Introduction to Fluid Mechanics
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Lecture – 60
Principle of Similarity and Dynamical Analysis-Part-II

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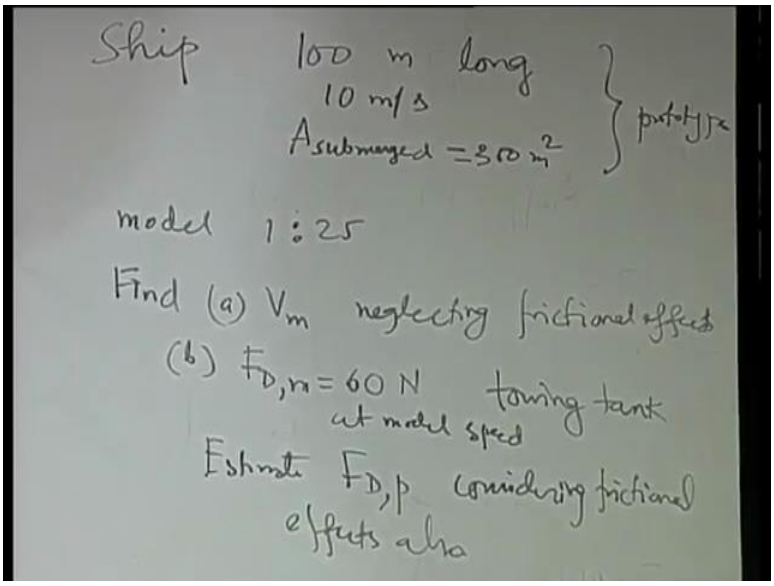

Estimate the total drag force on the prototype considering frictional affects also, there are some other data given for the problem, but we will come into that one by one. First let us broadly look into the at least the first part of the problem, second part there is some extra data that is given.

Now, before going into the problem, let us try to have an understanding of the problem. So, when there is a shape, now, what are the important resistances affects which are there? So, one of the important resistance is of course, the frictional resistance is there the viscous affect the other important resistance is called as wave making resistance. So, because of formation of the water waves and there it is a sort of a gravity dependent phenomenon. So, that wave making experiment resistance and there may be a third resistance which is because of formation of local ads and so on, but that is usually much much negligible as compared to the other 2.

So, here you have 2 types of important resistances, one is the resistance because of the wave on the wave making resistance and the other is the frictional resistance. So, when you consider, let us say that when you consider the wave making resistance. So, if you consider the wave making resistance then what are the important forces which will be important? Inertia force and gravity force. So, then the similarity will be determined by the Froude number ratio of the inertia force and gravity force.

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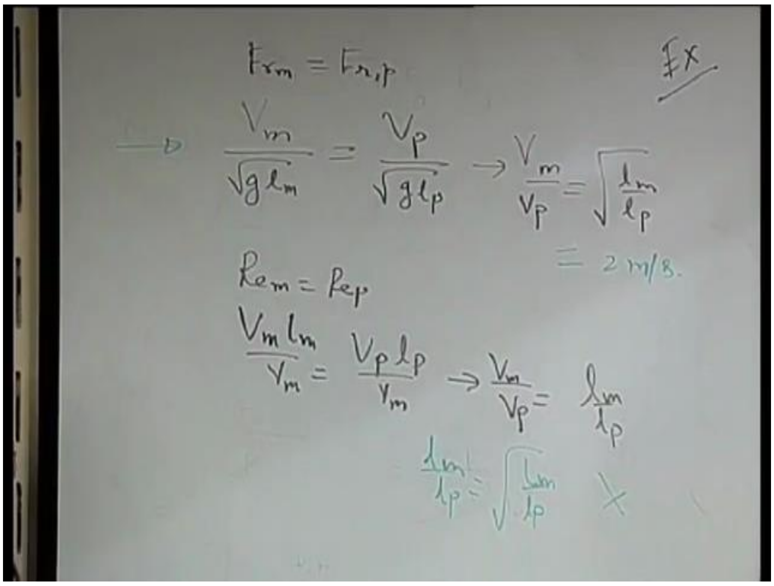

So, if you consider that that similarity then you have V model by square root of g L model is equal to V prototype by square root of g L prototype, this is for a that is considering the wave making resistance.

