Introduction to Fluid Mechanics
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Lecture – 59
Principle of Similarity and Dynamical Analysis-Part-I

In this lecture, we will discuss on the topic of principles of similarity and dimensional analysis. Let us first get of background on motivation of starting this topic. Let us say that we are trying to have an idea of the how to design an aircraft and we know that if you want to design an aircraft, you want to have a clear idea of the lift and drag forces as some of the fundamental entities to design it for the real system. At the same time, we know that it is somewhat prohibitive to have many many experiments on aircrafts of real sizes. So, if you have aircraft of real size and if you want to test it, it is not only very expensive, but also there are many other drawbacks associated with such experiments

So, one might be interested to have a reduced model that is a model of aircraft of maybe a reduced size and then tested in a wind tunnel. So, in a wind tunnel say keep the aircraft model and have a control flow of air relative velocity between the aircraft and the air and then from that if you measure the pressure distribution, you may also measure other parameter say velocity distribution and so on and you will get a clear picture of, what is the flow around the aircraft? The question is can you extrapolate these to the behaviour of the real aircraft that is a very big question? So, the first question that you would like to answer is that given a study on the basis of a model of a different size how or whether you can extrapolate the results of those experiments to predict what is going to happen in reality in the real situation.

So, in the real situation whatever is the entity that is being used that is considered to be a prototype and the model is a version of a prototype, a scaled version of the prototype, in this example the model is smaller than the prototype it is obvious because if you have a real aircraft that is quite large you want to reduce its size. It is not always true that models have to be smaller than the prototype, sometimes the prototype itself may be in conveniently small and you might want have a model a bit larger than that. Question will be that whether that scaling will affect your results or not that is one of the important thing that you want to answer and if the scaling does not affect your prediction then the next question comes that how can utilize the results from the experiments with a model to predict what is going to be for the prototype that is a first question.

The second question is let us say that you are doing experiments with a model you may have many parameters which are influencing the results of your analysis or the results of your experiment. Now how could you reduce the number of parameters to a fewer, but more effective ones, may be more effective non dimensional parameters? Why non dimensional parameters are important because sometimes you may parameterize the result as a sole function of certain non dimensional parameters. So, as an example, if you have say flow through a pipe, you can have many experiments with different links, different diameters or may be different densities of fluid different viscosities of fluid.

But if you parameter is the result in terms of Reynolds number then if you keep the combinations of these such that the Reynolds number is unaltered the physical behaviour is unaltered; that means, we in some in such a case may reduce the parameters from say 4 parameters to an equivalent single non dimensional parameter. So, the big exercise or the big understanding is; how can we make the parameterization of the experiments in terms of the reduced number of parameters using certain non dimensional parameters? So, these are the important question that we would like to answer through the study of principles of similarity and dimensional analysis.

Now, when we say similarity, what kind of similarity we look for? The most intuitive form of term of similarity that appeals to us is a geometric similarity. So, whenever we first studied about similarity in high school, we only studied about geometric similarity. So, if you have a figure and another figure which is geometrically similar you have basically ratios of the equivalent size as identical or equivalent lengths as identical. So, basically then you have a similarity in the length scales that is what we essentially found for geometric similarity.

So, geometry similarity is something which is intuitive that is if you want to study the flow fast and aircraft you may get small, but geometrically you would always like to make it similar in terms of the actual big aircraft.

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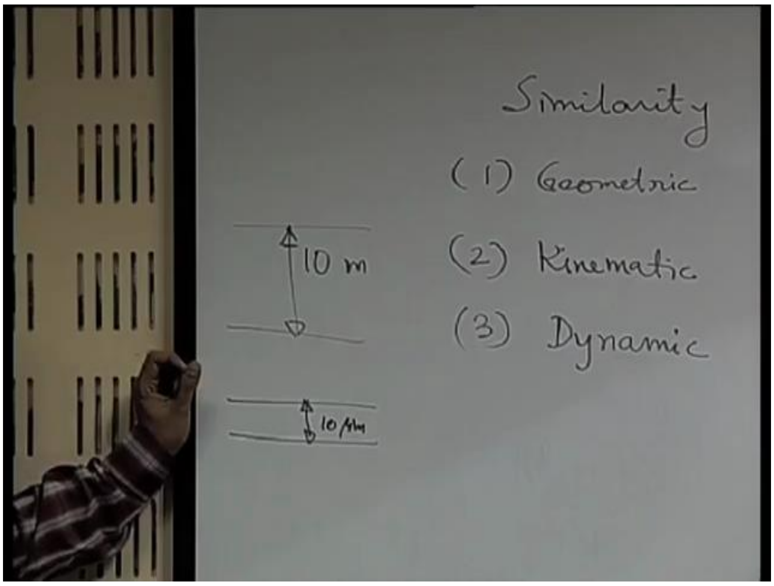

