

**NPTEL**  
**NPTEL ONLINE CERTIFICATION COURSE**

**Course**  
**on**  
**Spur and Helical Gear Cutting**

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**Lecture 19: Gear Hobbing- V**

Welcome viewers to the 19th lecture of the course spur and helical gear cutting.

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**Spur and helical gear cutting**  
**19<sup>th</sup> Lecture**

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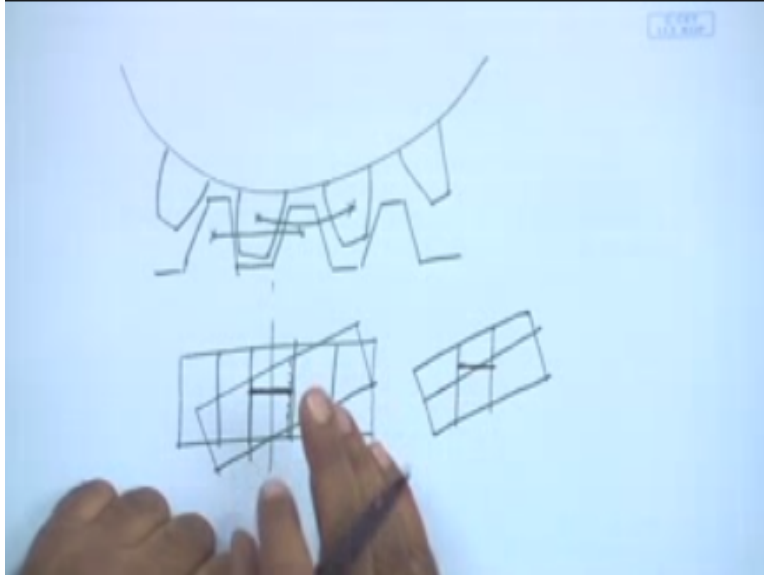
So last day when we were discussing we were talking about the you know the hand of the helix and the relation between the circular pitch of gears to be cut and the pitch of the hub etcetera, so let us start right away with those discussions.

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- For cutting a 4 mm module straight tooth spur gear in gear hobbing machine, normal pitch of single start hob thread rounded to third place of decimal is (in mm)
  - (a) 12.566 (b) 10.566
  - (c) 14.566 (d) none of these.
  -

We were discussing last time this one for cutting a 4 millimeter module straight tooth spur gear in helical gear hobbing machine normal pitch I made a small correction here it was written previously pitch so I made a change, normal pitch of the single start of thread rounded to third place of decimal is in millimeters there are four options given, what do we exactly mean if you have a look at the sheet of paper.

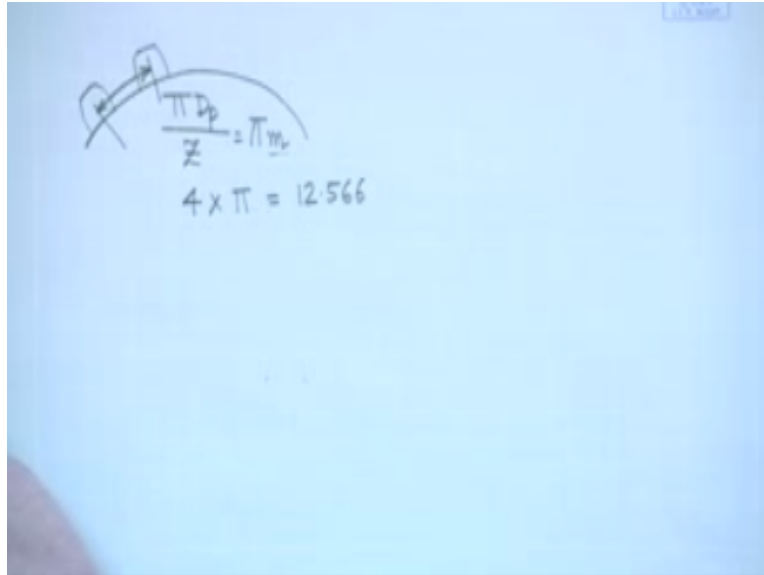
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We have this sort of a relation existing say this is the basic gear okay, and with this one we are drawing these being the gear teeth we have the hub which is actually a worm with cutting edges, sort of a relation is existing which means the circular pitch is equal to the linear pitch but if you remember in the other view the worm gear being like this is being a spur gear the teeth are of this type and the hob has to be at an angle so that on that side it matches with its thread matches indirection in alignment with the teeth of the gear.