So, friction Froude number of the models same as Froude number of the prototype also if you consider the viscous resistances then Reynolds number of the model and Reynolds number of the prototype they should be identical. So, V model into l model by the kinematic viscosity of the model is equal to V prototype l prototype by kinematic viscosity of the prototype. So, from here what you get V model by V prototype is equal to square root of l model by l prototype, g is not changing then and from here what you get? V model by V prototype is equal to l model by l prototypes; still you are having the same water with same kinematic viscosity.

Now, can you do an experiment where you satisfy both? That means, if you want to satisfy this you must satisfy l m by l p is equal to square root of l m by l p.

Student: (Refer Time: 03:23).

But they are not one, 1 is to 1 model is no model, 1 is to 1 then model is a prototype. So, this you cannot satisfy, this is a very interesting situation that you come up with the important non dimensional numbers you see and you cannot satisfy. So, you have to come to a compromise that which one will you satisfy? Let us say that you satisfy these because I mean you may satisfy only one of these for your similarity. So, let us say you give it a priority and that is what is considered in the first part that is you find out a model velocity neglecting the frictional affects. So, if you neglect the frictional affects then this is the solely dominating factor for the similarity because then Reynolds number is not important if the frictional affects are not important.

So, if you consider that then the velocity of the model then it comes to be 2 meter per second now that second part of the problem and for that some extra information is given from experimental data and let us just note down those that extra information. So, what are the extra things?

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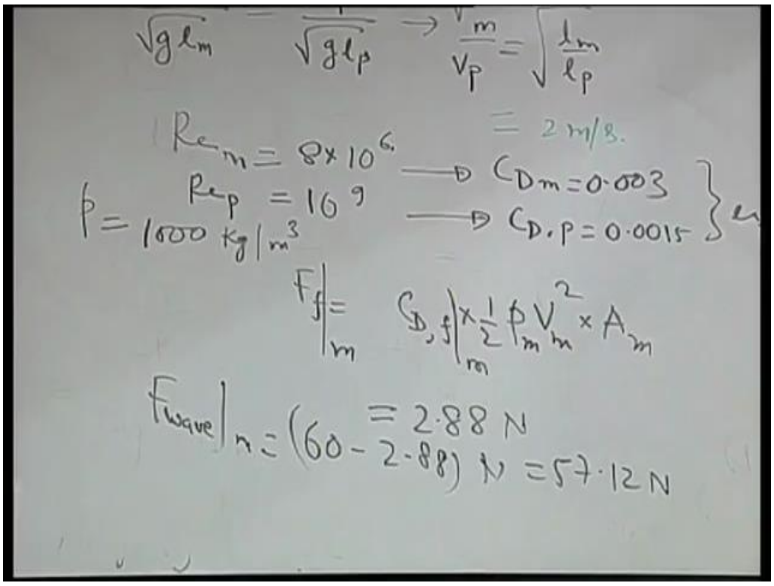

The Reynolds number of the model is 8 into 10 to the power 6 the Reynolds number for the prototype is 10 to the power 9 and you can clearly see that these 2 Reynolds numbers are different because you cannot simultaneously satisfy these 2 that is what we have seen and from the experimental data of C D versus Reynolds number from this, the C D is C D for the model is 0.003, these experimental data. So, what you are writing is whatever has been obtained from experiments.

Considering this calculations have been made with a consideration of the density as thousand kg per meter cube for water. So, now, the question is what is the frictional drag force on the model see when you consider this C D; this C D is a representative of what? This C D is a representative of the frictional drag.

Student: (Refer Time: 06:03).

It is not a representative of the wave drag there are 2 drags. So, out of the total drag force you can isolate the frictional drags or you isolate the frictional drag. So, frictional drag is the C D frictional times half rho.

