

The rotation will be $40 / Z$ that means 40 by available on the available whole circle if 70 hole circle is available you will be moving by 40 holes and automatically the Machine due to the presence of the gearbox will be converting it into $1 / n$ on the work piece. So $40 / n$ will give rise to one but enough so on a fourth on a 70 hole circle move by 40 holes and the rest will be taken care of all those gear boxes and other things that we have put in between that is it okay.

So the rotation is 40 holes out of 70 holes level now I will just introduce the idea of milling of helical gears we have studied about differential indexing instead of going for solution of

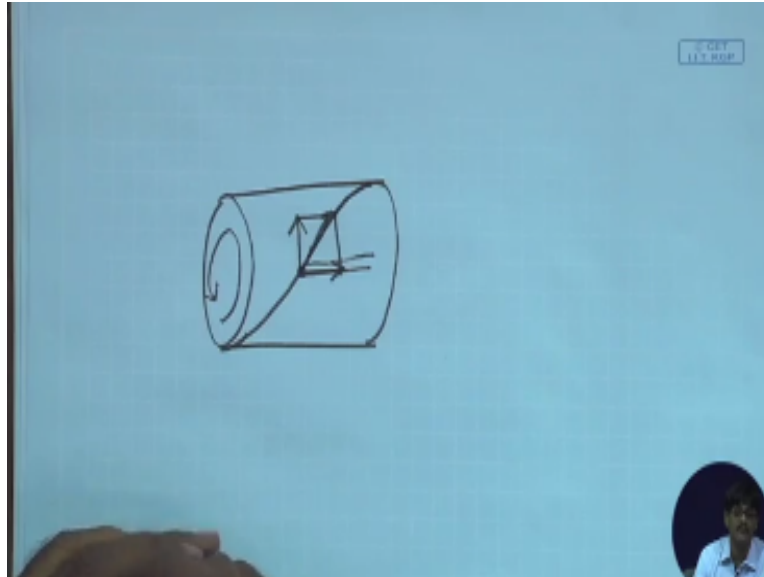
numerical problems I will introduce the idea of helical gear and later on in our tenth lecture ninth and tenth lecture we will solve numerical problems let us have a quick look what we are trying to you know trying to convey through this particular figure this is the table of a universal milling machine seen from the top this is a horizontal column in me type Universal milling machine why horizontal because this is the axis of the cutter okay axis of the arbor which is holding the cutter Arbor is the you know shaft which is holding the cutter this is the cutter.

Now how does it correspond to the cutter that we have seen in our drawings this is you know that rotary disc type cutter the disc is being seen from the top not the circular part I mean the width of the cutter is seen in this direction okay in the other view if you look along the arbor you will see a circular part with the teeth on its periphery. So how does the cutter move it rotates about the arbor to develop cutting speed and what are the other types of movement that it has relative to the table the table is going to have movement this way.

The table has movement this way what other movements are being provided here the cutter is being rotated the cutter is being rotated, so that means across previously we were having the cutter here and the table was straight okay and the table was simply moving to and fro so that the cutter cut right across cut a groove right across on the circular or curvilinear surface of the work piece the cylindrical surface of the work piece.

At this moment the vertical plane containing the cutter is at an angle defined by the helix angle okay is at an angle with the work piece why is this so because we have to cut a groove which is a helical groove on the surface, so for that let us have a quick look at this figure.

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The basic idea is if you have to say put a pencil mark on a cylinder in the form of a helix the helix is moving this way which means that at a particular moment it can be defined by a definite velocity this way and a definite velocity this way so that the resultant is always tangential to the helix. How is this possible? You can put a pencil here suppose I stick a pencil here give a rotation to this particular work piece or sorry cylinder and move the pencil actually at the same time okay I am holding the pencil I am moving the pencil this way and at the same time I am rotating this particular body you will have the cutting of oh sorry the tracing of a helical line on this one on this particular cylinder same thing for the cutting action during helical milling of gear teeth.

We have rotation of the work piece okay giving the rotation for the helical motion and straight-line motion of the work piece okay straight line motion these two combine to provide relative helical motion of the cutter with respect to the work piece but then why are we rotating the whole table about a vertical axis why is this particular thing inclined we could have done could well have done the whole thing with the cutter in this state with the table in this direction that we will discuss the next day please have a thought about it okay.

So this way we come to the end of the eighth lecture so after that we will be taking up the discussion of the sorry yeah this eighth lecture in the ninth lecture we will be discussing the rest part of helical milling and all the other things that are left behind for milling of spiral helical gears, thank you very much.