Introduction to Fluid Mechanics

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Lecture – 57

Pipe Flow-Part-III

Next what we will see is that we have till now discussed about the head loss, but how is the head loss related to the energy of the fluid; that may be interesting to us because in our in the very beginning of our course when we are talking about invisible flows, we were discussing about the Bernoulli's equation, and we found out later that the Bernoulli's equation sort of gives a mechanical energy balance for a system for flowing fluid. Now, therefore, here we have seen that even we might be tempted using the Bernoulli's equation, but because of certain losses that may not directly be applicable, there might be certain errors. So, these losses must have some relationship with the energy consideration in the pipe flow.

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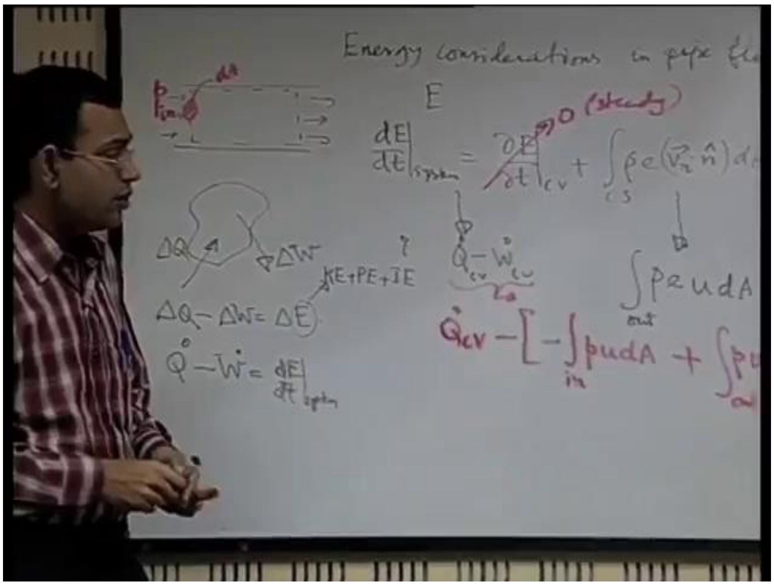

So, let us look into a bit more details of the energy considerations in the pipe flow. The objective will be to figure out that how the head losses are related to the total energy balance.

Let us say that you have a pipe of whatever section say circular pipe as an example, and we are looking for the fluid in the pipe and trying to write an expression for the energy balance. So, for any conservation equation we may start with the Reynolds transport theorem. So, let us start with the Reynolds transport theorem where E is the total energy of the system. So, we can write d E d t for the system where capital E is the total energy and small e is the energy per unit mass. So, now, let us make certain assumption for simplification that let us let us assume that it is a steady situation. So, the unsteady terms go away.

So, when you have the steady situation the next thing what you do, the next thing what you do is you try to write these expression for the d E d t of the system. So, that it is what it gives the total rate of energy change of energy of the system and so if you have a system like this some of whatever arbitrary configuration. The total energy change of a system of system is something of fixed mass and identity, and for that the total energy change is given by a particular form of the first law of thermo dynamics. So, you are having basically some interaction of heat and work. So, you have let us say there is a heat transferred to the system say delta Q, there is a work done by the system say delta W these are positive sign convention that we will follows. So, any heat transfers to the system we considered as positive, any work done by the system that is energy flowing out of the system because of what we considered as positive.

So, a let us say that some heat is transferred to the system and as an example some part of that is used to do work, the remaining will change the energy of the system. So, you have delta Q minus delta W is equal to the change in energy of the system. Obviously, I am not writing these terms in a very formal way whenever in the next semester when you will be starting thermo dynamics, you will be studying in details of how to formally write all these terms; but we are just trying to make use of this and I am just trying to be at you level so that we can proceed further.

Now, when you write this energy keep in mind that this energy is the sum total of kinetic energy, potential energy and anything else other than kinetic and potential energy which is the function of the internal configuration, which we call as internal energy. So, let us just symbolically write it kinetic energy, plus potential energy, plus internal energy. In books of thermo dynamics internal energy is given symbol of u as you have noticed may be earlier, but here because you already use u for velocity, we use just i as symbol for internal energy to avoid the confusion in the terminologies. Now this equation you can also write as a rate equation. So, you can write Q dot minus W dot is equal to d E d t of the system.