Student: (Refer Time: 06:23).

V square times the area. So, if you substitute these values. So, let us see that; what is the frictional drag force on the model? So, C D for the model rho M V M square A m. So, if you substitute all the values all these values are given actually you will get 2.88 Newton. So, with this one, if you, you know the length scale, so from the area of the prototype you can calculate the area of the model by the square of the lengths, they will vary velocity is are already known density known C D, also this C D important is frictional C D that has been calculated that is what you have to keep in mind.

So, this is the frictional drag force. So, what is the wave making drag force on the model the total is 60 Newtons. So, 60 minus 2.88 that is the wave making drag force. So, that is 57.12 Newton, what is the wave making drag force on the prototype? If we have the wave making drag force on the model, how do you calculate it? Yes?

Student: (Refer Time: 07:54).

For the wave making drag force you must have C D of the wave making drag for the model and prototype to be same.

Student: (Refer Time: 08:04).

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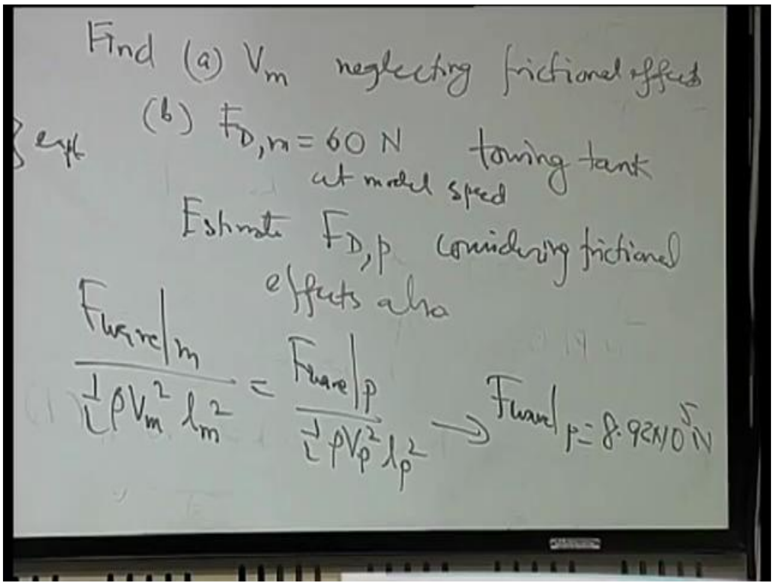

So; that means, the wave making drag for the model by half rho V model square into L model square is equal to the wave making drag for the prototype by half rho V prototype square into L prototype square. So, from here you can get that the wave making drag on the prototype if you calculate it. It comes out to be 8.92 into 10 to the power 5 Newton. I am just giving these numbers because you can calculate this at your leisure time and see. So, now, what is the frictional drag on the prototype what is the frictional drag on the prototype see your objective is to estimate the drag force considering frictional affects as well. So, this is the wave making resistance only, what is the frictional drag on the prototype?

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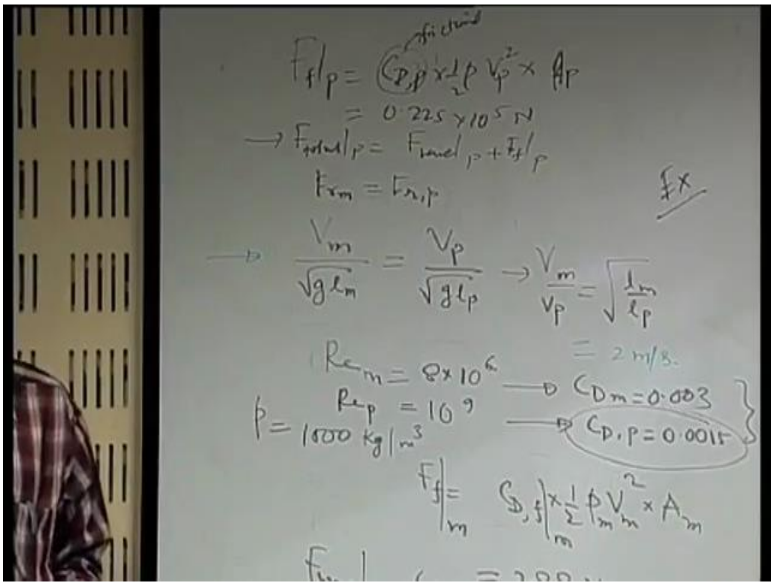