So, when we talk about similarity and the nature of similarity the first similarity that would come to our mind is geometric similarity and geometric similarity is the similarity in the geometry as obvious as that there is nothing more involved. The next type of similarity that we look for is known as kinematics similarity. So, kinematics similarity again from the name it is clear it is similarity of motion. So, may be what it is means is that if you have say 2 points; P 1 and P 2 and say the velocity here say u 1 and the velocity here is u 2, in a model in a in a prototype model like this where the equivalent point for P 1 is P 1 prime equivalent point for P 2 is P 2 prime.

Then and say the velocities are u 1 prime and u 2 prime then u 1 by u 2 will be identical to u 1 prime by u 2 prime. So, you will be basically having a sort of similar scale as the velocity now if you do not want to consider the velocity as such, but just have a more qualitative picture more qualitative picture, but a more physical picture may be obtained from the concept of the stream lines. So, if you have kinematics similarity; similarity in motion means the streamline which are there should also be geometrically similar because similarity of streamlines is an indicator of the similarities in the kinematics because streamlines relate to a visualisation of the kinematics of the motion or the velocity vector.

Now, remember that when you have streamlines the contour of a body is also a streamline as we have discussed because at through the contour of the body you do not have any penetrating flow. So, it is also a streamline; that means, that if you want to have similarities in streamlines, you must also have similarities in the contours of the bodies; that means, for kinematics similarity, geometric similarity is a must so; that means, whenever we say that there is a kinematic similarity that is prevailing implicitly we must understand that they are also geometrically similar additional restriction not be imposed kinematic similarity automatically ensures that just because the body of the surface or the contour of the body is itself a stream line.

The third important concept regard to the similarity is a dynamic similarity. Dynamic similarity is the similarity in forces; that means, if there are 2 types of dominating forces which have certain ratio in the model, the same ratio should be preserved in the prototype.

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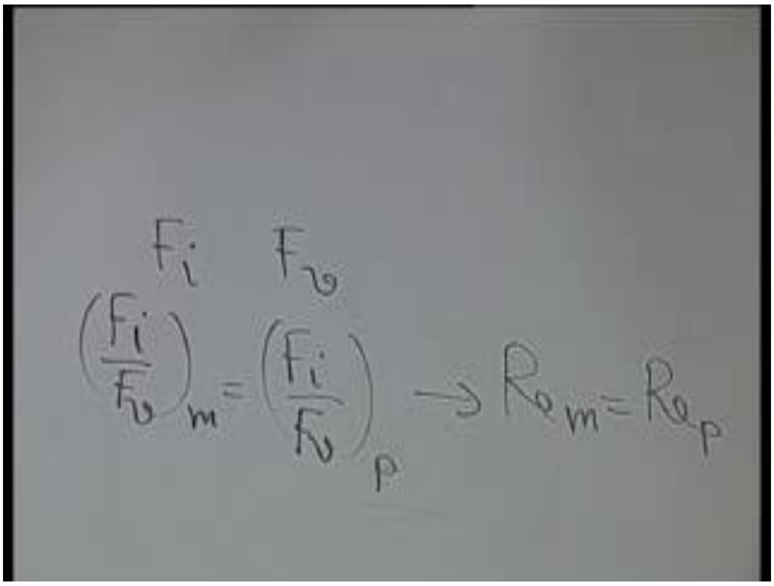

That means let us say that you have inertia force and viscous force. So, if you have these 2 as the important forces and competing forces then you must have the ratio of the inertial force and viscous force in a model same as the ratio of the inertia force by viscous force in the prototype or equivalently in this case; that means, Reynolds number in the model same as the Reynolds number in the prototype.

