Introduction to Fluid Mechanics
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Lecture - 07
Viscosity, Newtonian fluid

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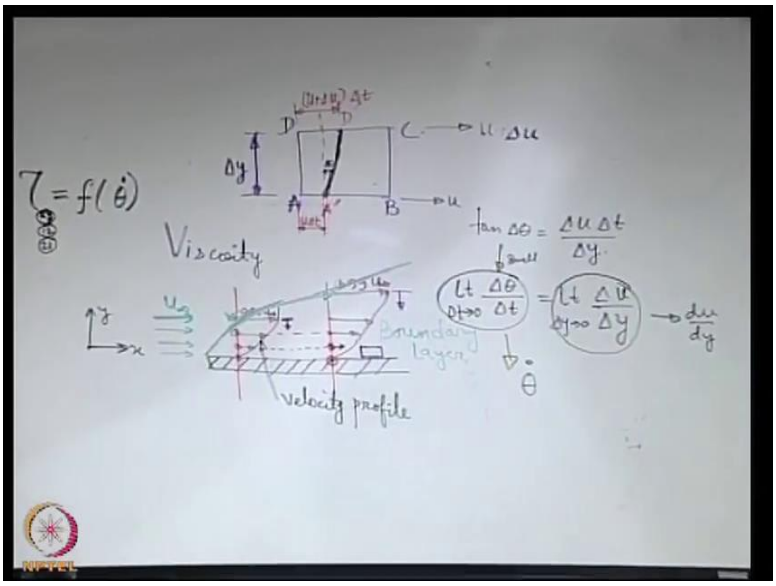

Which is a very important fluid property in the context of our discussions and we will try our best to understand it first qualitatively, and try to see that how we can mathematically express the fluid flow behavior in terms of the fluid property viscosity. We start with recalling the no slip boundary condition that we were discussing in the previous lecture. So, what was the consequence of the discussion that we concluded that in many occasions, the paradigm of no slip that is 0 relative velocity between the fluid and the solid at the points of contact of course that means, the tangential velocity component that particular situation gives rise to a boundary condition at the fluid solid interface known as no slip boundary condition.

We will try to see, what is the consequence of the no slip boundary condition? So, when we see the consequence, we have by this time understood that if the solid boundary stationary then a fluid which is coming from a far stream with a uniform velocity say u infinity and encountering the solid boundary what it will first do it will first have a disturbance, the disturbance is been imposed by the solid boundary. So, if we want to make a sketch of how the velocity varies with height at any section say at this identified section at the wall, if the no slip boundary condition is valid then since the plate is stationary, the fluid is also stationary. So, the fluid velocity is 0.

Next you consider a layer of fluid which is just above this one, this layer is subjected to 2 effects - one is the effect of what is there at the top of it, and what is there at the bottom of it. At the bottom, there is a plate and there is a stagnant layer of fluid molecules adjacent to the plate and this stagnant layer does not want the upper layer to move fast. On the other hand, the fluid which is above that layer is not feeling that effect of the wall directly, so that is trying to make the fluid move faster. Therefore, it is being subjected to a competition where the bottom layer is trying to make it move slower, the upper layers are trying to make it move faster, and it has to adjust to this.

So, where from this adjustments comes, if the bottom layer was not there then perhaps it would have not understood or felt the effect of the wall. Because what we are intuitively expecting is that there is some property of the fluid by virtue of which this message that there is a wall gets propagated from the bottom layer to the upper layers. And there must be some messenger for that and qualitatively that messenger is through the fluid property known as viscosity. So, viscosity is a kind of messenger for momentum disturbance. So, this is the disturbance in the momentum of the fluid. So, there must be some mechanism that place within that fluid by which a momentum disturbance is propagated, and because of this momentum disturbance what happens because of this momentum disturbance, there is a resistance in relative motion between various fluid layers. So, viscosity is also responsible for creating a resistance between relative motion against relative motion between different fluid layers.

So, let us see that how the relative motion takes place. So, first you have at the wall 0 velocity then as you go up you have a velocity higher than this one. It is not same as u infinity, but it is definitely greater than 0, because it does not feel the effect of the wall directly. If the fluid has no viscosity perhaps it would have never failed the effect of the wall, but now because the fluid has viscosity the effect of momentum disturbance is being propagated from the bottom layer to the top that is how this layer feels it not directly, but implicitly. And accordingly it slows itself down, but as you go too higher and higher positions, you see that the velocity is becoming greater and greater and eventually it will come to a stage when it reaches almost the u infinity, the free stream condition.