So, C D of the prototype into half rho V prototypes square into the area of the prototype right. So, this C D; this is the frictional C D; that means, which one? This one, so if you substitute that that will come out to be 0.225 into 10 to the power 5 Newton, some of these calculations may be veteroneous, but just I am outlining the procedures. So, just concentrate more on the procedure. So, then this is a frictional C D that we have to keep in mind and then the total drag force on the prototype is the sum of the wave making plus the frictional one.

Student: (Refer Time: 10:02).

So what you see here is that you get a velocity by neglecting frictional affects using that you use the similarity in terms of the drag coefficients where you consider the wave affects because these velocity was calculated by considering the wave affects on the top of that from the experimental data whatever you get you utilize that for calculating the frictional coefficients the frictional resistance and then add those together to get the total resistance that is the colloidal.

Now, we will come to the other important part of this discussion of the dimensional analysis that we have seen certain non dimensional numbers, but this non dimensional numbers how will we know that what are the important non dimensional numbers for a particular problem if you have a particular number variables and that is given by something known as Buckingham pi theorem.

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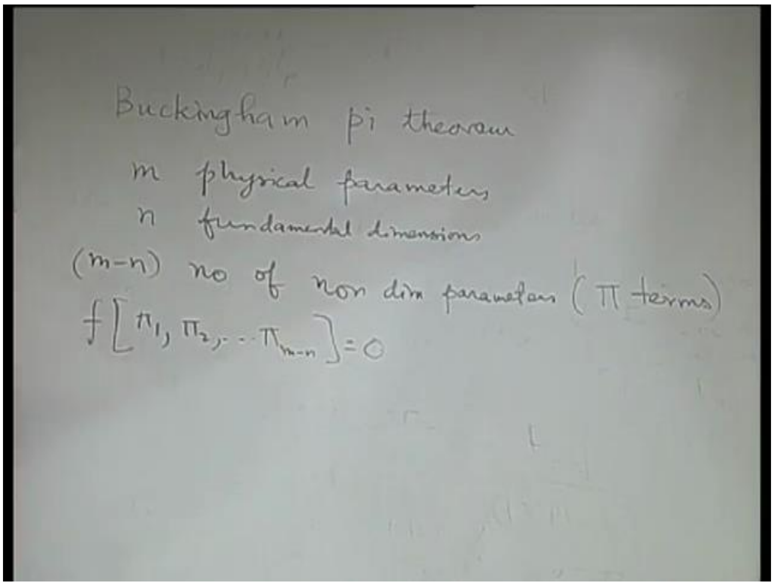

So, let us try to look into that. So, what we are looking for that if you have a many variables for a system, how you reduce those dimensional variables into some equivalent functional relationship between non dimensional numbers that this theorem tries to highlight how to do? So, it is a; that if there are M physical parameters and n fundamental dimensions then the functional relationship between all this may be written in terms of M minus n number of non dimensional parameters or these are sometimes known as pi terms that was the terminology used by Buckingham when he introduced it. So, this is as good as having a functional relationship between some non dimensional numbers pi 1 pi 2 up to pi M minus n that is equal to 0. So, you are reducing M number of physical parameters to M minus n number of dimensionless parameters how you do that the best way in which we will understand is through an example.