So, again it may be inferred that dynamic similarity should imply kinematics similarity because if you do not have kinematics then how can you have a dynamic similarity if you just think about inertia forces so; obviously, it follows the dynamic similarity should have kinematic similarity and kinematic similarity in terms should have geometric similarity therefore, it is as good as considering just the aspect of dynamic similarity it will aromatically ensure kinematic and geometric similarity. It is very easy to talk about these in theory, but when you go to experiments it may be difficult to achieve these types of similarities as we are theoretical intending, we will look into certain examples to illustrate that.

But before that we have to discuss about one important thing which we did not explicitly mention when we talked about these similarities. So, this discussion may give an indication that if you somehow have geometric kinematic and an obvious dynamic similarity; that means, then you have essentially all types of similarities between the model and prototype. So, whatever experiment you do in the model you can extrapolate that to the behaviour of the prototype that innocence true, but incomplete because the first and foremost requirement of a similarity is that the physics of the problem was identical for model and prototype.

So, let us say let us take an example, say you have a pipe of diameter 10 meter, now somebody says that I will have an experiment where I will have a geometry, similar thing with a diameter of 10 micron. So, it is just as if a geometric similar thing let us say by some way the Reynolds numbers are maintained to be the same. So, velocities are adjusted in such a way that the Reynolds numbers are the same. So, dynamitic dynamic similarities preserve and from the dynamic similarity it is possible to get a picture of the behaviour in these 2 cases. So, if one does not experiment with these may be one is intending to extrapolate it for this case.

It will be totally wrong because the physics of the problem has got changed it has got changed in many ways one of the most common way without looking into anything else is as you reduce the size surface tension effects become more and more important. So, capillary effects will have a strong role to play in terms of dictating the dynamical behaviour in this system where for such a large pipes the capillary effects will not be that important. So, physics of the problem has changed all together whatever, but the important physical aspects which were not important for which were not important for the largest diameter pipe for the much smaller size capillary it has become important.

So, no matter whether you maintain a Reynold number to be the same you will come up with the wrong conclusion because in the small scale the Reynolds number is not maybe the dictating factor because inertia force is not important. So, instead of going through the ratios of these certain forces or the non dimensional numbers, you have to first be ascertain whether this non dimensional number is physically relevant for the physics which is occurring over that scale or not. So, that is very very important. So, you should not change from a scale to another scale for predicting the relative between model and prototype in such a way that the physics of the problem changes all together and that is one of the very important tasks for an experimental designer that one should not design an experiment which changes the physics all together from what happens for the prototype.

Now, when you have and these forces the ratio of these forces let us just look into certain examples where we considered the ratios of different forces and those will give certain non dimensional numbers Reynolds number is one which we have already learnt and referred to many time.

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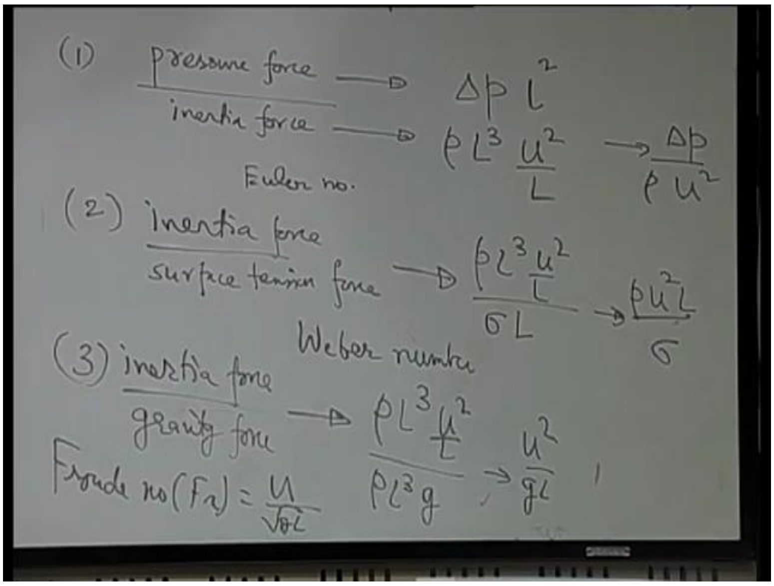

Let us look into some other ones let us say that we want to find out a non dimensional number. So, some examples of non dimensional numbers these non dimensional numbers are important because in terms of these you may reduce the number of parameters with respect to which you parameterized the result of your experiments.

