Introduction to Fluid Mechanics

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Lecture – 58
Pipe Flow-Part-IV

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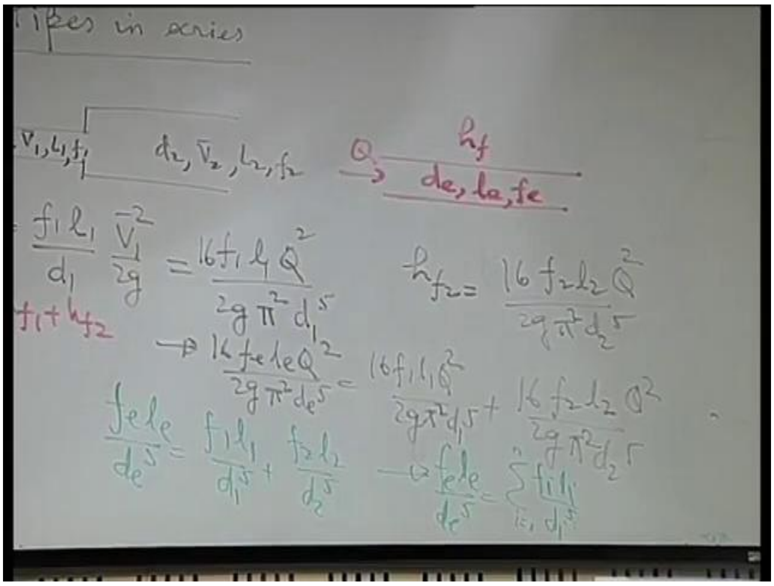

So, pipes in series it means that you have let us say that you have 2 pipes like this, the name series is obvious they are connected one after the other. So, you have let us say that the diameter of the first pipe d 1, the average velocity V 1, length l 1 friction factor f 1 and for the pipe 2 corresponding things are there.

What is the. So, when we consider this pipes in series and parallel in this analysis, the analysis that we are presenting as a theoretical a development we are not considering the minor losses we are considering the only the major losses. So, the head loss for the pipe 1 what is that? f 1 l 1 by d 1 into V 1 square by 2 g. What is V? V is 4 cube by pi d square. So, in terms of the flow rate; so f 1 l 1 V square will be 16 Q square, so16 f 1, l 1 Q square. Then 2 g pi square d 1 to the power 5, where Q is the flow rate which is going through each of these pipes. So, when there is series what is the common thing for them is the flow rate. The same flow rate is going through the 2 pipes. So, if you have h f 2 you have similar things 16, f 2, l 2 Q square by 2 g pi square d 2 to the power 5.

Now, what is the concept of an equivalent pipe? That is you replace these 2 pipes in series by a single pipe of some diameter let us say d e is the equivalent diameter, l e is the equivalent length, and f e is the equivalent friction factor such that you have the same flow rate and the same head loss. So, it is just like an electrical circuit where you are considering the same voltage and same current flowing through that. So, you find out an equivalent resistance sort of thing.

So, here it is like the head loss is like the pressure drop which is like a potential drop sort of thing, and the flow rate is like a current so to say. It is not exactly analogous mathematically, but is just another qualitative way of looking into it. So, when you have this h f as expressed as the head loss in this equivalent situation, then h f must be equal to the sum of h f 1 and h f 2.

So, if you write h f for the equivalent pipe it is a single pipe of length l e. So, from this you can write 16 f e, l e same Q is there by 2 g, pi square d e to the power 5 equal to 16 f 1, l 1 Q square by 2 g pi square d 1 to the power 5, plus 16 f 2, l 2 Q square by 2 g pi square d 2 to the power 5. So, from this what we can get, we can get a very important expression that f e l e by d e to the power 5, is equal to f 1, l 1 by d 1 to the power 5 plus f 2, l 2 by d 2 to the power 5.

So, in general you have if you have n number of such pipes in series, you have f e, l e by d e to the power 5 is equal to summation of f i, l i by d i to the power 5, i equal to 1 to n. So, as if it is like a equivalent resistance as the sum of the resistances, that is simple way of looking into it. Now let us look in to pipes in parallel.