So basically this distance that we have shown a circular pitch circumferentially this distance has to be equal to the to the distance between the threads and that means the normal distance between the threads okay, so if we look at the hog alone it is this distance not the axial distance so we are not talking of actual pitch we are talking about the normal pitch, normal pitch has to be equal to the circular pitch of the gear this is the equality which has remained in so that they will be in mesh this particular equality does not necessarily have to be satisfied in all cases but it is the general case yes, okay.

So we have the normal pitch equal to  $\pi$  into module how do we get that sorry, the circular pitch equal to  $\pi$  into module, how do you get that let us take a fresh sheet of paper.  
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Yeah, so circumference being equal to  $\pi$  into pitch diameter this divided by the number of teeth  $Z$  okay, gives us  $\pi m$  okay,  $\pi m$  so that means the circular pitch okay sorry, this being a spur gear this is equal to simply the module and this is multiplied by  $\pi$  this is the distance that we are talking about, this is the distance we are talking about these are the two teeth okay, this has to match with the normal pitch of the hub and therefore we have what is the module, 4 module,  $4\pi$  this one will have to be equal to the normal pitch and this is equal to 12.566 and some trailing you know numbers.

So be fine if we come back to the question now, the options are 12.566 millimeters 10.566, 14.566 none of these, therefore 12.566 is the correct answer, normal pitch of a single start of thread okay. Once we have understood this one let us move on to other questions.

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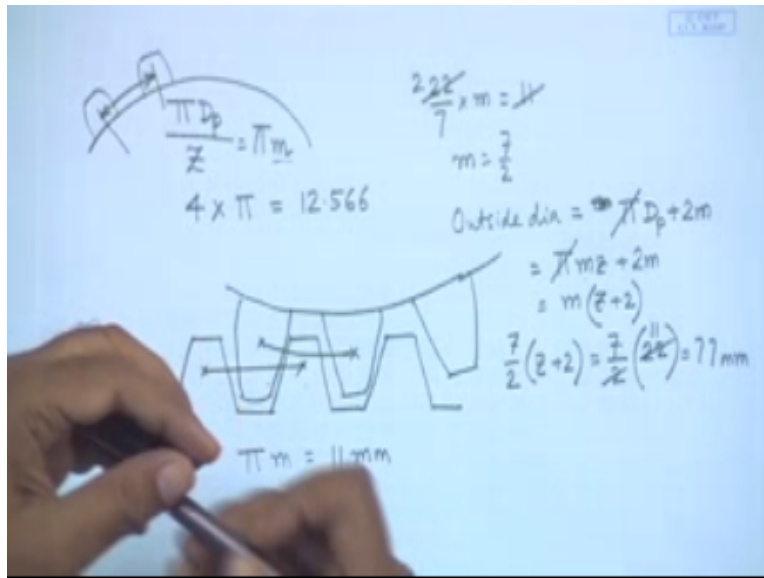
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- Linear pitch of a rack is 11 mm. It is in mesh with a pinion having 20 teeth. Outer diameters of the pinion is
  - (a) 67 mm (b) 77 mm (c) 87 mm (d) 70 mm



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Linear pitch of a rack is 11 millimeters okay, so let us draw a rack.

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It is in mesh with opinion having 20 teeth outer diameter of the pinion is so this is the pinion etcetera, so we need to find out what is the outer diameter of the pinion now as they are meshing therefore linear pitch of the rack must be equal to the circular pitch of the gear circular pitch once again is  $\pi m$  okay, so  $\pi m$  must be equal to 11 millimeters, let us use this part this is clear so we have  $\frac{22}{7} \times m = 11$  this and this they will cancel and  $m = \frac{7}{2}$  okay.

Outside diameter will be equal to okay, outside diameter will be equal to  $\pi D_p + 2m + 2$ ,  $D_p + 2m$  this is equal to  $\pi m z + 2m$  which means  $m$  being taken common I am sorry, answer if we drop this one okay, I am extremely sorry drop this one this is in the subcomponents so it is not in the diameter so  $D_p + 2m$ ,  $mz + 2m$ ,  $m(z+2)$  okay, so once we are knowing the module we write  $\frac{7}{2}(z+2)$  must be equal to  $\frac{7}{2}$  into what is the number of teeth 20, so 22 cancels out 11, so answer must be 77 millimeters, okay.