So, let us say that we considered the ratio of the pressure force by inertia force as an example. So, pressure force we will try to see, what are the scales just in the same way as we did when we came up with an expression for the Reynolds number. So, pressure force will be some pressure difference times and area this times L square inertia force inertia forces is for the mass that is rho into a L cube into acceleration.

So, acceleration is like u D u D x. So, u square by L. So, the ratio of these 2 becomes delta p by rho u square, sometime this is known as Euler number. Let us say we want to find out the ratio of inertial force by surface tension force. So, inertia force, we have already seen surface tension force if sigma is the surface tension force coefficient sigma into L. So, this becomes rho u square L by sigma, this is known as Weber number. Let us say we want to find out inertia force by gravity force. So, inertia force by gravity force will be rho L cube u square by L divided by gravity force is m g. So, rho L cube g so that is u square by g L. So, this is the non dimensional number classically the square root of this one is considered to be one of the important non dimensional numbers called as Froude numbers that is just u by square root of g L then some more examples let us go through.

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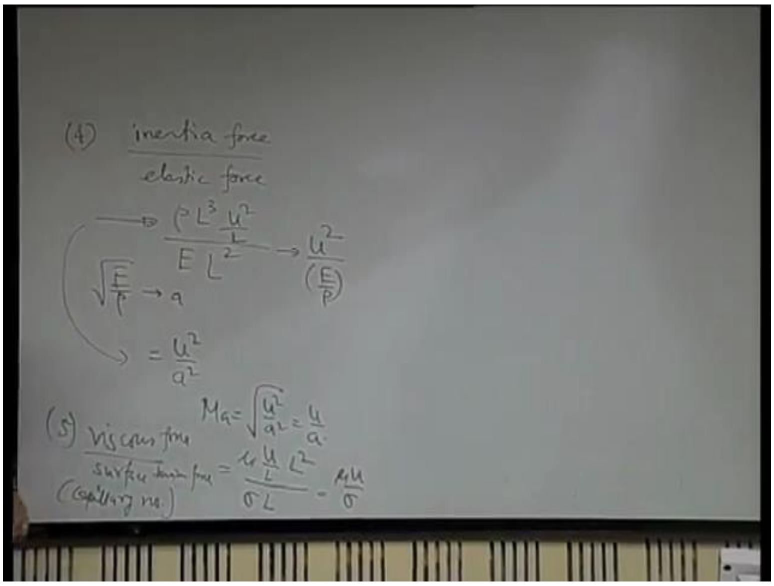

Let us say we consider inertia force by elastic force. So, rho L cube u square by L by elastic force if you have modulus of elasticity as E then E into area.

Student: (Refer Time: 18:40).

E into L square so that is equal to what? U square divided by E by rho. What is E by rho I mean what physical thing does it represent? It represents square root of E by rho represents what?

Student: (Refer Time: 19:06).

The sonic velocity through the medium so that is a; that means, this is nothing, but u square by a square where a is the sonic velocity or sonic speed through the medium and we know that sometimes the square root of this one which is the mach number which is a very commonly used mach number is the square root of this one that is u by a in another example let us say viscous force by surface tension force. So, viscous force viscous force is what if you just want to write it for a Newtonian fluid mu into the velocity gradient is the shear stress. So, mu u by L times L square shear stress in the area and surface tension forces sigma into L. So, that is mu by sigma sometimes known capillary number.

So, in this way many such non dimensional numbers are possible. In fact, hundreds of hundreds non dimensional numbers are there depending on the ratios of different forces, but we have just introduced some of the more common once which are which may be pertinent to an introductory level course now when we talk about similarity we have to understand one thing that whether these similarity is going to be maintained for all cases and; that means, that can you predict the real behaviour. So, 2 questions we want to answer can you predict the real behaviour without satisfying the similarity in certain cases the inverse that is if we do not satisfy the similarity then what would be the consequence or is it possible that in all cases we satisfy the true similarity and these type of interrelated questions would try to answers through some example or problems let us is look into that.