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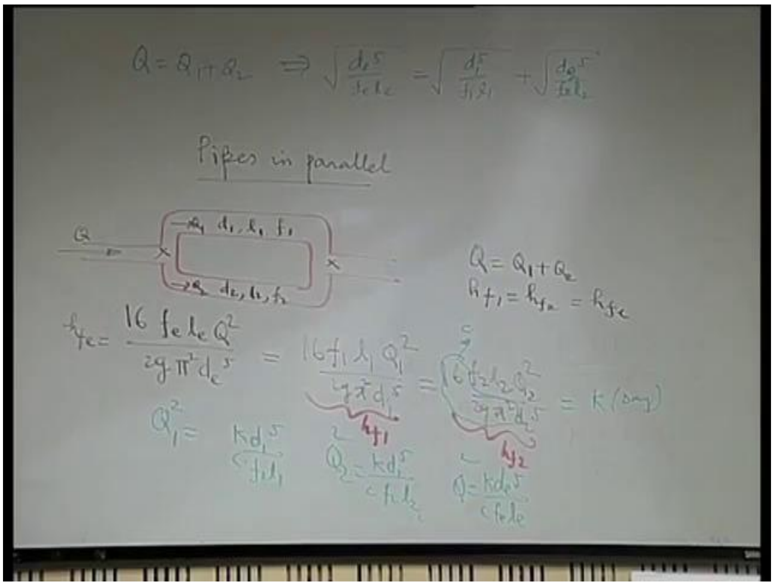

So, when you have pipes in parallel, let us try to make a sketch of a may be a situation like this. So, you have 2 pipes which has sort of connected in parallel; that means, they are branching of from just let me sketch it in a bit of a different way .

So, let us say that you have a pipes through which some fluid flow Q is coming there, now you have 2 pipes we say diameters d 1 length l 1. So, length 1 means not just a straight portion plus also the curved portion all those taken together, d 1, l 1 and the friction factor f 1 second pipe d 2 l 2 friction factor f 2. So, now, so these pipes both are connected across these 2 points which has shown as cross.

So, what you can say that let us say that Q 1 is the flow rate through this one, Q 2 is the flow rate through this one. So, you can say that Q is equal to Q 1 plus Q 2; if you consider the node which is given by the cross just like Kirchhoff's current law. So, the Q is distributed as Q 1 and Q 2 then what about the head loss.

Student: Head losses are the same.

Head losses are the same, because eventually you are talking about the difference in energy between these 2 points, no matter whether you traverse by the upper pipe or the lower pipe eventually we end up at the point, and the loss of energy are therefore, should be same as what you calculate from here or what you calculate from here. So, you have h f 1 is equal to h f 2. So, these are basic equations and from that you can find out the equivalent length of the pipes. So, you have and for the equivalent pipe you have say h f equal to h f 1 equal to h f 2 and Q equal to Q 1 plus Q 2. So, what is the h f of the equivalent pipe 16?

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F e, l e Q square by 2 g pi square d 2 the power 5 right. This is the h f of the equivalent pipe; this is equivalent to h f 1 that is 16, f 1, l 1, Q 1 square by 2 g pi square d 1 to the power 5, and this is also equal to h f 2. So, this is h f 1 this is h f 2. Let us say that each is equal to some constant k, and this 16 by 2 g pi square this is a term which is like a constant for all let us call it as C. So, you can write this is Q 2 sorry. So, you can write Q 1 is equal to Q 1 square is equal to k into d 1 to the power 5 by f 1, l 1.

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Right similarly Q 2 is equal to k into d 2 the power 5 by c f 2, l 2, and Q is k d to the power 5 by C f e, l e.

Student: (Refer Time: 10:04).

Hm.

Student: (Refer Time: 10:05) Q 2 square.

Q 2 square he is said since you have Q equal to Q 1 plus Q 2, you have from this expressions root over d to the power 5 by f e, l e is equal to root over d 1 to the power 5 by f 1, l 1 plus root over d 2 to the power 5 by f 2, l 2 the other terms gets cancelled out. So, these are expressions for the equivalent the relationship between the equivalent and original once in terms of the respective diameters on the friction factor.