So, one of the important understanding is that if you draw the locus of all this velocity vectors, you can make a sketch which will represent how the velocity is varying over the section, which is taken along this red line. And this type of sketch we will encounter many times we in our course known as velocity profile. So, it is giving a profile of variation of how the velocity varies over a section. This velocity profile comes to a state where beyond which you really do not have any significant variation in the velocity that is it has almost reach reached u infinity.

What does it mean? It means that say beyond this if you go this has reached 99 percent of u infinity. So, beyond this if you go, it will be only little change or for all practical purposes no change that means, beyond this the fluid does not directly feel the effect of the wall. It does not feel the effect of the wall at all that does not mean that the fluid does not have viscosity it has viscosity, but the momentum disturbance could propagate only up to this much. So, we can see that we may demarked the physical behavior one is below this threshold location, and another above this one, below this the fluid adjust itself with the momentum disturbance above this it does not feel the momentum disturbance.

Let us consider a second cross section. So, let us say that we go to this cross section another one, a cross section like this. So, when we want to plot a velocity profile at a wall because of the no slip boundary condition, it is 0 fine. Now, let us say that we are interested about plotting the velocity at the same location at this one. So, now, you tell me whether it will be more or less then what was here, what should be the common sense intuition?

Student: less.

Professor: Less, why do you feel that it should be less?

Student: (Refer Time: 07:52)

Professor: So, it is like now more and more fluid is being in under direct effect of the plate, so there is the greater tendency that the fluid is being slowed down more and more.

When the fluid first entered only a few fluid elements were subjected to the effect of the plate, now that more and more fluid has been subjected to effect of the plate, the effect of slowing down is stronger. So, you expect that here the velocity will be maybe somewhat less than what it was here. In this way, in all sections it will be like this what it will imply is that it will take a greater height here to reach the almost the u infinity because of a greater slowing down effect. So, it may reach u infinity say at a height here, it is not exactly u infinity, but say 99 percent of u infinity. And we are happy with that because for all practical engineering purposes that is as good as u infinity for us. So, again we may have a velocity profile here whatever.

And then we can make a very interesting sketch. What type of sketch, see at every section we are having a demarcation between a position below which viscous effects are strongly important and beyond which these effects are not so important. So, accordingly we may draw a demarcating line between this 2. So, when you consider this particular section, you say that this is the location up to which viscous effects are strongly creating gradients in the velocity so to say so here up to this much and so on.

So, if you join this with an imaginary line this is not that such a line in the fluid, but it is just a conceptual demarcating boundary between a region close to the wall where viscous effects are very important. And this region is thinner as this velocity is higher. We will see that later on that if this velocity is very small, this region actually propagates almost towards infinity, but if this velocity is quite high, of course there are other parameters involved we will see what those are. But qualitatively if this velocity is very high this region is thin and this region is changing, it is not of a fixed dimension. So, this imaginary line demarcates between outer region where viscous effects are not important so to say and an inner region where the viscous effects are important. And the inner region where these viscous effects are important that region is known as a boundary layer.

So, we will be discussing about the details about the boundary layer through a separate chapter later on, but we are just trying to develop a qualitative feel of what is the boundary layer, because it has a strong consequence with the concept of viscosity. And the other thing is that this is the clever way of looking into the problem. If you want to analyze the problem where of course, the fluid is having some viscosity then outside the boundary layer, you may not have to care for the viscosity. So, it is almost like a fluid without viscosity as it is behaving because the momentum is not further getting transmitted to create a change in the velocity.

So, if you have a viscous flow analysis within this region that may be good enough coupled with a ideal flow analysis or fluid flow analysis without viscosity outside this region, so that is why conceptual this boundary layer is a very important concept. Not only that most of the interesting physics in the flow takes place within this layer and therefore, it is very important to characterize this particular behavior. So, we have loosely seen the no slip boundary condition and its consequence, before we more formally look in to the viscosity, let us look into one or 2 animated situations, where we try to understand the implication of the no slip boundary condition.