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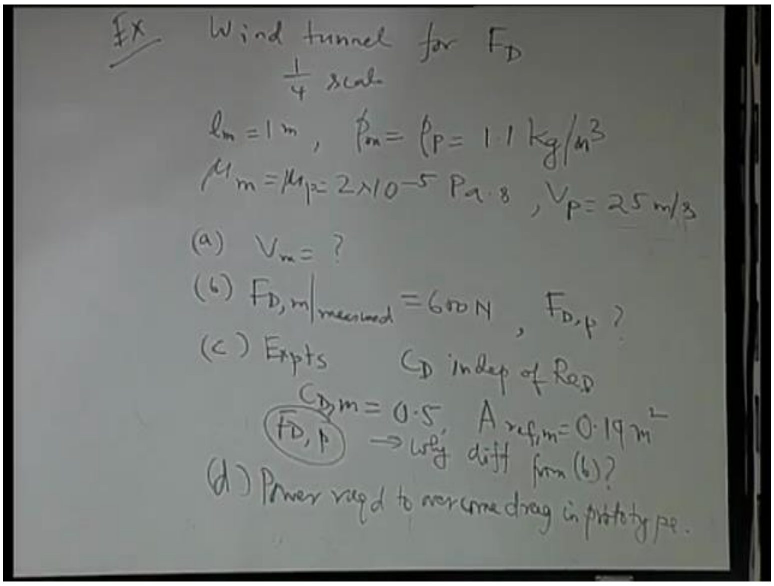

Let us say that we have one wind tunnel experiment wind tunnel for dark force determination the scale is 1 4th scale and it is a test on an automobile with the length of the model as one meter density of the model and the density of the prototype they are. So, this is when we say density of the model and prototype these are not density of the solids actually these are densities of the fluid flowing around. So, in loose sense we right the density of model and prototype does not mean the car density of the car it is basically the density of the fluid that when are talking about the air then similarly the viscosities of the air conditions 2 into 10 to the power minus 5 Pascal second and the velocity of the prototype is 25 meter per second.

The first part of the problem is calculate the velocity of the model second part is the drag force on the model is measured to be 600 Newton, calculate the drag force on the prototype then the third part that experiments indicate that the range in which we are operating the drag coefficient is independent of Reynold number and it is equal to C D for the model is 0.5 with a reference area as 0.19 meter square. Calculate the drag force on the prototype and then from that you find that why it is different from what is predicted in part b and the 4th part you find out what is the power required to overcome the drag force in a prototype. These are the parts of the question.

To look into it one by one, so the physical scenario just try to get a picture of this that you have you want to design a car and you want to have the car design for speed of 25 meter per second and then for that you are having model experiment where you are having the size of the model car 1, 4th of that of the prototype one and the wind conditions etc at the same the land scale of the model is 1 meter.

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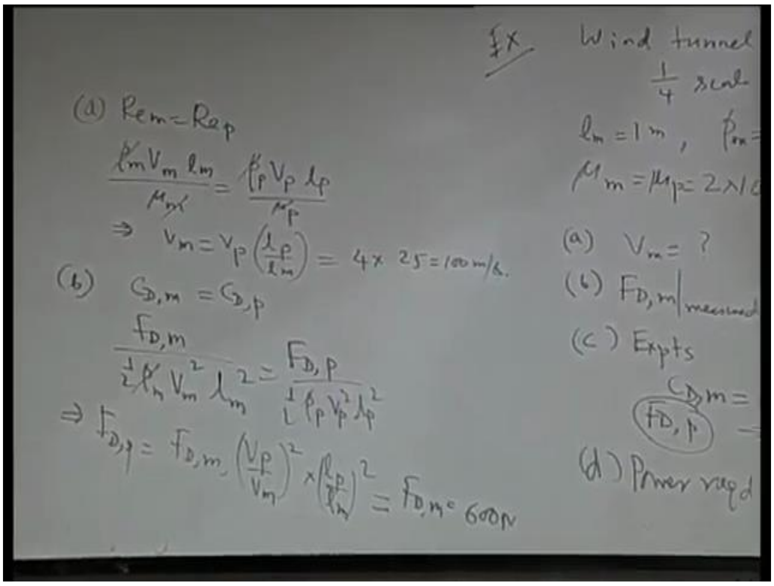

So, what is the velocity of the models? So, here inertia forces and viscous forces are important because this is a live Reynolds somebody strongly dictating it. So, for the dynamic similarity you must have Reynolds number of the model same as Reynolds number of the prototype so; that means, you have rho model mu model or we are calling V. So, we model l model by mu model is equal to rho prototype V prototype l prototype by mu prototype. So, the densities are the same, the viscosities are the same. So, v m is equal to v p into l p by l m. What is l p by l m? 4, so this is 4 into 25 that is 100 meter per second.