So, with this background let us try to work out a few problems, where we have the pipes connected in may series or parallel.

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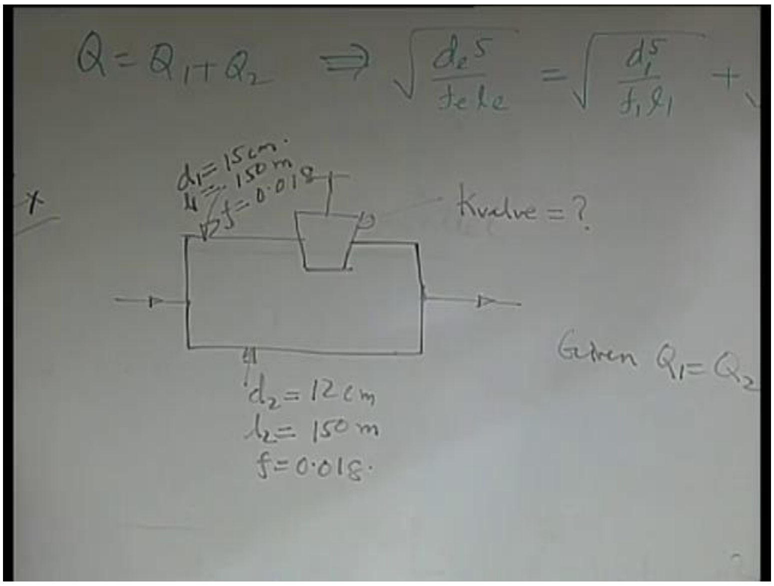

So, you have 2 pipes, 2 pipe lines and these 2 pipes the upper one is d 1 is equal to 15 centimeter, and length is 150 meter, the friction factor is a constant which is 0.018. The other pipe is the diameter d 2 is 12 centimeter, the length l 2 is 150 meter and the friction factor is the same 0.018, it is given that Q 1 is equal to Q 2.

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We have to find out what is the last co efficient of this valve. So, the approach is very straight forward, you see why I am illustrating this problem is the whole idea is never get tempted to use the formula which is ready made available with you. There is a formula which is ready made available with you and you might be tempted to use that what should prevent you from being tempted with that, is that here you have a minor loss that is not considered in this formula ok.

So, if you use that formula it will give you erroneous solution, but obviously, the concept of pipes in parallel you may use. So, what are the things you have h f 1 equal to.

Student: h f 2.

H f 2, not just the h f it is the total head loss, so not just the friction loss.

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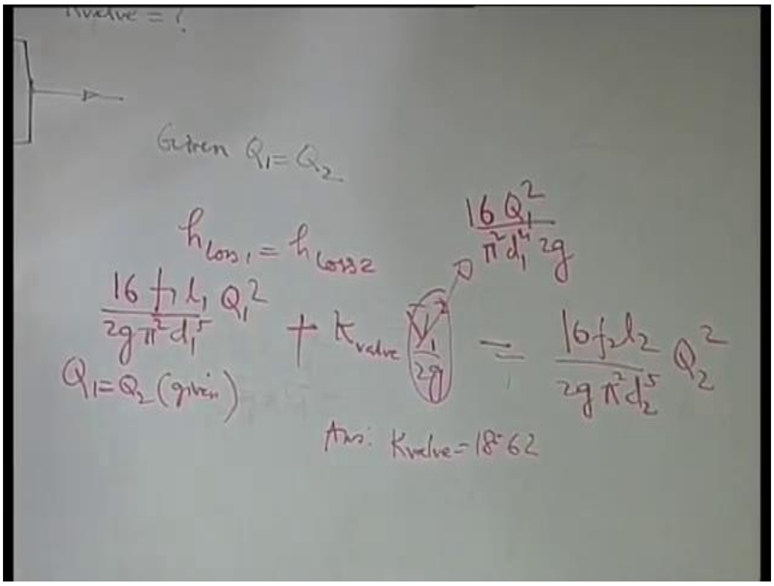

So, h loss 1 is equal to h loss 2. So, what is h loss 1 you have f 1, l 1 by d 1 or we write in terms of Q 16, f 1, l 1, Q 1 square by 2 g pi square d 1 to the power 5; plus the k valve into V square by 2 g. So, V 1 square by 2 g is as good as V 1 is 4 Q by pi d square. So, 16 Q 1 square by pi square d 1 to the power 4, 2 g that is V 1 square by 2 g is equal to the head loss at 2 that is 16 f 2 l 2 by 2 g pi square d 2 to the power 5 into Q 2 square right; and it is given the Q 1 equal to Q 2 given. So, you can cancel that from the 2 sides and get the value of the K valve straight away a very simple exercise the answer is.

