Introduction to Fluid Mechanics
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Lecture – 08
Kinematic viscosity, Reynolds number

So, if we try to identify the dimensions of the viscosity.

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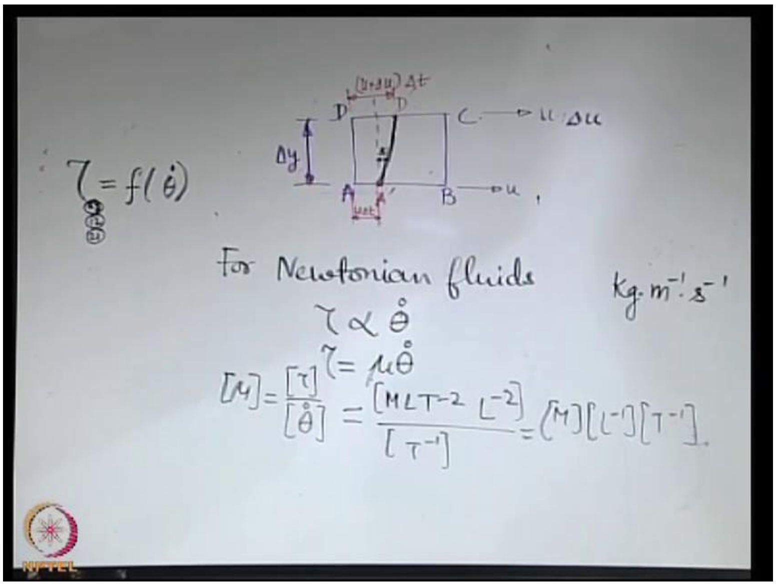

So, it is a dimension of shear stress divided by the dimension of the rate of angular deformation. So, let us write it in the M L T dimension. So, this is stress, force per unit area right; force is mass into acceleration. So, you tell that what should be this, whatever you tell I will write that; so first the force M L.

Student: (Refer Time: 01:07).

T minus 2, then this is what this is force.

Student: Force.

Divided by area, so another L to the power minus 2; then this is T minus 1, so it is M L minus 1 T minus 1; so in SI units you can write this as kg per meter second. But, there are different styles in which this is written also in SI units; of course kg per meter second is one of the styles. But you can also write it in terms of force units, and those are typically more common units. This, if you write it in terms of force units or units of pressure so to say you can also write it as Pascal second. Because, you see stress is like a pressured unit and this is second inverse. So, that goes in the numerator it may also be written as Pascal second. So, these are alternative units for the viscosity; alternative expressions for units of viscosity all in SI units.

So, in our course we will be always following SI units, and therefore it is important that we get conversant with these units. Of course, in other units there are expressions, like in serious units this particular is unit is given the name as poise and reason is like these names have come up to honor the very famous scientists or mathematicians who developed the subject of fluid mechanics.

The subject of fluid mechanics has been initially developed mostly by mathematicians. And, it is important to honor them in various ways. One of the ways is to give units in their names. So, by giving honor to the Poiseuille, to the famous scientist Poiseuille; the name poise came because Poiseuille had many seminal contributions towards a better understanding of viscosity.

So, this is regarding the units of viscosity in SI systems. Now we will learn a concept which is closely related to viscosity and that is known as Kinematic viscosity. That is also a property of the fluid.