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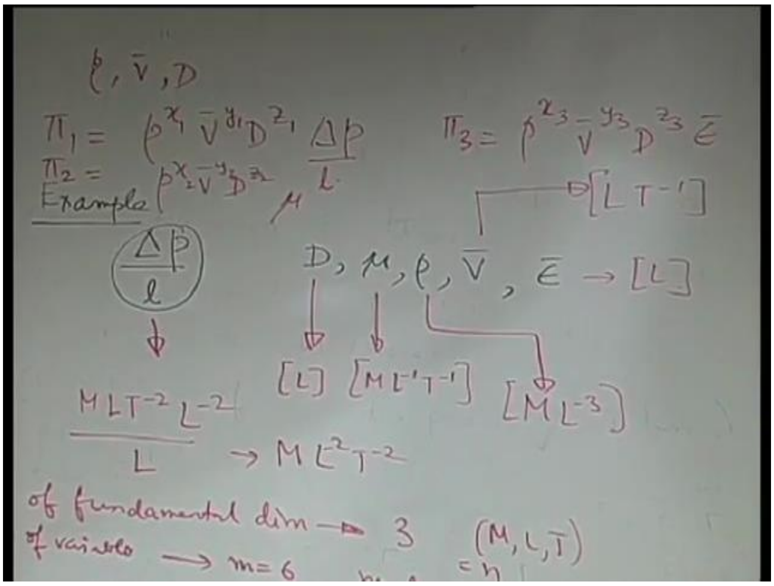

So, let us take an example through understand that how we reduce this example let us say that you want to estimate the.

Student: (Refer Time: 13:02)

Pressure drop in a pipe of a given length l, what is important for us is the pressure drop divided by the length because we know that D p D x is physically what is the important parameter it is a dependent parameter it is a dependent parameter it is a function of which variables? So if you have a pipe of say diameter D in which a fluid is flowing then what are the parameters from which this should depend?

Student: (Refer Time: 13:32).

The diameter of the pipe that is 1, then what fluid properties?

Student: (Refer Time: 13:39) viscosity.

Viscosity, density.

Student: (Refer Time: 13:41).

Then.

Student: (Refer Time: 13:42).

Average velocity.

Student: (Refer Time: 13:45).

Then.

Student: (Refer Time: 13:49).

The surface roughness; average surface roughness, so this, so first of all you must have a idea of the problem. So, you cannot do a mathematical exercise without having a physical idea of what are the important parameters. So, we have identified the physical parameters. So, this is the sort of a dependent variable and these are the independent variables. Now we want to see that what type of functional relationship should hold through for that. So, for that we will write the dimensions of these parameters. So, what is the dimension of this one?

Student: L.

L, what is the dimension of viscosity M L to the power minus 1, T to the power minus 1, viscosity sorry; density M L to the power minus 3, velocity L T to the power minus 1 epsilon.

Student: L.

L, what is the dimension of delta P by L?

Student: (Refer Time: 14:56).

So?

Student: (Refer Time: 15:01).

So, delta P is what? Delta P is.

Student: (Refer Time: 15:05).

Newton per meter square Newton; Newton is mass into acceleration. So, M L T to the power minus 2 that is the; so that divided by this one that is the pressure and then divided by L. So, this is M L to the power minus 2 T to the power minus 2. So, now, how let us see that how many numbers of fundamental dimensions are there? So, number of fundamental dimensions what 3 M L and T and how many so that is equal to n for the Buckingham pi theorem and how many number of variables are there 1, 2, 3, 4, 5 and 6.

So, number of pi terms on the dimensionless terms is 6 minus 3 that is 3. So, we will just show that how to find out this pi terms. So, first to find out the pi terms you have to select some variables known as repeating variables what are the repeating variables you must have certain variables. So, number of repeating variables is same as the number of fundamental dimensions; that means, here you have 3 number of repeating variables.

How to choose the repeating variables there are certain important things first of all out of the 3 repeating variables you choose none of those should be the dependent variable that is a first thing none of those should be dimensionless and none of those should be having same dimension and may be the most important thing is collectively, they should contain all the dimensions that is your object is to select 3 variables out of this in such a way that none of these are dimensionless none of these are of the same dimension and when they are considered collectively they will contain all the dimensions.