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K valve is 18.62.

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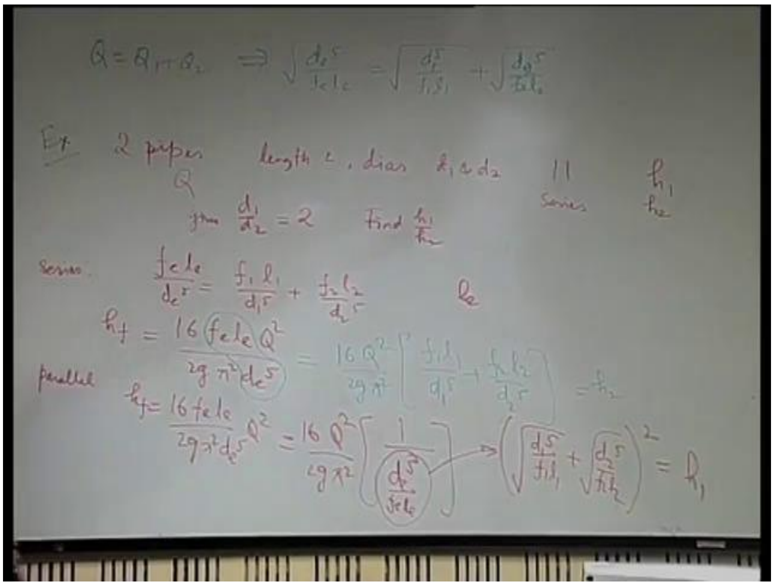

Next we work out another problem you have 2 pipes of length l and diameters d 1 and d 2, and they are arranged in parallel. So, when they are arranged in parallel, the loss of head for a particular flow rate Q, Q is the flow rate the loss of head is h 1, and the same pipes when they are arranged in series the loss of head is h 2, it is given as d 1 by d 2 is equal to 2 fine; h 1 by h 2 neglect the minor losses and assume a pump stand friction co efficient to be the same for all the pipes.

So, the 2 important assumptions that minor losses are neglected and number 2 friction co efficient or the friction factor is a constant, and that constant value is same for all the pipes; under which conditions friction factor you have a constant virtually?

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For very high Reynolds number highly turbulent flow, it will become only a function of epsilon by d. So, but here the diameters are changing, so we are assuming that epsilon is also different from the 2 pipes such that the epsilon by d remains the same so that the friction factor remains the same. So, when the 2 pipes are connected in series. So, you can work this out through the equivalent resistance concept. So, when they are in series you have what is the condition for the equivalent f e le by d to the power 5, is equal to f 1 l 1 by d 1 to the power 5 plus f 2 l 2 by d u to the power 5.

This is for the series and now the equivalent things equivalent thing has combinations of 3 parameters, and see it is not important what are the individual values of this parameters, it is important that you collectively choose them to satisfy this constant that should be good enough; that means, you may chose your equivalent friction factor or equivalent length in such a way that you will get some equivalent diameter or you may choose equivalent friction factor and equivalent diameter has to be something so as to get some equivalent length. So, you may take any of these out of 3 2 very freely and the third one you get from this expression.

Let us say that we assume that the 2 pipes are of the same length right. So, let us consider that l e or. In fact, if you see that it is f e, l e by d to the power 5 that is going to be solely important for the head loss. So, even if you do not assume any particular value that will not matter. So, if you consider the head loss what is that? 16 f e, l e Q square by 2 g pi square d to the power 5 right. So, in place of, you can clearly see that you get an expression where you have f e, l e by d to the power of 5.