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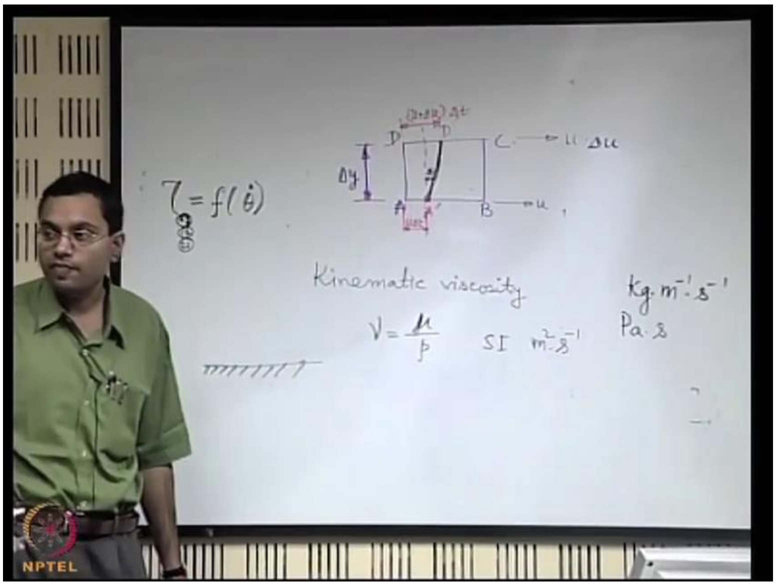

So, when we say Kinematic viscosity we define it with a symbol nu which is the viscosity divided by the density; nu by rho, very simple definition. We will see that what is the physical significance of this definition, but first let us look into the units and so on. You can see that if you write its SI unit; what will be its SI unit? So, this was the viscosity unit and rho is kg per meter cube, so it will come to SI unit of meter square per second.

It does not have any maths unit involved with it, and that is why the name Kinematic. Because when you have a maths unit involved it is as if like you are thinking of a forcing situation, where the kinetics also come into the picture. So, here it is solely like dictated by units determining the motion. So, that is why the name Kinematic viscosity. Or that is something more superficial but the concept is more subtle that, how we can utilize the concept of these to get a physical feel of what is happening within the fluid.

Again take that example where you have like flow or a plate or any solid boundary. Now what is the viscosity that tries to do? It tries to defuse the disturbance in momentum; the solid boundary creates a disturbance in momentum, viscosity is a fluid property that tries to defuse the disturbance in momentum to the outer fluid. So that is there in the numerator; in the denominator what is there? In the denominator there is a fluid property which tries to maintain its momentum, because it is density so it is directly related to the mass and mass is the measure of inertia.

So, what it tries to do, the denominator represents a physical property by virtue of which the fluid tries to maintain its momentum. And the numerator is a property by which it tries to disturb its momentum or propagate the disturbance. So, it is an indicator of the relative tendency of the fluid to create a disturbance in momentum as compare to its ability to transmit a momentum; or rather to maintain its momentum not to transmit it to maintain its momentum through its inertia. So that is the physical significance of the Kinematic viscosity.

So, it is not just solely the viscosity that is important the density is also important, because when you are thinking of the characteristic of fluid in transmitting momentum you must also try to compare it with a situation where it is maintaining its own momentum and not responding to a change in momentum. That is why this is a very critical an important ratio.

The next question that we will ask again qualitatively that what is the origin of viscosity, that where from such physical property originates; is it just by magic or where from it occurs.

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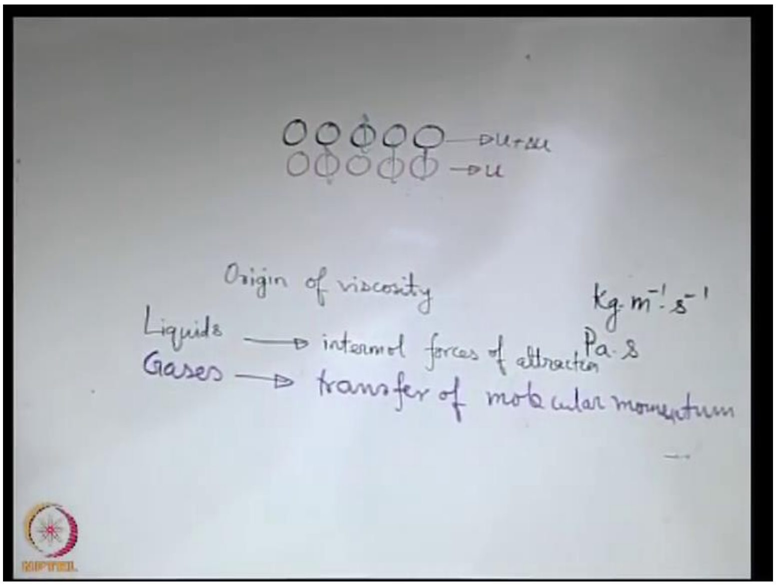