So, these we can write Q dot minus W dot. Q dot minus W dot of what? Q dot minus W dot for a system, but in the limit as delta t tends to zero when it is derived this is same as this for the control volume as well. So, this is as good as Q dot control volume minus W dot control volume; what is the control volume say you have chosen some control volume which is across which some fluid enters in the pipe, and it leaves the pipe and the boundary of the control volume is shown by this dotted line.

Now, let us concentrate on the right hand side; first of all this control volume is stationary, so that the relative velocity and the absolute velocity they are the same. So, this will be equal to integral of rho e. So, integral of E dot n d t A for the out flow it will be positive and the inflow it will be negative. So, we can say that integral of rho u d A for out flow boundary minus integral of rho e u d A for the inflow boundary right because we have now loss the vector sense. So, we have put the proper algebraic sign to represent the vector sense.

Now, next is to split different terms based on the like what are the important effects. So, heat transfer. So, heat transfer of course, there may be some heat transferred to the system or away from the system say you are heating the wall of the pipe is heated. So, there may be a heat transfer from the surrounding to the system, if it is not heated then also there may be a heat transfer because of the temperature difference between the ambient outside and the fluid that is there in the pipe. And how such temperature difference may be created we will try to see.

Now if you consider concentrate on the work done; so what is the work done here by the fluid in the control volume. First off all you have the fluid let us consider the inflow the fluid is entering with the pressure p. So, it is putting some energy to the control volume as it displaces the some fluid and enters it. So, what is the corresponding work done see we have related this with the flow energy or flow work, so that is work we are preferring here. So, if you have let us say a small element of area here say d A. So, the elemental work done is P in into d A into the displacement; the here we are writing the rate so the rate of that displacement that is the velocity.

Student: velocity

So, P in d A into u integral of that over the entire area sin plus or minus.

Student: plus plus plus.

You see this consistency of the first law of thermo dynamic; see any energy in the form of work if it is transferred from the inside of the system to outside it is positive, here the energy is been put into the system. So, that is negative work in terms of the work so; that means, you have Q dot minus. So, you have minus integral of P u d A for the in and for the outflow it will be plus. So, that is the left hand side expression that we are having. Now the next is we are assuming that rho is a constant for this problem or for this discussion. So, when rho is constant we can take rho out of the integration in place of E must we can write this is energy per unit mass.

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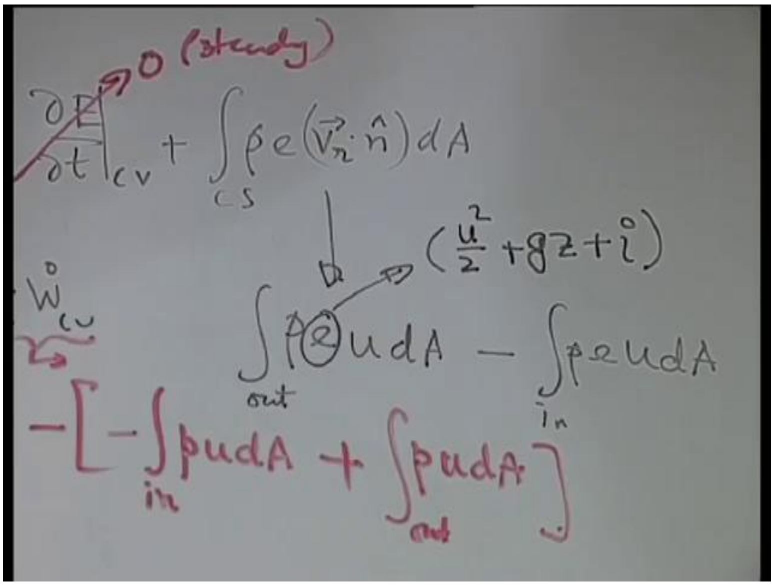

So, first say if you write first kinetic energy u square by 2, potential energy g z and internal energy per unit mass i. So, if you collect all the terms what you get at the end. So, you get Q dot c v, let us say that you take this terms of integrals of P u d A to the right hand side. So, if you take these terms to the right hand side you will see that it will club up with this 1 u square by 2 g z i plus that you will have 1 P by rho because rho is there as a multiplier this is just P alone to adjust with that you will have one rho multiplier outside. So, in the bracket what we will enter is P by rho.