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So, let us look into that. So, this is a going to be a representation through a colored dye that; what is the visual representation of a no slip boundary condition? So, if you look in to it carefully, you see that if you focus your attention on the region which is there at the interface between the fluid and the solid, the entire dye was concentrated on that may be let me play that again, so that you can see it again. So, carefully see what happens on the surface, you see that there the fluid almost tries to adhere to the surface; and at the end that will be cleaned off at the end of the movie. Just to show that there it was the fluid was almost like a stagnant one because of the no slip boundary condition.

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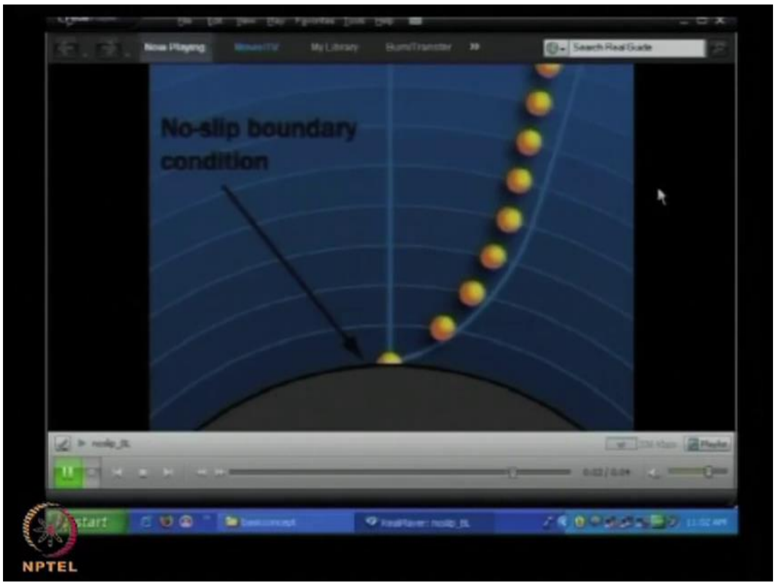

So, to have a different and may be a more artificial point of view let us look in to this. So, at the wall because of the no slip boundary condition you have the so called particles or molecules sticking, but as you go outside you see that as you go further and further you see that there is a velocity profile that is being developed there are velocity gradients which are being developed. So, this is a very important concept.

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And when we discuss about this concept in a greater details may be before that, let us see another one where you have 2 cases see qualitatively we are trying to understand what is the effect of viscosity. So, the upper panel represents the case with low viscosity and the lower panel case represents a case with high viscosity. So, if you see the case with low viscosity, you can find out a very important demarcation between the upper and the lower case. Visibly, what is the demarcation?

Student: (Refer Time: 14:54).

So, the boundary layer for this so called low viscosity case is very thin whereas, for high viscosity case it is thicker; that means, what the high viscosity case is trying to do it is trying to propagate the disturbance of momentum imposed by the plate to a greater distance. So, effect of viscosity is in terms of also to the extend, by which the effect in disturbance of momentum is propagated into a medium. Of course, we will go in to the mathematical quantification of this, but my first intention is that we first develop a qualitative feel of or the physical feel of what we are going to discuss about.

Now, whenever we are going to discuss on this concepts, obviously we will not always be having a molecular picture. And as you recall in a continuum hypothesis that if you consider molecules maybe just like an isolated particle, a fluid is a collection of such isolated particles. And whenever we are going to discuss about the behavior of the fluid in terms of its viscous nature here, we will be mostly bothering on the continuum nature.

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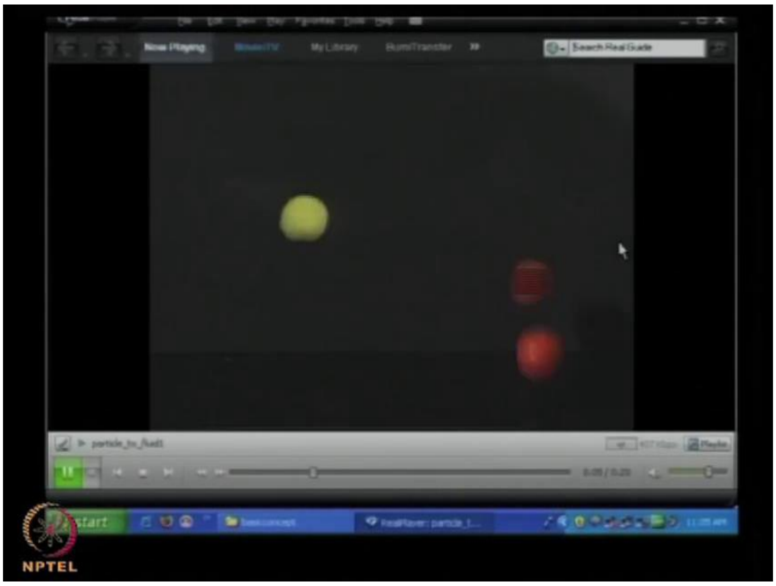