So, will see; what is the origin of viscosity in a very qualitative way. Let us first concentrate on liquids: why we separately concentrate on liquids and gases is a very straight forward reason, that the molecular nature of gases and liquids are different. And therefore, the viscosity or the viscous behavior is going to be different. And it is important to appreciate that at the end, because through a continuum description, like viscosity is a continuum description of a fluid property we are usually abstracted of the molecular nature, but we have to keep in mind that is the molecular nature of the fluid that has eventually given rise to these properties.

So there is a direct relationship, it is like an up scaled version of the molecular behavior so that you are abstracted from it, but you have to keep in mind at that it is the molecular behavior that has given rise to these properties. So, when we talk about liquids. So, when we talk about liquids, the main origin of viscosity in liquids is inter-molecular forces of attraction. So, there are different types of forces of attraction, like if you have like molecules, if you have unlike molecules you might have accordingly different types of forces like cohesion and adhesion and so on.

Fundamentally if you have 2 different types of molecules or 2 molecules of the same type there will always be some attractive and repulsive potential which are acting amongst themselves. The net effect is an intermolecular force of attraction that binds the molecular configuration together, otherwise molecules will just escape. And for liquids it is the intermolecular forces of attraction that gives rise to the viscosity mainly; so intermolecular forces of attraction.

However, if we want to give this logic for gases it sounds to be weak. Why it sounds be weak? Because we know that gases are much less densely populated systems, and therefore intermolecular forces of attraction are not that strong. So, what gives rise to a strong viscosity in gases many times? Let us look into an example. Let us say that you have a system of gas molecules. So, one layer of gas molecules which is at the top of another layer which is like this, what is the difference between these 2 layers; the bottom layer is moving at a slower velocity and the top layer is moving at a faster velocity. And we have seen that how such velocity profiles occur in a system of fluid.

Now for gases, these molecules have strong random motion with respect to their mean positions. So, these are vibrating with respect to their mean positions. In these way what is happening? It is very likely that a molecule from the slower moving layer joins the group of a faster moving layer. And what we will try to do? It will try to reduce the velocity of the faster moving layer. On the other hand it is very likely that because of these random thermal motion this motion is because over thermal energy of the system. So, if the temperature is absolute 0 it will go to a stop, but in any other case this thermal motion will be there. So, from the upper layer there will be molecules which are joining the bottom layer and these will now try to enforce the bottom layer to move faster.

So, that is how there is an exchange or transfer of molecular momentum. And this gives rise to the viscous properties of gases. So, for gases it is mostly because of transfer of molecular momentum. If we say now that there are certain factors which are affecting the viscosity for gases and liquids then we should be able to explain how those factors influence the viscosity through this basic understanding. Let us concentrate on one of the factors as examples; let us say temperature. So, viscosity of fluids is generally a strong function of temperature.

Now let us ask ourselves a question; if we increase the temperature of a gas and a liquid intuitively would you expect the viscosity to increase or decrease increase?

Student: Decrease.

Let us take one by one, say for liquids?

Student: Decrease.

It is expected to decrease, why, if you increase the temperature the intermolecular forces of attraction would be overcome by the thermal agitation and therefore the basic origin of viscosity will be disturbed. So, it is intuitively expected that for liquids the viscosity should decrease with increase in temperature. However, for gases what will happen? If you increase the temperature there will be more vigorous exchange of molecular momentum. And therefore, for gases it is expected that the viscosity will increase with an increase in temperature. Of course there may be exceptions, but we should not discuss exceptions these are more rules than exceptions.