So coming back here let us see (b) is the correct answer, 77 millimeters so once we arrive at that let us move on to the second question and in third one.

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## Calculations without differential

- Suppose in a particular hobbing operation, the number of teeth to be cut is 20 and the lead of the helix is  $L=1000$  mm. What should be the gear ratio  $U_i$  if no differential is used? The feed in mm of Hob vertical movement for every rotation of work pc is  $s=0.2$  mm/rev.



Calculations without differential largely we had identified a problem that is the differential really necessary and we have had a theoretical discussion at the end of which we had reached the conclusion that if we use, if we have a machine without differential there will be gear ratios which will be very difficult to implement okay. So now we will follow this up with a numerical problem, numerical problem that is why is the differential becoming absolutely essential for the gear humming machine.

Suppose in a particular hobbing operation the number of teeth to be cut is 20 and the lead of the helix is 1000 millimeters what should be the gear ratio  $U_i$  if no differential is used. The feed in millimeters of vertical movement for every rotation of the work piece is  $s= 0.2$  millimeters per revolution of the work piece, so what do we have we have a helical gear being cut it is number of teeth is 20 the lead of the helix is 1000 and  $U_i$  has to be found out no differential is used and the feed in millimeters of vertical movement is 0.2 millimeters per revolution.

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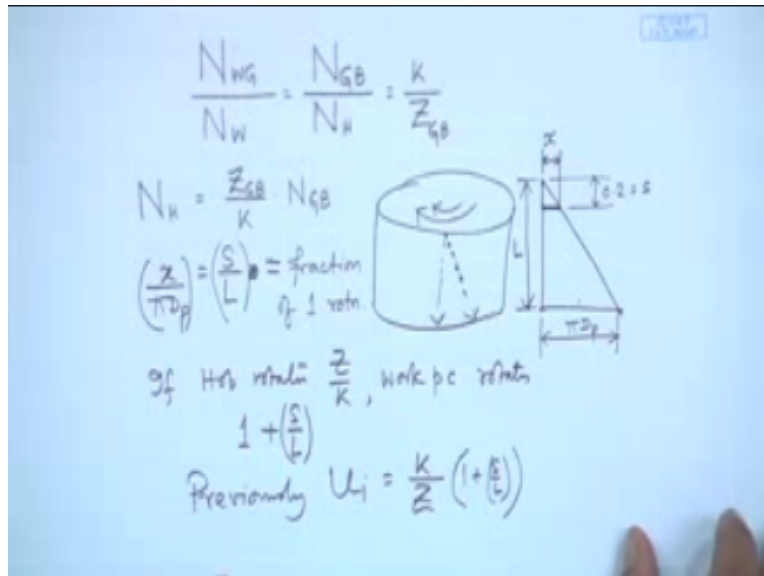
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- For spur gear, for each rotation of the work pc, the Hob rotates  $Z/k$  times.
  - For helical gear, the w/p rotates a bit more or a bit less than one rotation for  $Z/k$  rotations of Hob.
  - How much more / less ?
  - This extra amount of rotation is  $\rightarrow$  1 full rotation while the cutter moves down by 1 lead = L



So we what we say is that if we are cutting a spur gear of 20 teeth then for every location of the work piece the hub rotates is that by t times now you might ask me what is that, how do we get that.

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So if we come to the piece of paper we have  $N_{WG}/N_W =$  let us use  $N_{GB}/N_H = K/Z_{GB}$  this is accepted  $K/Z$  okay. Now if we are cutting a spur gear of  $Z$  number of teeth with  $K$  number of starts of the hob in that case I can say that therefore  $N_H$  will definitely be equal to  $Z_{GB}/K \cdot N_{GB}$  so if this is 1 for every rotation of the gear blank  $N_H$  will then definitely be equal to  $Z/K$  into 1, so if I have to have one rotation of the gear blank I have to rotate  $N_H$  that is now the hob rotation has to be equal to  $Z/K$  this is understood for spur gears we have done it so many times.