So, let us look in to a sequence of animated pictures to see that how you can have a transition from a particle nature to a flow nature. So, this is the behavior with one particle, there may be 2 particles isolated. So, these particles are like balls. So, these are idealizations do not think that this are like real fluid particles, these are just to develop certain concepts. So, you see that you can have more number of particles isolated particles.

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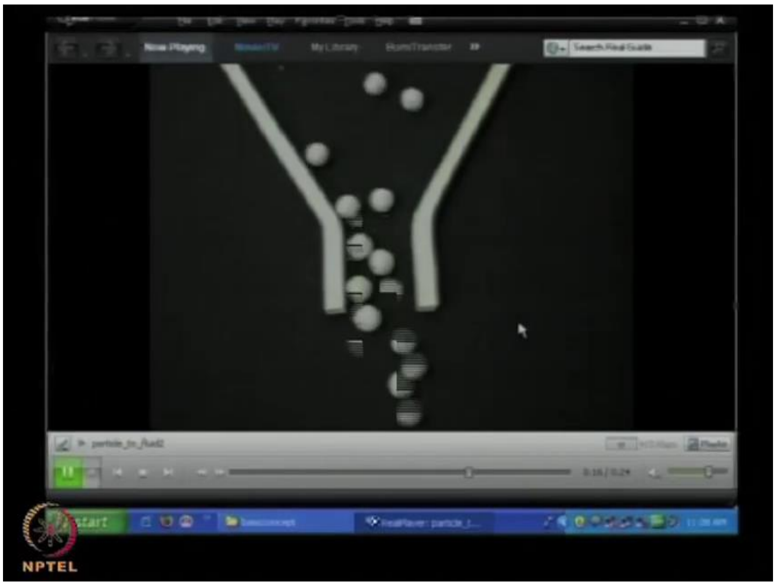

Then let us look in to the next sequence of images. So, now you are dropping those particles through this funnel, this group behavior is enforced by. Now, let us see the next in this sequence. So, if you see a third example, we look into a fourth example straight away, some problem with playing this. So, we will continue, but if you just look into the small part of the small version of the figure, you can appreciate that if you have more and more number of particles very densely populated together, you will see that it is almost like as if there is a continuous flow that is coming out of the funnel.

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So, depending on the compactness and the nature of the particles, you may start with the particle nature and at the end you may come up with the fluid flow nature, so that is the whole hallmark of a continuum that in a continuum.

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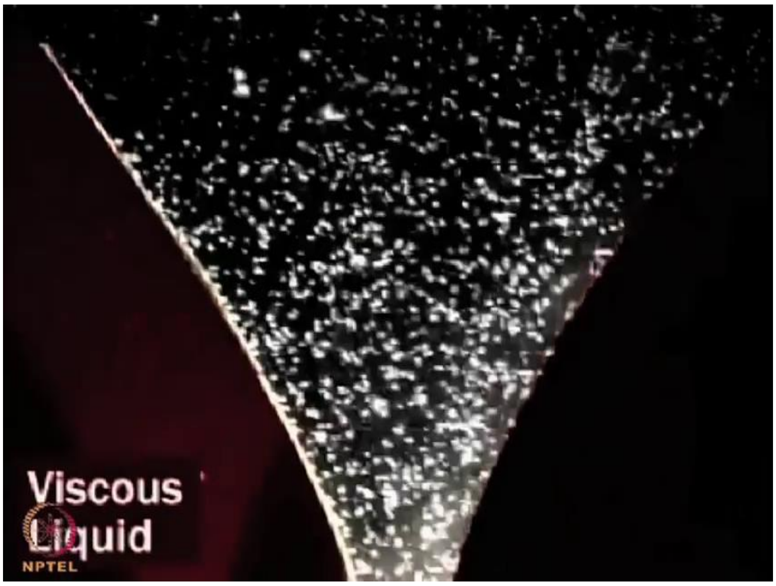

We are basically looking for a continuous distribution of matter, and we will first try to analyze the viscosity behavior through that continuum understanding. So, let us say that we take a fluid element. So, we take a fluid element at a particular location and that particular location maybe say close to the wall within the boundary layer. So, let us say that we are taking this fluid element, which was originally rectangular. Now, let us say that what happens to the nature of the fluid element or its geometrical characteristics, if it is subjected to viscous effect.