So, what is the basic understanding, see whenever we learn a particular concept it is important to learn the basic science that leads to the concept. So, if we just learn it like a magic rule that viscosity of a liquid decreases with increase in temperature, it serves no purpose only in your examination you may answer it well but the next day you forget. And many times in the examination also you may confuse between these 2. But if you recall what is the correct physical reasoning that goes behind that it is very easy to appreciate that why this should be more intuitive than not.

Now just as an example for gases let us try to see that whether we may quantify the viscosity may be for ideal gases through the concept of exchange of molecular momentum.

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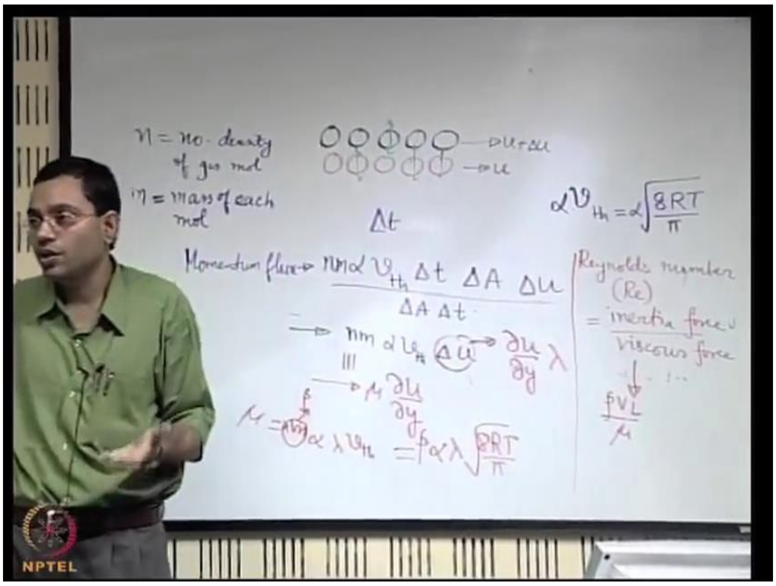

So, let us say that you have n as the number density of the gas molecules. Let us say n is the number density. Number density means: basically, the number of gas molecules per unit volume. So, with this number density and with this type of interaction let us see that; what is the exchange of molecular momentum? Let us say that m is the mass of each molecule. Now let us say that these molecules are having a random thermal motion and the random thermal motion is being characterized by a velocity which if you follow the kinetic theory of gases it may be expressed as square root of 8 R T by phi for ideal gases.

Now of course, this is a random thermal motion, so not that this full motion is utilized for just the transverse velocity component because the molecules have degrees of freedom thermal degrees of freedom in all direction. So, let us say that a fraction of that alpha is what is utilized for the exchange of transverse molecular momentum in the y direction. So, if we consider a time of delta t then with the time of delta t what is the distance that is swept in the vertical direction that is alpha into this characteristic velocity into delta t; that is the distance swept in the y direction. And what is the number of molecules which is taking part in that interaction? So, this is the distance in the y direction, if you multiply that with the cross sectional area say delta A of the phase that we are considering across which these molecules are moving along y. So, this represents the total volume.

And what is the mass within the total volume? So first n is the number density, so n into this is the number of molecules in the total volume that if you multiply with m that is the mass of each molecule; then this is, what is the total mass in this volume? So, with this total mass there is a molecular momentum of the upper layer this into u plus delta u of the lower layer is this into u. So, the net exchange of molecular momentum is the difference between that 2 and that is this into delta u.

Now if you want to find out that; what is the rate of exchange of this molecular momentum; say per unit time per unit area that is known as flux. So, any quantity which you are estimating as a rate in terms of per unit time and per unit area normal to the direction of propagation of that, then that is known as a flux. So, we call it as a momentum flux. So, for the momentum flux dash this should be divided by delta A into delta t.