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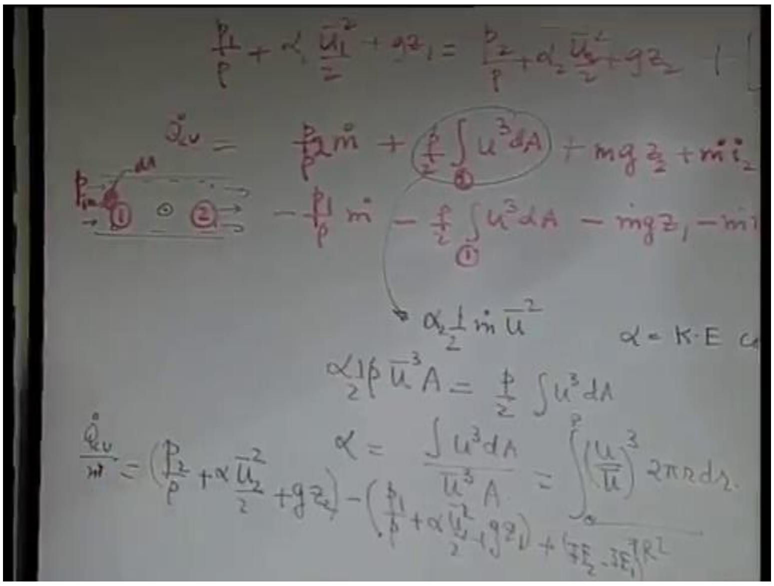

So, you will have this equal to say if you take rho outside then integral of P by rho, plus u square by 2 plus g z plus i, into u d a same thing for the inflow.

Now, the next important thing is the integration of whatever integral appears in the 2 inflow and the outflow boundary terms. So, when you write this integration you have to keep one thing in mind that you have to be careful whether properties are varying over the cross section or not because there is integrals, so what the cross section. So, let us assume that the pressure is not substantially varying over the cross section, and that in the way through that we have seen that the measure pressure gradient is along the x direction. Then u will definitely vary with cross section because you have a velocity profile it is not a uniform flow. The potential energy effect that also you may consider that the pipe diameter is not so large, that there will be a great difference in potential energy effect.

So, certain terms of this you may very confidently take out of the integral assuming that those are constants. So, like you can for example, write say you divide Q dot c v by rho. So, one you have P by rho or let us do one thing you will divide by rho in the next stage, we will now see that if you take P by rho out of the integral then what you have left with in the integral is integral of u d A that is the volume flow rate, that multiplied by the density is the mass flow rate. So, we call it m dot; next is you have rho, rho by 2 integral of u cube d A. So, this is remember we are writing for the outflow, so let us give some names to this areas; 1: for the inflow area and 2: for the outflow area. So, this P 1 by rho this is integral over the section 1, and then similarly this is m dot g z. If you assume that the temperature is also not varying over the section then the internal energy you may assume to be constant over the section. So, plus say m dot i

Student: (Refer Time: 14:24).

Oh this is 2 right yes m dot g z this is 2 i 2; then minus similar terms for one. So, minus P 1 by rho m dot, minus rho by 2 integral of u cube d A for the 1, minus m dot g z 1 minus m dot i 1. Now let us say that we neglect this effect we do not neglect this effect of velocity variation out write, but we somehow make up for negligence. See if we do not consider this effect all together I say that the consider that the same velocity, velocity same as the average velocity is there then what approximation to this term could be half m dot into u average square, because this is what this is like kinetic energy.