So, what will happen? To understand it, let us zoom its picture. Originally, let us say that name of this fluid element is A, B, C, D. The layer A B has the tendency say to move with the velocity u. Let us assume that the velocities are all along the positive x direction. And therefore, since this moves it to velocity u, if we consider a small time interval of delta t, a will move to a position say a prime with velocity of u, so that the displacement is u into delta t. Now, the upper layer says it is moving with a velocity of u plus delta u.

Now, we can understand that why it should be different because we have qualitatively discussed that there is some effect which propagates the disturbance in momentum through the fluid. And therefore, these 2 layers are expected to be of different velocities. And let us say that the fluid element that we have taken is quite a thin one with a thickness of delta y. So, now, let us see that where does D go, D will definitely go to a location D prime which is what which is somewhat advance than what was the location a prime. So, these you can say is like u plus delta u into delta t.

So, if you now consider that what is the relative displacement of D in comparison to A? Why that is important, that is important because if you now try to sketch the new location of A D, which is like this. You will realize that it has not only got displaced linearly, but it has undergone an angular deformation which is the so called shear deformation. How is this shear deformation quantified, it is quantified by this angle say delta theta. If the time interval is small then obviously, this is expected to be small fluids under shear are continuously deforming.

So, here we understand that this kind of deformation is possible if the fluid is under shear. So, when the fluid is under shear and it is continuously deforming, you allow more and more time this angle will be more and more. So, we restrict to a small time interval, when this was very small and this can be quantified as tan of delta theta is equal to what this net displacement here is delta u into delta t divided by delta y. For small delta theta tan of delta theta is roughly like delta theta and then you divide that by delta t right side you have delta u by delta y, and you take the limit as delta y tends to 0 as well as delta t tends to 0. So, this is nothing but d u d y, if there are other components of velocity we are assuming that it is having only a component of velocity along x that is not the reality, but to introduce the concept, we have started with such a simple understanding.

So, if it is having only one component of velocity then this is the case; otherwise it could be represented by some partial derivatives. We will come across the more detail understanding of the deformation of the fluid elements, when we talking about the kinematics of fluid flow in a separate chapter. But just for introductory understanding, this in the right hand side is a sort of gradient of the velocity which comes from the velocity profile and that is representing what this is like theta dot. So, it is representing the state of angular strength of the fluid, so a rate of angular deformation so to say. So, when we say rate of deformation of a fluid, it might be linear deformation or angular deformation. If you do not detail it with a further qualification, we implicitly most of the times mean that we are talking about angular deformation. So, this is rate of angular deformation of the fluid.

Now, who is responsible for this rate of angular deformation, a shear stress. So, there is a shear stress and the shear stress is very much related to the disturbance of momentum that was imposed because of the presence of the plate. So, shear stress may also be interpreted as the momentum flux. We will see that how it may be interpreted with a different example, but important thing to understand is that this theta dot must be related to the shear stress. So, this is the kind of straining. So, this is rate of shear strength or rate of angular deformation. So, these are the terminologies which are commonly use to quantify this one or 2 exemplify this one. So, we call this rate of deformation or rate of angular deformation or rate of shear strength.

In fluids strength itself is not important because as we have seen if we allow time it will be straining more and more. So; obviously, if you want to quantify strength, it becomes a kind of a redundant exercise you allow more time under shear it will be straining more and more because fluid is continuously deforming under shear. What is most important for a fluid is like the rate at which it is shearing; and for that this quantification is very, very important. What is responsible for that again is the shear stress. So, there must be some relationship between the shear stress and the rate of deformation.

Why such a relationship is to be present, because one is like a cause and another is like an effect. And it is the behavior of the material of the fluid that will decide that how it will respond to a situation and have an effect of deformation and that type of behavior in general continuum mechanics is known as constitutive behavior or constitutive relationship that means, the fluid has the constitution. So, in a particular disturbance, in a particular situation, it responds to that and the manner in which it responds, it comes from its own constituting behavior it comes from the material property. So, therefore, some relationship, which should relate the rate of deformation with the shear stress.