But now the situation is changing, the situation is changing in this manner that in order to cut a helical gear we are deciding that every rotation has to have an additional rotation of the blank so that this cutting will not proceed this way but the cutting will be proceeding in this way and in that case we will be putting some extra rotation of the blank and that will you know lead to the cutting action the downward motion and it will be proceeding next point of cut will be here.

Next point of cut will be here like that, what is this extra rotation in the theoretical discussion that we had we came to the conclusion that this extra rotation is related to the downward motion in this manner this being equal to  $\pi D_p$  this means equal to lead if you move down by  $s$  that means a 0.2 millimeters if you move down point per rotation  $0.2 = s$ , then this much is the amount of rotation that is required to be added.

So with every rotation sorry, with the downward movement taking place this way the extra rotation is this much I am moving down this you maintain this extra rotation so that at the end of the day I move down by  $L$  and the extra rotation is one rotation, so mind you this is extra rotation

this is already rotating to maintain you know  $N=Z/K$  and work piece rotation=1 that is already maintained the extra rotation takes place this way for rotation that is taking place I am moving down this much you add this much okay.

So how much is this, so from similar triangles we can easily find out this  $x$  amount of rotation must be equal to  $S/L$  okay, that is  $x/\pi D_p=S/L$  so this must be  $x/\pi D_p=S/L$  so that means this is the actual fraction of one rotation, this is a fraction see this is one full rotation a part of it so the fractional amount of rotation is nothing but  $S/L$ , so if this be so therefore now if we can make a statement if of rotates  $Z/K$  work piece rotates  $1+S/L$  that is it, this is the extra amount of rotation in expressed in fractions of rotation one full rotation fraction of it.

Hence the gear ratio previously,  $U_i$  was equal to  $K/Z$  okay,  $U_i$  was equal to you know if you started from hub and you went through all the gearings and say all the other gears and belts and bevel gears excreta 1:1 you would have had this to be  $K/Z$ , output by input now there is an extra rotation added because the output is not 1 but  $1+S/L$ ,  $U_i$  is equal to output rotation which means gear blank rotation by hob rotation and gear blank rotation is not one anymore it is this thing  $1+S/L$  okay.

Previously we were putting 1, so in that case let us take a practical example this particular numerical problem all these things we have already discussed okay.

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- $U_i = (1/20) \times \{1 + (0.2/1000)\} = \{1000.2/20000\}$

- $= 10002/200000 = 10001/100000$

- This ratio is quite difficult to achieve through ordinary gear box and use of differential becomes necessary

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$U_i = 1/20$  that means  $K/Z$ ,  $K$  is single start worm and number of  $K$  to be cut is 20 okay, this is  $20 \times 1 + S/L$  oh my god, so 1000 comes in the denominator so that we have this one to be equal to 1000.2 in the numerator and 1000 at the denominator this 1000 gets multiplied by 20000,  $1000.2/20000$  I cut it I remove the decimal so it becomes  $10002/200000$  which is equal to  $10001$  wait a minute have I made a mistake yeah, there is a small mistake here please look at this.

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$$\frac{\cancel{1002}}{\cancel{20000}} = \left( \frac{5001}{100000} \right)$$

We have finally 10002/200000 I intend to cut it with 2 I am sorry please make this correction 5001/100000 this is quite a difficult you know fraction to attain, how do we attain this fraction by gear ratio now first of all we will try to divide it into products if they allow and after a lot of you know permutation combination the gear ratio it will be difficult to obtain, instead of that if you employ the differential system that means there will be lead change gears and the differential the problem of this I am professor achieving a particular number of teeth and problem of achieving a particular helix angle they become totally different okay.

They become they do not really remain an integrated problem but they become totally different from each other they become mutually exclusive, so this is the reason to avoid such difficult gear ratios we go for finally the differential instead of a known differential system.

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## Willich Equation

$$\frac{N_o - N_a}{N_i - N_a} = e$$



- $N_o$  = Output RPM     $N_a$  = Arm RPM
  - $N_i$  = Input RPM
- $e = -1$  for the 4-bevel differential  
So, if  $N_o = 0$ ,  $N_a = 2 N_i \rightarrow$  this is what we took

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Next the weight equation or the Willich equation there will be the pronunciation are not pretty sure the four bevel differential what did we do when we came across this particular mechanism we said that this is going to be there we divided it up into two different cases where we said that if this be this is written slightly upside down  $N_i$  input and giving the input here I am giving second say input through the arm rotation and I am getting an output here, I can also give this one as a second input this one is the first input and get the arm rotation as an output this is what we had done in the actual numerical problem on gear hobbing that we had discussed.