Because 1 rho into u d into d i is like m dot; and that u square is like this one, but this is unanimous why is unanimous because this is not exactly same as this one, because m dot is what rho into u average into a. So, u average into u average square is u average cube that is not same as integral of u cube d a. So, that is an error and that error has to be adjusted with the multiplying factor say alpha, which we call as kinetic energy correction factor. So, this alpha at the section 2 this may be the different at different section because velocity profiles may be different in general over different sections. So, this alpha is known as kinetic energy correction factor. So, what is this correction factor all about? This is the correction factor to correct the kinetic energy from a hypothetical consideration that it is based on the absolute velocity to the real kinetic energy that is there integrated over the cross section.

So, kinetic energy correction factor will be what. So, you have alpha into m dot is rho u average into A into half into u average square. So, u average cube is equal to rho by 2 integral of u cube d A. So, you can now write a expression for alpha as integral of u cube d A this one. So, if it is for a circular pipe. So, this is as good as u by u average whole cube d A is 2 pi r d r by pi r square from 0 to i. So, you can calculate the kinetic energy correction factor given the velocity profile. Now can you tell whether it will be more for laminar flow or turbulent flow?

Student: laminar flow (Refer Time: 18:08).

Laminar flow; why it should be more for laminar flow?

Student: (Refer Time: 18:20).

So, it depends on the u by u average.

Student: right (Refer Time: 18:25).

Right. So, u by u average it is u divides from u average significantly more for laminar flow. So, you will have more significant value of this one deviated from one. So, if u was equal to u average throughout, then kinetic energy correction factor would be one. If it is slightly deviates from u average then it will be very close to one, but if it is largely deviating from u average say consider the fully developed laminar flow through a circular pipe.

So, u is u central line is 2 into u average. So, you can see there is a large difference and that 2 factor will be there if you consider this kinetic energy correction factor. So, it will be large value. So, I will leave it you leave it on to you as an exercise that you calculate the kinetic energy correction factor for fully developed laminar flow through a circular pipe. Just substitute the velocity profile the u by u average equal to 2 into 1 minus small r square by capital R square, and then just do the integration. Now you see that this kinetic energy correction factor if you put let us see that what equation you will get at the end. So, now, let us divide all the terms by m dot ok.

So, if you divide all the terms by m dot you have Q dot by m dot is equal to P 2 by rho plus say alpha into u 2 square by 2, because you have already divided by m dot which is rho into u bar then plus g z 2, internal energy term will just write separately; minus P 1 by rho plus alpha u 1 square by 2, plus g z 1, plus internal energy 2 minus internal energy 1 right. So, we just rearrange it little bit to write that you have P 1 by rho plus alpha u 1 square by 2, plus g z 1 is equal to P 2 by rho plus alpha u 2 square by 2, plus g z 2 plus internal energy 2 minus internal energy 1 minus this one right. So, many times when you say alpha may be it is write alpha 1 and alpha 2, now if you consider these alphas as 1 this will look like a modified Bernoulli’s equation, that here you have the total mechanical energy at 1 here you have the mechanical energy at 2, and you have term here the correction term. This correction term if it is zero, then it is just like the Bernoulli’s equation that you have studied earlier.

So, sometimes this is known as modified Bernoulli's equation again that is a very wrong concept this has nothing to do with the Bernoulli's equation except the form. Because Bernoulli's equation you are writing between 2 points, here you are writing the equation between 2 sections 1 and 2. So, be very very careful this is a very important miss conservation people have; many times you see that in some of the industrial applications even the kinetic energy correction factor is omitted and then still it works. It work beautifully because many of the engineering flows are so turbulent that kinetic energy correction factor is very close to one; that means, considering that or not considering that does not matter, but its matter of negligence or understanding.

So, if you net if you understand the it has to there, but for a highly turbulent flow you neglecting that is one thing, but the very bad thing is you do not know or do not understand that it has to be there. So, that misconception has to avoided. So, do not take it as a modified Bernoulli's equation, better we just call it as an energy equation which looks like a modified form of Bernoulli's equation with the terms in the top Bernoulli's equation adjusted with something. So, what is this now? We will now concentrate on physical meaning of this. So, internal energy 2 minus internal energy 1 what is this so.

Student: (Refer Time: 23:36).