Next the drag force on the prototype, see what is the important coefficient that what is the important relationship that should dictate the equivalence of the drag force, it is a drag coefficient should be same as in the model and prototype. So, you must have C D of the model same as C D of the prototype. So, you have the C D is what the drag force divided by half rho v square into the area that is l square is equal to F D of the prototype by half rho p v p square into l p square. So, you have F D of the prototype, what is the drag force on the prototype that is a drag force on the model into v p by v m square into l p by l m square densities get cancelled out. So, what is v p by v m? That is one-fourth.

So, 1 by 16 and l p by l m is 4. So, it will be what? What would be the drag force on the prototype?

Student: (Refer Time: 28:01).

So, these 2 get cancelled out. So, this is the drag force on the model, which is 600 Newton then let us considers the third part experiments indicate that for the range of Reynolds number in which one is operating for this case is independent of Reynolds number that we have seen that we have discussed about the physical situation under which is like that. So, then C D of the model is 0.5 area of the area of reference for that corresponding C Ds 0.19 meter square. So, you can calculate that what is the corresponding drag force on the model and corresponding drag force on the prototypes if you calculate that let me just tell that what you get?

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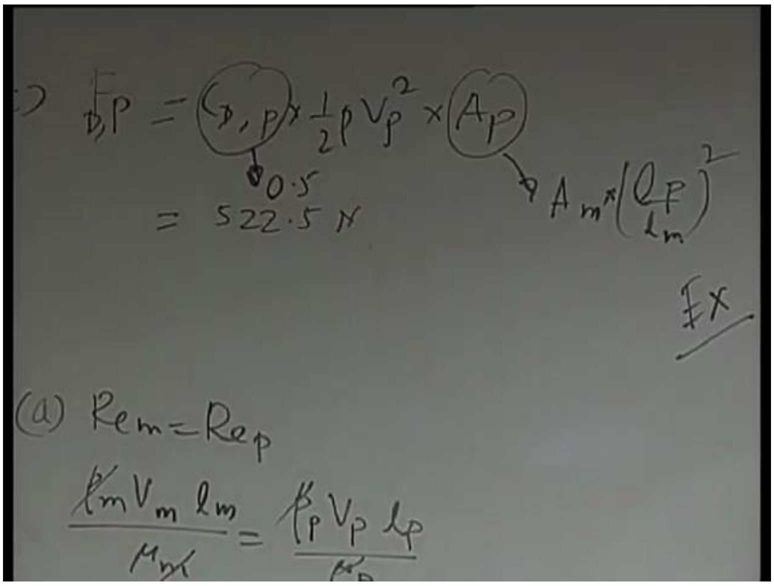

So, the drag force on the prototype that you get as of course, C D of the prototype into half rho V prototype square into area of the prototype and you have the test on the area of the model. So, you can write this as area of the model into what?

Student: (Refer Time: 29:25).

16 l area of the prototype by area of the model is l prototype square by l model square. So, it will be area of the model into l p by l n square. Velocity of the prototype you are

already given, the drag force of the prototype is same as drag force of the sorry; the drag coefficient of the prototype is same as drag coefficient of the model and that is obtain experimentally as 0.5. So, from these if you calculate the drag force on the prototype this comes out 520.5 Newton.

Now, interesting is not what is the exact calculation, but why these 2 predictions are different. So, the drag force prediction from part b is 600 Newton from this one, it is 522.5 Newton, this is experimentally obtained. So, this has more authenticity because the drag coefficient for model and prototype same that you have used this velocity is known and this is just from the model area with a scale ratio. Now why you feel that this may be different see the key is just try to use a common understanding that C D is independent of Reynolds number; that means, you may have C D of the model equal to C D of the prototype without satisfying Reynolds number of the model same as Reynolds number of prototype.

So, when you are assuring this and when you have in the range C D Independence of Reynolds number maybe you could have achieved it with the 34 Reynolds number, but still your prediction goes well because C D becomes independent of this is a case without satisfying the so called dynamical similarity you are able to come up with a prediction by exploiting the physical behaviour over that that origin that C D is independent of Reynolds number. So, these are critical tit bits of similarity not always like you blindly look into the similarity, but also looked in the context in which it is being applied then the 4th part of course, that is very very obvious power required to overcome what is the power required to overcome the drag force. It is the drag force times the velocity so that is 13062.5 watt that is the answer.