So, you have a choice say, let us consider rho V and D as an example. So, if you consider rho V and D. So, rho contains M L V contains L and T, it is good enough to consider rho V and D they together contain M L T all and none of these are of same dimension and none of these are dimensionless now in this way you could have many such possible combinations out of these once and that you are free to choose. So, if you choose this then what we do the first pi term is written as say rho to the power x into V to the power y into D to the z in to one of the variables say delta P by L similarly pi 2 you will have rho to the power x V to the power y say let us call this x 1 y 1 z 1 say x 2 y 2 z 2 into the other remaining variable other remaining variable is mu and pi 3 as rho to the power x 3 V to the power y 3 D to the power z 3 into epsilon.

Then how would you calculate pi 1 p 2 and pi 3? So, you have to keep in mind that this is dimensionless so; that means, let us take an example, let us calculate the pi 2 just to show you as an example. So, rho to the power x 2 is what n to the power x 2 into L to the power minus 3 x 2 then V to the power minus y 2. So, V to the power minus y 2 is L to the power minus y 2 T to the power minus y 2 then D to the.

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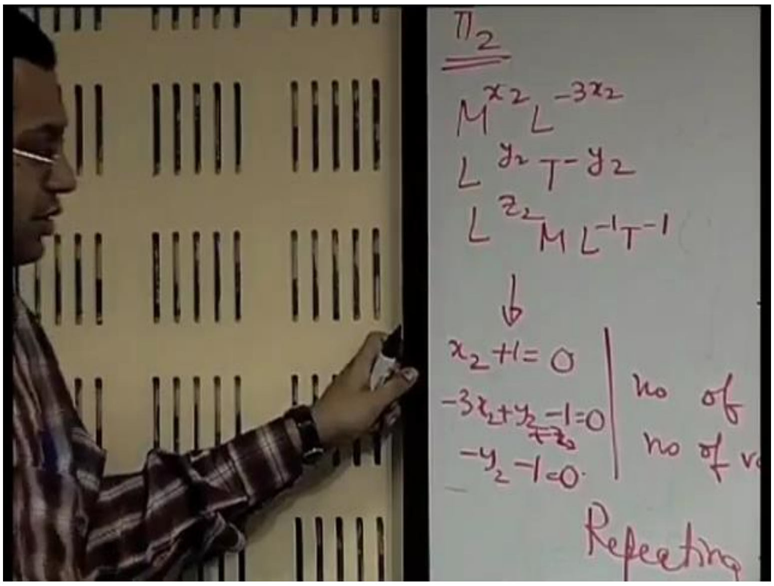

Student: (Refer Time: 19:58).

Oh! Sorry, this is not minus, I have confused it, this was a over bar, sorry.

Student: (Refer Time: 20:03).

So, this was as just the bar g bar. So, M to the power x 2 into L to the power minus 3 x 2 then L to the power y 2 into T to the power minus y 2 that is for D to the power y 2 and then

Student: (Refer Time: 20:22).

D to the power.

Student: (Refer Time: 20:24).

Z 2, so that is L to the power z 2 into mu, what is mu? Mu is;

Student: (Refer Time: 20:32).

M L to the power minus 1, T to the power minus 1. So, this should be dimensionless; that means, from this what you get? X 2 plus 1 is equal to 0 for M for L minus 3 x 2 plus y 2 minus 1 is equal to 0 for T minus y 2 minus 1 equal to 0.

Student: (Refer Time: 21:03).

Which?

Student: (Refer Time: 21:06).

Oh, plus z 2 oh sorry, sorry, in the second term you have z 2. This is plus z 2 second term because of that l. So, from these you can calculate h 2 equal to minus 1 y 2 equal to minus 1 and what is z 2? Z 2 is also minus 1. So, what you get as pi 1 sorry pi 2? Rho V bar mu by rho V bar D, you can easily recognize that it one by the Reynolds number see when you get a dimensionless number now what dimensionless number you will use in practice it depends on the convention. So, you may use 1 by pi 2 also and that is what we actually use. So, in this way you can calculate pi 1 pi 3 what will be pi 3? Can you tell like this is just by common sense you can say?