So, have basically let us say that you have. So, at the section 2 say at the section 1 fluid has entered, at the section 2 let us say that heat transfer is zero let us say that you have insulated the wall of the pipe so that there is no heat transfer across the control volume. So, then what do expect that term to be positive or negative? This is pure physical understanding do not try go for any mathematics to describe it.

Student: (Refer Time: 24:09).

Think about this you have because of viscous effects the relative motion between various fluid layers; it is as if like you are rubbing one of your palms with the other. So, you have the frictional resistance between because of relative motion between various fluid layers; and because of that frictional to overcome that frictional resistance what will happen there? There will be some work that is necessary, but that work is not an useful work. So, the entire work is dissipated and where it is dissipated it is it is dissipated in the form of inter molecular form of energy. So, the fluid gets over heated. So, it increases temperature of the fluid.

So, because of viscous action whatever work is necessary to overcome that that is eventually manifested in the form of at increased temperature this is known as viscous dissipation. So, whenever you are will be studying may be heat transfer later on, you will go through the greater details of viscous dissipation, but it is very important to have a qualitative understanding of it. That is velocity radiant between fluid layers, and there is a relative motion between the fluid layers to overcome that resistance some energy has to be spend upon some work we done, but that work is not manifested in the form of useful work. So, that what it only does is it increase the temperature of the system through the internal energy rights.

So, we can conclude that these term is always positive; now you may say that I will have a heat transfer which is more than this one, but see spontaneously that effect is not going to there. In the limiting case what may happen see when the system is over heated you have a hat temperature in the surrounding. So, you will have a heat transfer. So, this spontaneous heat transfer is what? This spontaneous heat transfer itself is negative. So, here you have to remember that this was put with the sign convention that heat transfer to the system is positive; here the system is getting over heated, so there will be heat transfer from that to the surroundings. So, that itself will become negative. So, the sum total of that will be positive; that means, what you can say that these represents the total mechanical energy at section 1 this represents the total mechanical energy at section 2 this is the positive term; that means, the total mechanical energy at section 2 is less than total mechanical energy at section one.

So, there is some loss of energy that is manifested in the form of the head loss that we have seen. So, these expressed in the form of head that is if you divide all the term by g, then it is it is expressed in the form of unit of length or head that is nothing, but the head loss that we have calculated for the pipe flow problems. Now consider one important thing, if we ask you that what is the work done by the fluid to overcome the wall shear stress, the shear stress at the wall what will be that.

Student: (Refer Time: 27:20).

Yes what should be the work done to overcome that? Keep in mind one thing how do you calculate the work? Some force or here the rate of work, so some force multiplied with some velocity. So, where we are concentrating at the wall? At the wall there is some shear force what is the velocity of the fluid relative to the wall zero; that means, there is no work done to overcome wall shear stress at the wall. Again this is the very very important thing, because these are loose wrong concepts that because there is consciously there is some work done to overcome wall shear stress that is why fluid is losing energy all those bla bla things people gives as explanations, but you have to be very very particular in this where is the work if the velocity is zero.

So, there is no work done to overcome the shear stress at the wall that work is zero; only whatever is the work internally that manifested in the form of this internal energy change, but not sort of a useful work by the displacement at the wall; obviously, because it is a no slip boundary condition.

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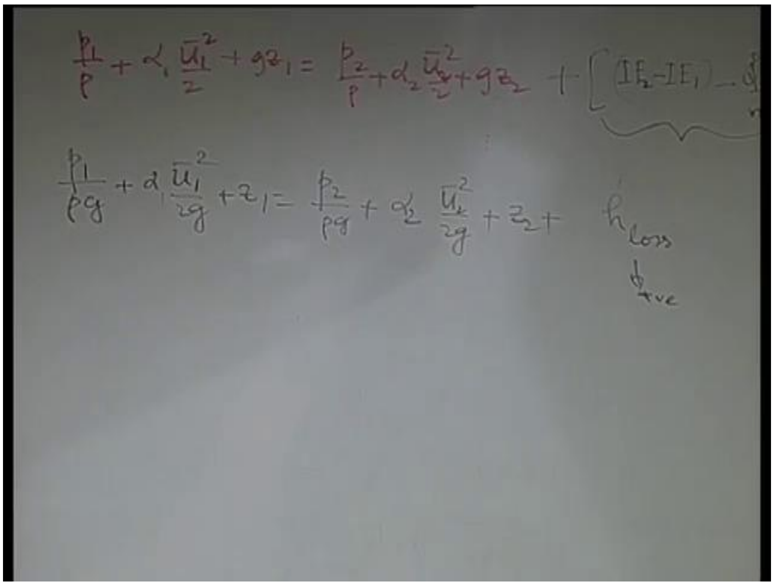