So, the momentum flux is nothing but nm alpha v thermal into delta u. So, this delta u we may express in terms of a gradient; let us think that we are having interactions between 2 layers of molecules. When there interaction is possible what is the characteristic link that should separate them, which should make them interact. That is the mean free path, because that is the distance over which characteristically one molecule should traverse before colliding. So, these may be expressed in terms of a gradient like this.

So, this is like the rate of change that multiplied by the distance over which it is traversing, that is like the characteristic delta u. So, this lambda is the molecular mean free path. Now we have seen that the molecular momentum flux that is nothing but the shear stress in a fluid. So, these if it is a Newtonian fluid these should also be expressible as equivalent to some mu into the velocity gradient along y by the Newton’s law of viscosity.

This is identically equal to shear stress. Shear stress is nothing but like in this case the molecular momentum flux. So, if you equate these 2 then what you get out of this, mu as n into m into alpha into lambda into a v thermal. N into m is what? It is basically the density in terms of mass of the fluid; so it is rho into alpha into lambda into this thermal velocity. So, it is something which we can just write in this way. So, this is like roughly an expression that talks about the viscosity behavior of ideal gases. And you can clearly see that as the temperature increases the viscosity increases. Of course, for real gases it is not so simple, for real gases you also have to consider other interactions, but this just gives a qualitative picture of the entire scenario.

Keeping this in mind what we will just do, we will briefly define a few non dimensional numbers with which we will try to correlate this behavior. The first non dimensional number that we will be defining is something which you have heard many times is the Reynolds number. So when the fluid is flowing, the fluid is being subjected to different forces. One of the forcing mechanisms is the inertia force. So, the fluid has inertia, because of that inertia it tries to maintain its motion.

On the top of that there is a resistance in terms of viscous force which tries to inhibit the motion. So, we may try to gate a qualitative picture of what is the ratio of this inertia force and viscous force which will give us an indication of the extent of; the effect of the viscous force in terms of influencing the fluid motion when it is subjected to an inertia force. So, the inertia force is like mass into acceleration. So, mass into acceleration if you write m into a this is the inertia force, and the acceleration in terms of velocity can be written as like v square by L; this is also an unit of acceleration just expressed in terms of velocity.

So, mass into acceleration this is the inertia force. And the viscous force you can express like using the Newton's law of viscosity. Let us try to express the viscous force; the viscous force is the shear stress times the area; shear stress is mu into the velocity gradients, so mu into v by L that is the shear stress by Newton's law of viscosity that multiplied by area, so we are just trying to write it dimensionally; that into L square is the viscous force. So, this is inertia force and this is viscous force.

So, if you find out the ratio of these 2 what you will get. Now when you write the mass into acceleration, you have to keep in mind that the mass is the density times the volume. So, the mass is rho into L cube in terms of dimension. So, if you find out the ratio of these 2 forces what you will get is rho into v into L by mu. So, these L is what it is a characteristic link scale of the system. We have earlier discussed that; what is the characteristic link scale of the system? So, in a system with a particular characteristic length scale the Reynolds number expressed in terms of that length scale, and we will see; what is this velocity? This is also a characteristic velocity; as system may have different velocities at different points; so this kind of characteristic velocity taken.

And with these characterizations it may be possible to have a quantification of the ratio of inertia and viscous force. Even when the inertia force is absent this tells us a way by which we can have the concept of Reynolds number utilizable that not the inertia force by viscous force, but if it was possible to have an acceleration by having a driving force what would have been that equivalent inertia force. The ratio of that equivalent inertia force by viscous force may be interpreted as a Reynolds number in cases when the fluid is not accelerating. If the entire energy of the fluid was utilized to accelerate it what would have been that equivalent inertia force that by viscous force would still then be interpreted as an equivalent Reynolds number.

So, we will try to stop here today. And in the next class we will try to utilize the concept of this non-dimensional number and relate it with the viscous behavior for gases.

Thank you.