Now what are we trying to do we are saying that at that time we had roughly estimated the rotation of the arm you know by a thumb rule method we can use the Willich equation in order to actually calculate what we need the arm location if one of them is held steady suppose this  $N_i=0$  and  $N_o$  is equal to some particular value. The arm location in our numerical problem was the output, so the Willich equation says that the output rotation with respect to the arm rotation divided by the input rotation with respect to the arm rotation will bears a constant ratio which is obtained by keeping the arms steady and rotating the system.

So if I rotate  $N_i$  looking from this particular direction suppose it is rotating clockwise and this is rotating counterclockwise let us draw a figure and show it okay.

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The image shows a handwritten derivation on a blue background. At the top, a fraction  $\frac{10002}{20000} = \left(\frac{5001}{10000}\right)$  is written. Below it is a diagram of a differential gear mechanism with three gears and an arm. To the right of the diagram, the equations  $\frac{N_i - N_a}{N_i - N_a} = e = -1$  and  $N_o = 0$  are written. Below the diagram, the derivation continues:  $\frac{+N_a}{N_i - N_a} = +1 \Rightarrow N_a = N_i - N_a$ , followed by  $\Rightarrow 2N_a = N_i \Rightarrow N_a = \frac{N_i}{2}$ . A small circular inset in the bottom right corner shows a person's face.

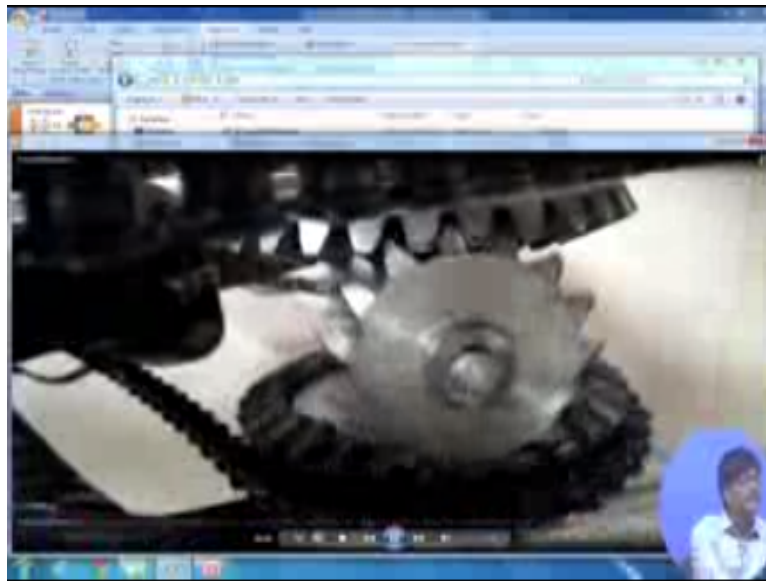
Suppose this is gear 1, this is gear 2 and this is gear 3 so if it is rotating say it this way if it is rotating this way, this will rotate this way so that if I look from this direction I will find that these two are having different directions of rotation different sense of rotation, so coming back to this problem  $e$  is defined to be you know if the arm is kept steadied they will simply have the same rate of rotation but the sense will be opposite so that  $e$  will be equal to  $-1$  for this particular four bevel differential rather different the Willich are all of the same number of teeth okay.

So  $e = -1$ , so if  $e = -1$  okay, and if we keep one of the bevel gears to be fixed so I put  $N_o = 0$  for example, so I am giving an input  $N_i$  I am keeping  $N_o$  to be 0 and I am trying to find out the arm rotation and you will find it will be  $-N_a/N_i - N_a = -1$  therefore if you solve it, it will become let us solve it quickly.  $N_o - N_a/N_i - N_a = e = -1$ ,  $N_o$  we put to be equal to 0 and therefore we have  $-N_a/N_i - N_a = -1$  if you cross multiply you will get  $N_a$  first of all we remove this negative sign  $N_a = N_i - N_a$  therefore we get  $2N_a = N_i$  therefore  $N_a = N_i/2$  that is what we had taken up at that time and by applying Willich equation we can actually calculate it out to be this much.