So, from this analysis what we understand is that we can cast the energy consideration between 2 sections 1 and 2 as P 1 by rho g, plus alpha into u 1 square by 2 g plus z 1 is equal to P 2 by rho g, plus alpha 2 into u 2 square by 2 g; remember all these terms are average velocities z 2 plus head loss which is the positive term. This is sometimes known as modified Bernoulli’s equation or better to say this is an energy equation, modified mechanical energy equation which still represents the conservation, but there is a loss.

Now, what could be these losses what could be the sources of this losses? One of the sources of the losses is because of the viscous effect that we have already discussed that is the head loss h f; we could characterise it that how it is different for laminar flow turbulent flow so on, but there also could be other types of losses and other types of losses are possible because of other changes present in a piping.

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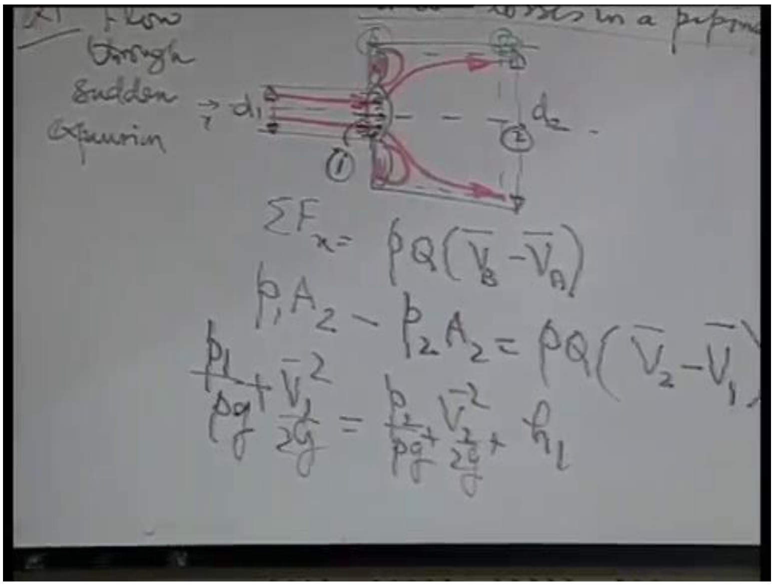

For example you have a small pipe now that small pipe is getting change to a larger diameter now this is something where there is the loss why there is loss see of course, the diameter change is there, but why diameter changing gives us the loss.

So, if you consider the stream lines like this the stream lines because of the sudden changing cross section will be having there curvature in this way. So, locally what will happen? There will be eddies form in this way these eddies do not participate in contributing to the energy of the main flow. So, whatever energy is there associated with the rotation of this eddies there is a loss so far as the main flow transmitter is concerned. So, this is also a loss this loss was not taken into account for calculating the h f. similar things may be for like if you have holes in the piping system because those are creating some resistance if it wall see the wall is totally close the fluid cannot flow. If the wall is partially close then partially opened the fluid may flow so; obviously, there may be other forms of resistances and those losses because of the other forms of resistances known as minor loses in a piping system. So, we now look into minor losses in a piping system; again this minor loss is a misnomer because sometimes.

So, when there is something minor then there is something major. So, what is this major? Major is this head loss due to the friction this we call as major loss, but in many practical considerations minor loss becomes much much greater than the major loss. So, it is the name major and minor should not be confused i a in a literal sense. So, just because originally the loss considerations where there form the pipe friction consideration and in one way it is major because in a pipe line if there is nothing else at least the physical resistance because of fluid friction is there over a length. You may not have wall you may not have a sudden change in cross section, but the length of the pipe itself is present. So, the reason of naming the major loss this loss will be there, other losses minor losses are losses they may be there may not be there depending fitting on what fitting that there in a piping, but in if their sometimes they are much much more important than the major loss. So, the relative importance need not be understood.