And in a general functional form we can write that the shear stress should be a function of the rate of deformation of course, when we are writing a shear stress here. What should be the correct subscripts if we want to write it in terms of tau i j to represent it say if this is x-axis and this is y-axis may be one 2 or 2 one because tau 1 2 and tau 2 1are the same. Or even you can write tau x y or 1 2 or 2 1 whatever, but here will just omit the subscript because here we are looking for only one particular component of the stress tensor, other components are not relevant for this. So, we will just call it tau just to be simple enough in the notation.

Now, this function or relationship may be linear, non-linear whatever, try to draw an analogy with the mechanics of the solids that you have learnt. So, you have learnt that in most of the solids, which have elastic properties, you have stress related to strain, and that behavior may be linear non-linear whatever. But if you have a elastic material then within the proportional limit you have stress is proportional to strain and that proportionality is being connected with an equality through a material property known as modulus of elasticity. Of course, all materials are not linear elastic materials, but this particular law, which is the Hook's law, is very popular one because many of the engineering materials will obey that behavior within proportional limits; and many times in engineering, we are working within those limits.

Similarly, for fluids, very interestingly most of the engineering fluids that we encounter and typically the 2 common engineering fluids we always encounter are air and water, and these fluids will also obey that type of linear relationship between the shear stress and the rate of deformation. So, for those fluids which obey the linear relationship between the shear stress and the rate of deformation we call those as Newtonian fluids.

So, Newtonian fluids are those fluids for which the shear stress is linearly proportional to the rate of angular deformation or shear deformation or shear strength.

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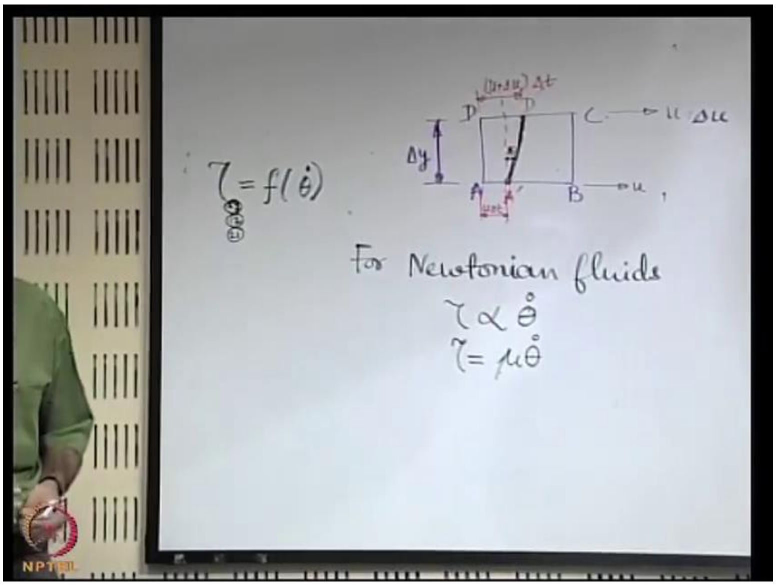

So, for Newtonian fluids you have tau is proportional to theta dot. This proportionality again should be breezed up with the equality through a material property here the material is the fluid, so that is expressed by an equality through a fluid property mu which is called the viscosity of the fluid. So, this is the formal definition of the viscosity of a fluid. Of course, if it is a Newtonian fluid then only this definition works. So, for a fluid which is not a Newtonian one, then this definition does not work, still one may cast the relationship in this type of pseudo form, but that is not real viscosity that is called as apparent viscosity. Because a real in a true sense the viscosity definition should be following the Newton's law of viscosity, but all fluids do not obey the Newton's law viscosity. The fluids, which do not obey the Newton's law viscosity, are known as non-Newtonians fluid.

It is a entire branch of science which deals with how the material should respond in terms of it shear deformation behavior and linear relationship is just only a small part of that. The entire science is known as theology where you are basically dealing with the constitutive behavior of say fluids against various forcing mechanisms, but here we will confine our scope mostly to Newtonian fluids, and we will briefly touch upon one or 2 examples of non-Newtonian fluid just to appreciate that there may be interesting deviations from the Newtonian behavior. To do that first, we will concentrate on this fluid property viscosity, and we will try to formally find out its unit dimensions and so on.