So this is the level differential and this is what is used there and when you have two different rotations given suppose  $N_i$  is not 0 in that case what will be the arm rotation as a result it will be simply equal to  $N_i + N_o/2$  so it will be basically as we had stressed at that time an algebraic addition of the inputs divided by a particular factor as we have understood here the factor is 2 that means just a moment excuse me,  $N_a = N_i/2$  I have written it wrongly here please correct in the calculations that we have done it has been done correctly when I distribute this you know PPT in PDF form I will make this correction extremely sorry, okay.

So we understand that arm will be always equal to half of this input and in the same manner armed be equal to half of this input if this is put on and this is put off and both of them then they are acting it will be the algebraic addition of the halves of these two input RBM's okay, so as we had claimed this does a sort of addition this they will differential okay, not sure we will take a short break from here to have a look at how this actually operates.

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So let us see this is the arm and that is one of the beverages at the top this is put off and the bottom one is rotating okay, once again let us see yeah it is rotating see the arm this is rotating and the bottom one is not rotating with it but it is having a separate rpm. Unfortunately, I could only get hold of this particular differential where the V the bevel teeth they are not the same that means the bevel gear diameters are not the same but anyway something is better than nothing okay, so this is what we had ever taken considered in our setup coming back let us start again so this one is now understood and accepted.

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## Hand of the helix

- Hold a machine element (screw / worm / hob / helical gear) away from you.



- A right handed helix twists clockwise while moving away from you. The left side one is left hand and right side one is right handed.



Now comes we had a discussion on the hand of the helix, how do we decide upon the hand of a helical member. The basic procedure is all the machine element it might be a screw a worm hob a helical gear away from you that means this particular arrow is designating that you are here you are here and you are holding this away from you it is going away from you okay, if it is a right handed helix it twists clockwise okay, while moving away from you okay.

So this is moving away from you but in which direction is it twisting obviously it is twisting counterclockwise so this cannot be a right handed helix what about this one, this one is rotating this way clockwise while moving away from it so this must be right handed so the left side one is left-handed and the right side one is right-handed and they can all be you know mutually changeable if you put it the other way around it does not become right-handed. No, so this one is right and this one is left hand helix.

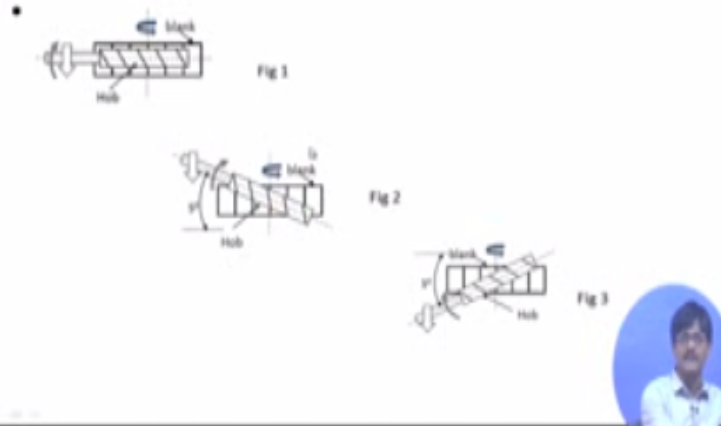
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The helix angle of a hob cutter (right hand helix) is  $5^\circ$ . It is used for cutting straight tooth spur gear. Axis of the hob cutter should be

(a) As in fig 1 (b) As in Fig 2 (c) As in fig 3 (d) none of these.