So, we will look into some examples for the which give rise to the minor losses. So, the first example is flow through sudden expansion. So, here what is important is we are able to see that there is some loss, let us say that the diameters of the smaller and the larger pipe say d 1 and d 2. We are interested to find out the loss because of this flow through sudden expansion and when we find the loss due to sudden expansion, we isolate the effect of the loss due to the length of the pipe. So, we only consider the loss due to this sudden expansion effect not the length effect. So, what we do we just take a control volume and try to apply the Reynolds transport theorem momentum conservation.

So, let us say that we take this section as section A and take this section as section B. So, for this control volume, if you want to write the Reynolds transport theorem; so the resultant force along the x direction if it is x, I am just writing the final simplified form because we have discussed about such problems many times. So, we assume it the a steady flow and V B minus V A right. Now what are the forces acting on the control volume. So, you have a pressure force on the 2 ends right. So, when you have the pressure force on the 2 ends see also there is a shear force here at the wall, but we are neglecting that effect because that effect is already considered in major loss ok.

So, that does not mean that that effect is not there, we are isolating that effect from the minor loss effect. So, the pressure force what is the pressure distribution at one; let us one at a. So, we have a section one which is say for here a stream as the section say 2 which at a downstream let.

Let us consider that the velocity profiles are approximately uniform at 1 and 2, when is it possible? It is possible when it is almost the very highly turbulent flow. So, let us assume that the flow is highly turbulent, so that the velocity profiles are almost uniform that is the kinetic energy correction factor is not important. Now when you have a say you want to write the force due to pressure here, see it is important to note that the pressure is acting here in this way and this is let us assume that this pressure at the pressure this not greatly different, and that will not be greatly different because this kinetic energy are greatly different and this length we are not considering so large, that there will be a huge loss of head because of the friction.

So, you here you have the pressure at this one, roughly same as P 1 if P is not located for away from the section a if it is quite close then possible. So, what we are considering is that in addition to the pressure acting over this part, the same pressure also acts over the upper and lower parts this is an assumption. So, under that assumption is well justified because the change in pressure from this one is not felt so easily by this one, because this is just a small recirculating region, so the change in pressure is failed only when the proper bounding stream lines are making it feel. So, here it is just local recirculation this does not understand so easily that what would be the change in pressure from this to the subsequence section.

So, you have P 1 into. So, the entire pressure here is like P 1 and the area is like A 2. So, that you have to understand it is not the area of one, but the total area over which as it this P 1 is acting. So, it is basically P 1 into area of the section a area of the section a and b at the same. So, this minus P 2 into A 2 is equal to rho Q now V b and V a. So, V b is like V 2 that is fine what about V a; see the velocity is here are not contribute to the energy. So, V a is V a average is like roughly like it is taken as V 1 average, again it is it is an approximation. So, what it considers is that the average velocity is like this is the part of the section where the velocity effects are important, at this part of the section just has the velocity which is the same as the average velocity as this one and this is totally distributed uniformly. So, that is what is. So, there is lots of approximation you want, but many of these approximations are not so bad if the low is highly turbulent. Now that is number 1 number 2 is you can write the difference.

So, practically you are considering this as like a section 1 and this as like section 2 section one is this part this small part. So, if you now write the energy equation like you can write P 1 by rho plus V 1 average square by 2, is equal to P 2 by rho. So, kinetic energy correction factor we are not considering because we are assuming it close to 1 plus V 2 square by 2 plus head loss also the points are so located we are neglecting the change in potential energy.