So, let us say that you write in place of that 16 Q square by 2 g pi square, then you write f 1 l 1 by d 1 to the power 5 plus f 2, l 2, by d 2 to the power 5 this is given as h 2 this is series if they are in parallel.

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Again h f formula is the same, but expression for. So, this you have 16 Q square by 2 g pi square, then you have 1 by d to the power 5 by f e, l e right and that you can substitute in place of this 1 right that is d to the power 5 by f e, l e and this is given as h 2 sorry this is given as h 1, just you divide by this 2 and you will get a ratio when you divide you will get a ratio of d 1, d 2 by d 2 and l 1 and l 2 are the same. So, that ratio will give a number. So, this when you divide you will just get a number f 1 and f 2 are the same. So, those f x will cancel and it will be expressed solely in the as a function of d 1 by d 2. If you write h 1 by h 2, so the h 1 by h 2 the answer is 0.02188 that is the answer; let us work out maybe another problem.

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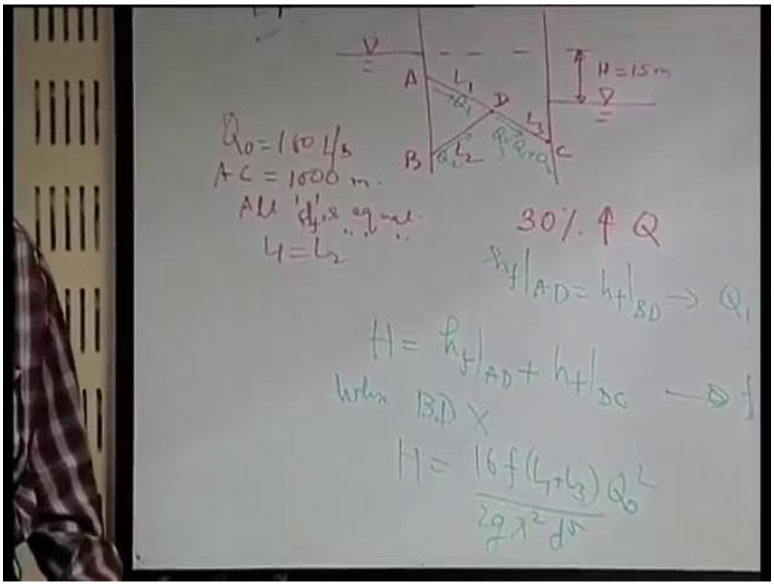

The problem statement is like this that initially. So, you can see that here there are pipes A, B, B D and D C. So, this is just shown by schematics, so not with being shown. So, here initially only the part AC was there, there was no branch B D and then the flow rate was 100 liter per second that is given. So, Q 0 is 100 liter per second and the length of AC is 1000 meter that is 1 kilo meter. To increase the flow rate another pipe BD is added. Estimate the length of the new pipe that is the problem all diameters are equal. So, all diameters are equal and assume the same length for all the pipes not for all the pipes that is L 1 equal to L 2 that is same length for the 2 parallel pipes, and same friction factor for all pipes. So, friction factors are also equal and it is given that there is a 30 percent enhancement in the flow rate because of this.

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So, you have to find out basically L 1 and L 2 that is a question. So, let us say that there is a flow rate Q 1 to L 1 and Q 2 through L 2; and the total Q is sum of Q 1 and Q 2. So, then you can write. So, the head losses if you neglect this elevation difference, the head losses should be what? The head loss for AD and head loss for BD they should be the same; they are like pipes in parallel. So, if their head losses are same head loss is function of Q f and l. So, you have f and L are same therefore, Q should be same. So, h f AD equal to h f BD that will give you Q 1 equal to Q 2 and therefore, you have Q 3 which is either equal to 2 Q 1 nor 2 Q 2 all the same.

Then what is the total head loss that is capital H. So, we will not write the modified equation in all details you have just seen that this capital H should be compensating the total head loss; so the head loss in AD plus the head loss in.

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D c, right. So, this will be a function of Q 3, because head loss in a d is a function of Q 1 Q 1 may be expressed as function of Q 3, and head loss in d c is a function of Q 3. And the head loss when this branch system was not there, still the head loss would be the same right. So, when b d is not there then the head loss is the head loss for the length AC with the original flow rate as Q 0.