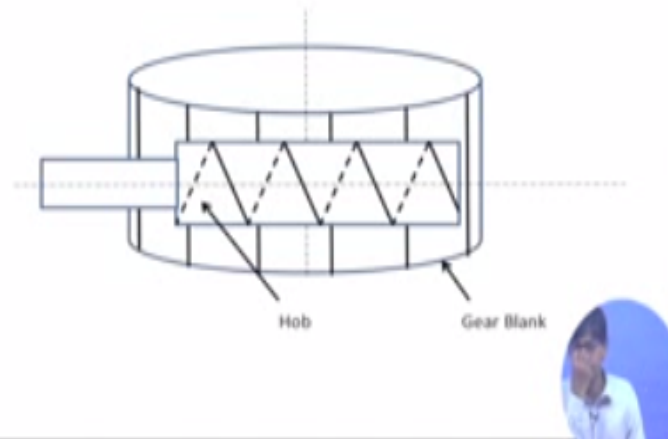
Let us take a problem I hope you can read it the helix angle of a hop cutter a right-handed helix is  $5^\circ$  it is used for cutting straight tooth spur gear axis of the hob cutter should be as in figure one, as in figure two and as in figure 3 which one is correct or none of these, so let us look at these this one the axis of the hub is crossed with the axis of the blank that is at  $90^\circ$  but this would not do because the threads of the hub has to align itself in the direction of the teeth being cut. Here they will be at an angle, here the threath appears to be aligned with the teeth of the gear.

But the problem is these are the threads that you can see through there away from the gear, the other side of the thread which is you know facing the gear get it that is not aligned with the gear teeth why because it is moving this way it is crossed it is not vertical, so this is also not correct this one is correct because here this is the part of the thread which is not visible to us but it is definitely in line with the teeth where it is meeting the teeth okay, in contact with the teeth and hence the answer is as in figure 3, as in figure 3.

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## RH Hob with straight spur gear how to determine the rotation direction



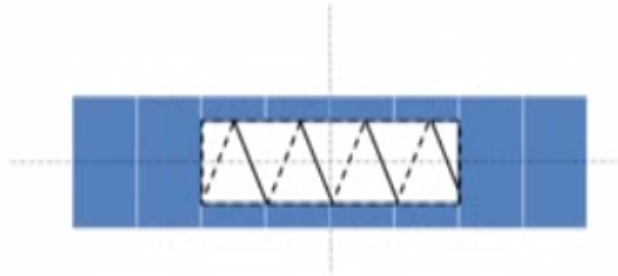
Now coming to this how do we decide this problem also we have identified, how do we decide whether a particular hub is correctly aligned with the work piece or gear blank, for that let us see we are taking the case of our spur gear with straight teeth and this is a right-handed helix so the dotted one is the one which is touching or on the side of the gear teeth in contact with a gear teeth.

The basic rule is that these dotted threads okay, traces of the thread which is in contact with the teeth after proper you know proper positioning or orienting of the hub these dotted lines should align themselves with the potential teeth okay, the teeth which are which are going to be cut they have to get aligned I mean the thread has to get aligned the dotted thread has to be get it has to be aligned with those teeth. So here obviously the dotted thread should be vertical, now let us try this out is this direction correct no not at all it is becoming more and more horizontal the threads the dotted threads so this is not allowed.

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RH Hob with straight spur gear  
rotation angle is Helix angle of worm

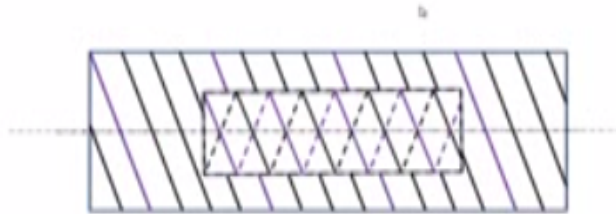


Second case let us try it out the blue one is the gear to be cut and these white vertical lines they are the traces of the teeth. Now let us give it a rotation and see what happens yes, this seems to be it therefore the dotted lines are matching with the direction of the teeth being cut and therefore we say that for cutting a spur gear with right hand hob the rotation angle is simply the helix angle of the worm okay.

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RH Hob with LH helical gear  
Rotation angle is Helix angle of Hob + helix angle of gear



Second case, there can be so many combinations I am having a right-handed worm sorry, right-handed hob and I want to cut a left-handed helical gear, so this is the axis of the hub of been rotating about this axis and what are these threads they seem to be different yet to add to the variety I have incorporated a hob with two starts can you see there are two start it hardly makes a difference you can definitely use to start hub there is nothing wrong with that and in fact you will be more confident after going through that sort of a problem.