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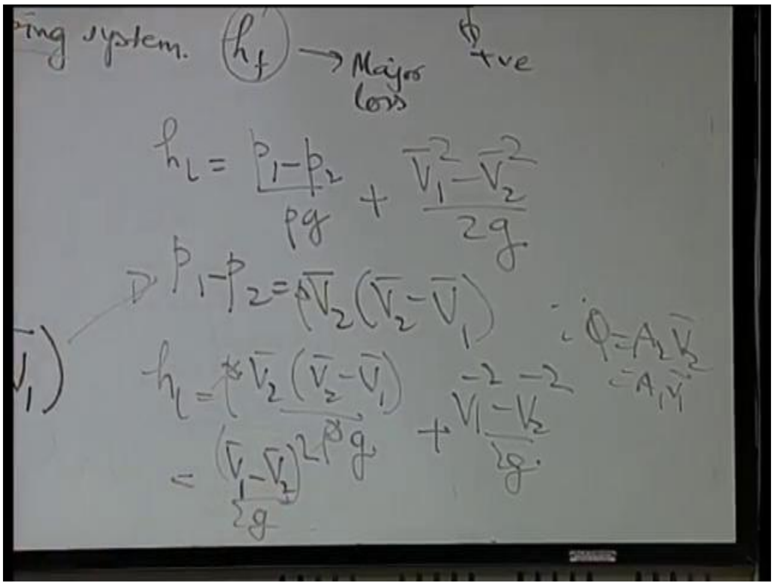

So, the head loss we can calculate. So, divide by g into call it a unit of a head; head loss is P 1 minus P 2 by rho g plus z 1 square minus V 2 square by 2 g.

Now, P 1 minus P 2 you can write; in place of Q you can write A 2, V 2 right. So, that a 2 gets cancel from the 2 sides. So, you get this as V 2 into V 2 minus V 1. So, this is seems Q equal to A 2 V 2, which is same as A 1, V 1. So, that is from this equation. So, then you can substitute that h l equal to in place of P 1 minus P 2 you will have P 2 into V 2 minus V 1 by . So, there is a rho here that there is a rho here right rho into this one. So, rho gets cancelled out then plus V 1 square minus V 2 square by 2 g. So, if you simplify this it will be V 1 minus V 2 whole square by 2 g, you can clearly see that just 1 step. So, what we can get from this one, is a very interesting thing that I am this shows that it has to be always positive thing. So, the head loss is positive, and it is a function of the difference in the average velocities over the 2 section. One special case is that let us say this is the small pipe entering into a very large reservoir. So, you have small pipe like this it is entering a large reservoir.

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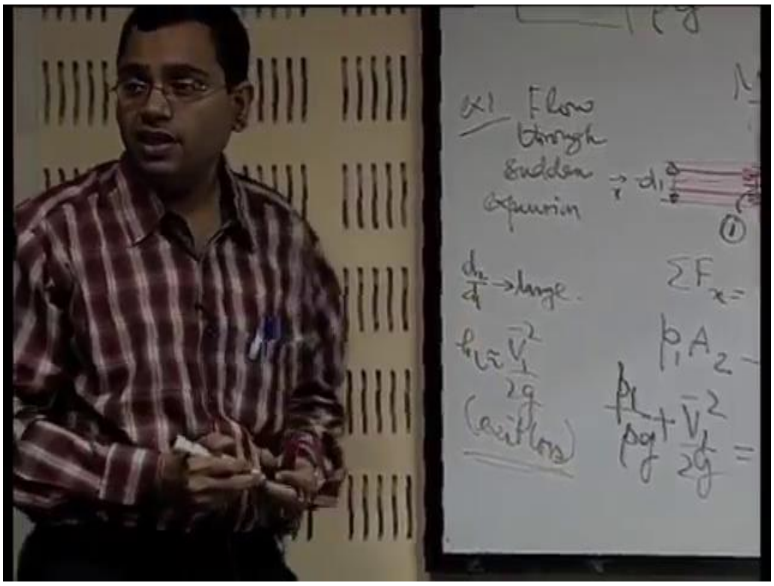

So, what is happening is fluid is exiting from the pipe to a reservoir and then that is the special case of this one with d 2 by d 1 very large; and then what does it become? This becomes approximately V 1 square by 2 g this is known as exit loss. So, exit loss is very important in engineering, because it signifies the exit of the fluid from a pipe line to a reservoir. So, it is a special case where the ratios of these 2 sizes are grossly different otherwise this is the formula straight away you can use. So, this is one example of minor loss in the next class we will look into some other examples of a minor loss, let us stop here.

Thank you.