Student: (Refer Time: 22:24) epsilon.

Epsilon by D, other terms will go away. So, pi 3 will be epsilon by D.

Student: (Refer Time: 22:30).

And pi 1, so if you write say delta P by L, so, pi 1 can you calculate it?

Student: (Refer Time: 22:50).

It will be of course, there will be some V square by 2 g type of thing that will come there because delta P is there. So, you will have a V square by 2 g which has a unit of what? Unit of length.

Student: (Refer Time: 23:11).

And there is also a D.

Student: (Refer Time: 23:15).

U, it may be non-dimensionalized by D, so this.

Student: (Refer Time: 23:21).

This will be your important non dimensional parameter.

Student: (Refer Time: 23:24).

But then you have to see that you also have a delta P by L. So, how the delta P by L combines with that I am using it you only as an exercise, we do not have much time left for this lecture, but the exercise that I will leave on you is.

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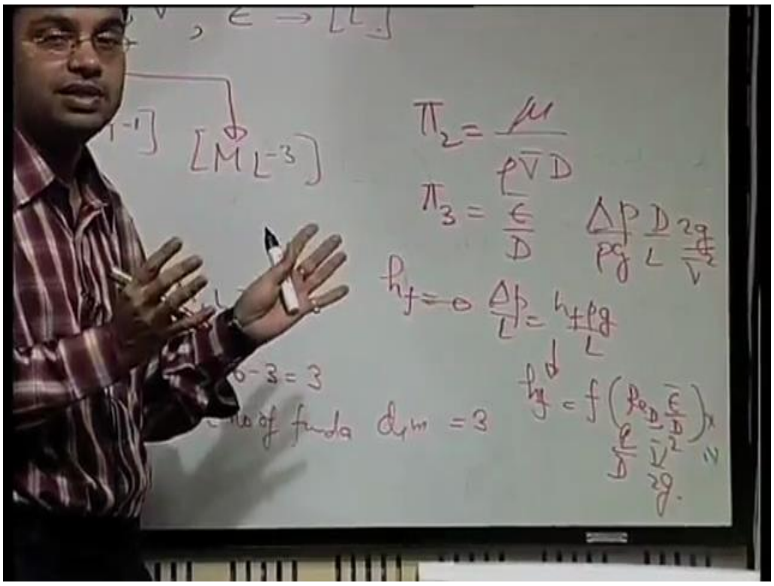

And then you have to show that this boils down to that h f which is equal to which is expressed as delta P by L is nothing, but h f rho g by L, from that h f will be a function of.

Student: (Refer Time: 24:00).

What?

Student: (Refer Time: 24:02).

Reynolds number and.

Student: Epsilon.

Epsilon by D or rather you may better way in which you write that delta P by L or even h f form is fine, this small f is a non dimensional number, this into L by D into V square by 2 g that form. So, it is a non dimensional function of Reynolds number and epsilon by D times this one. So, from here you can get actually what is pi 1 that is h f is delta P by rho g and delta P by rho g into D by L into 2 g by V square or its inverse that you will get here or basically you might get the inverse of this one.

Student: (Refer Time: 25:16).

So, this is the form which is the (Refer Time: 25:19) back equation see that you get this equation not in the exact value time in the form of an exact value, but you get the functional dependence and reduce as your number of experiments and makes you to come up with a dimensionless parameter and the choice of the dimensionless parameter is it depends on the physical situation. If pi 2 is a dimensionless dimensionless parameter and pi 3 may be pi 2 in to pi 3 is a dimensionless parameter, pi 2 to the power half is a dimensionless parameter. So, important is you have 3 independent dimensionless parameters you may make many other dimensionless parameters sorry dimensionless parameters using this one.

So, the dimensionless parameter that you make it depends on your physical situation let us stop this lecture here.

Thank you.