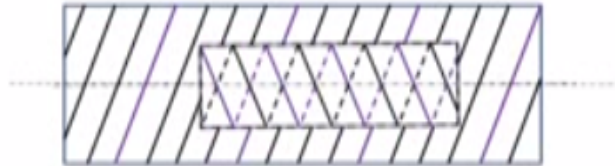
So is this right-handed hub yes, it is moving away from you as it rotates clockwise the dotted part is on the other side of the hob, what about the gear if you are here, if you are here as it moves away from a you it rotates counterclockwise so this must be a left-handed helical gear and the axis of rotation which has not been drawn it is in this direction okay, so how do I rotate it the Golden Rule once again is that get those dotted lines aligned with the direction of the teeth.

Yeah, so this is the correct rotation so you have to give wider large amount of rotation here which is which happens to be equal to a written above angle of the hob, helix angle of the hob plus helix angle of the gear please remember helix angle of gears they are expressed with respect to the slant of the teeth with the axis of the gear, so this is four right hundred hop with left-handed a little gear okay.

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RH Hob with RH helical gear  
Rotation angle is Helix angle of Hob - helix angle of gear



Another combination right handed hob with right handed helical gear here the teeth to appear to be almost aligned and if their helix angles are equal they will be perfectly aligned, but that might not be the case the helix angle of the hob might be you know 2 or 3 or something like that while the helix angle of the gear might well be different, so the angular rotation ultimately is the difference in these helix angles. See the slight amount of rotation which is required to make these mesh exactly and I get aligned with the gear teeth that is it.

So it is a difference which has to be the amount of rotation taking another case what is that left-handed hob, how I know it is a left-handed hob while it is rotating and moving away from you it is rotating essentially in a clockwise direction okay, that is good and what about this particular thread that is a right answer because while moving away from you it has to have you know rotation in a clockwise manner when you are looking at it from this side that is good, so how much is the rotation let us try this out.

Yeah, so once again quite a bit of rotation is required and this comes out to be the helix angle of the hob plus helix angle of the gear okay.

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## MCQ

- In the **gear hobbing** process, the 'Hob' shape is similar to
  - 
  - A spur gear with cutting edges
  - A reciprocating rack with cutting edges
  - A worm with cutting edges
  - None of the others



So let us end up with a couple of multiple choice questions, in the gear hobbing process the hob shape is similar to a spur gear with cutting edges. No a reciprocating rack with cutting edges not exactly. A worm with cutting edges yes, and of course none of the other others in that case is not applicable so in this case the correct answer is a worm with cutting edges.

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- In gear hobbing, indexing gear ratio  $U_i$  depends on
  - (a) number of teeth on the gear blank,
  - (b) module of the hob cutter,
  - (c) RPM of the hob cutter



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In the gear hobbing, indexing gear ratio  $U_i$  depends on number of teeth on the gear blank module of the hob cutter and rpm of the hob cutter let us see, what is it dependent upon let us write down here.

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$$N_H \times U_i = N_{GB}$$

$$U_i = \frac{N_{GB}}{N_H} = \frac{K}{Z}$$

We know that starting from the hub rotation, suppose how much rotation of hub multiplied by  $U_i$  must be equal to and we are considering that there are no other machine elements in between no gears, no belts, no bells, no worm and worm gear etcetera, hob  $U_i$  we must be getting gear blank okay, therefore  $U_i$  must be equal to rotation of gear blank /rotation of hob and this we know has to be equal to  $K/Z$  okay, so now we understand that  $U_i$  is equal to this one it is a constant  $K/Z$ , so now if I say that it depends upon the number of rotations of the hob I can definitely say no it does not you change the rotations of the hob.

$N_{GB}$  will also change so that this will be untouched if you  $2N_H$  do you mean to say that  $k/Z$  will change no so it does not depend upon the number of rotations of the hob, does it depend upon  $Z$  yes, if you are cutting a different number of teeth you will find that  $U_i$  will change so let us come back to the problem does not depend upon the number of teeth on the gear blank yes, module of the hob cutter well  $K/Z$  did not contain module of hob cutters our team of the hob cutter just now we showed that RPM of the hop cutter is not going to affected is only equal to  $K/Z$  okay.

So the correct answer is number of teeth on the gear blank so with this we come to the end of the 19<sup>th</sup> lecture we have one more lecture left and we will pick up our loose ends in that one, thank you very much.