So, 16 f L, L is L 1 plus L 3 into Q 0 square by 2 g pi square into d to the power 5; and it is given as that there is a 30 percent enhancement in Q; that means, Q 3 by Q 0 is 1.3. So, from that you can find out the missing length, you have to keep in mind that total L 1 plus L 3 is 1000 meter. So, just assume these as some x and this is 1000 minus x and this is also the next you can solve for that remaining things are be given.

Let us maybe look into another problem very briefly. So, let us say that you have 2 pipes or a pipeline it has a diameter say d 0 and the velocity V 0.

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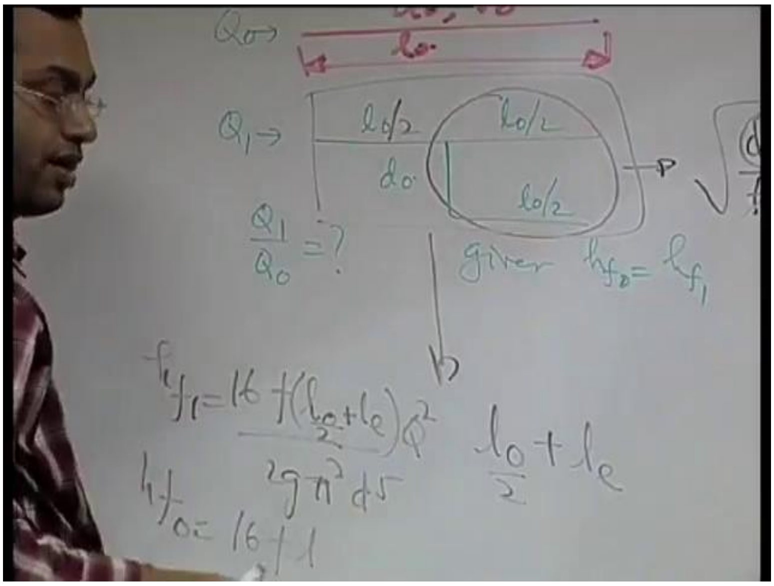

It is having some length say l 0; to increase the flow rate a new arrangement is made what is the new arrangement? The new arrangement is a branch is taken away from the midpoint of this one. So, this is l 0 this is l 0 by 2 and this is l 0 by 2. So, the diameter is the same the diameter is d 0 for the second arrangement as well, you have to find the change in flow rate say here flow rate is Q 0 here the flow rate is Q 1. So, you have to find out what is Q 1 by Q 0 given h f 1 is equal to h f 0 is equal to h f 1. So, this is a straight forward pipe series parallel problem. So, only thing is what you do you replace this by an equivalent pipe. So, if you replace this by an equivalent pipe these are 2 pipes in parallel. So, root over d to the power 5 by f e, l e is equal to root over d 1 to the power 5 by f 1, l 1 plus root over d 2 to the power 5 by f 2, l 2 here all f's are the same.

So, let us consider that the equivalent friction coefficient also the same, l 1 is what l 0 by 2, l 2 is l 0 by 2, d 1 and d 2 are the same which is equal to d same diameter pipes. So, this is d to the power 5, this is d to the power 5. So, let us say that the equivalent diameter is also d. So, you can find out an equivalent length in terms of as a function of l 0 right. So, then this entire pipe as if it is replaced by a pipe of length l 0 plus l equivalent; and you have h f 1 is equal to 16 f, l 0 plus l equivalent by into Q square by 2 g pi square d to the power 5 and h f 0 is 16 f sorry this is l 0 by 2, l 0 by 2 plus l equivalent sorry this is l 0 by 2 just correct it this is l 0 by 2 half, half. So, 16 f l 0 Q 0 square by 2 g pi square d to the power 5. From here since these 2 are equal you can find out what is Q 1 by Q 0, the answer is that the increment is 26.48 percent. So, this is just very simple equivalent pipe system analysis.

So, let us stop here today or for this lecture and we will continue with a next lecture with a new topic.

Thank you.