Introduction to Fluid Mechanics
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Lecture – 56
Pipe Flow-Part-II

So, now, as engineers what are the important thing that we have to understand is what is the friction factor because if you see this Darcy-Weisbach equation you can estimate the head loss completely if you know what is the friction factor.

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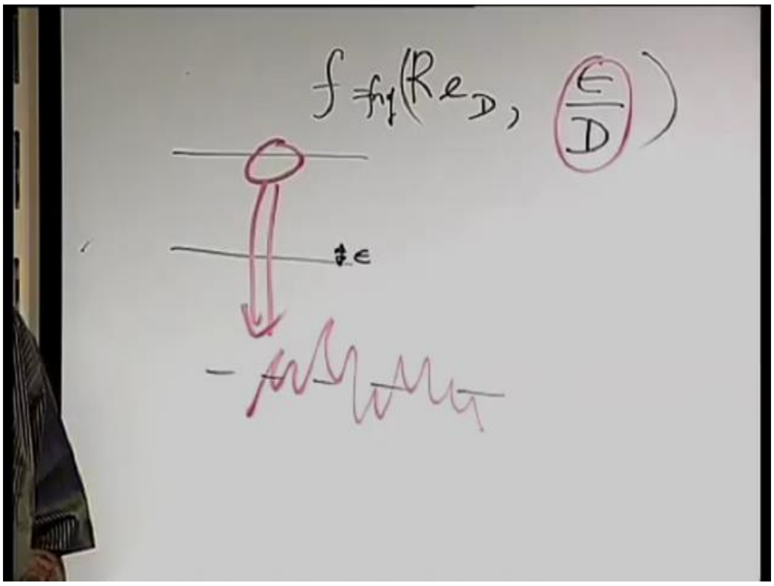

So, the friction factor how do you know for a fully develop laminar flow we may determine it exactly by a very simple analysis which we have already done through exact solution of the Navier stokes equation. As the flow becomes turbulent we have seen that such simple exact solutions are not possible because you may only statistically operate on certain quantities and you may get an estimate of the velocity profiles with certain fitting parameters as much as that, but not something which is very very exact.

Therefore for turbulent flows the better way in which the friction factor was understood was by doing a lot of experiments and we have to find out or we have to understand that what should be the important parameters that govern the friction factor if you are doing an experiment. So, what should be the important parameters? See the friction factor one of the parameters we have seen it should be the Reynolds number right, the other parameter should come into the picture more importantly for turbulent flows because turbulent flows are often triggered by disturbances; disturbances are sometimes triggered by the roughens effects of the wall. So, if you considered a pipe like this and if you say glow up one of the locations and the wall of the pipe, the wall of the pipe in a very powerful microscope we will look very undulated because no manufacturing process we will allow you to have a real atomically smooth wall.

Now, this roughness should have a strong role to play in terms of triggering the onset of turbulence by having a disturbance when the fluid is going beyond the critical when the flow is beyond the critical Reynolds number. What is, how to characterize this roughness. So, always one of the important characterization says you have the center line and you consider the average roughness with respect to the center line of these like undulation. So, let us say that you have an average roughness called as epsilon which represents the roughness average roughness of the wall.

Now, it is not the absolute value of this that is important say this average roughness is one millimeter. Let us say we have two pipes one pipe is having a nominal diameter of one centimeter another pipe is having a nominal diameter of kilometer. So, one millimeter of roughness will be more influential in which case?

Student: (Refer Time: 03:13).

The smaller one, because it is important that on an average what is the roughness relative to the characteristic length scale of the system. So, it is not the epsilon, but epsilon by D what is important. For our non circular section you may replace it with a hydraulic diameter, but it is basically the epsilon by D. So, what you expect is that the friction factors should be a function of Reynolds number and epsilon by D right. For a laminar flow it is not a functional epsilon by D, why? Because for a laminar flow whatever disturbances are created by the wall these disturbances get dampened out almost instantaneously (Refer Time: 03:54) are not allow to grow and that is why the effect of the wall disturbance is not at all important. So, for fully develop laminar flow the friction factor is independent of the surface roughness, but if you go to a turbulent flow they freely surface roughness or the role to play

So, by keeping this physical considerations in mind there was a German engineer called Nikuradse who performed lot of experiments to characterize the friction factor as a function of Reynolds number and epsilon by D, and these experiments where later on summarized in the form of a very nice chart by an engineer called as Moody and that chart is known as Moody's diagram.

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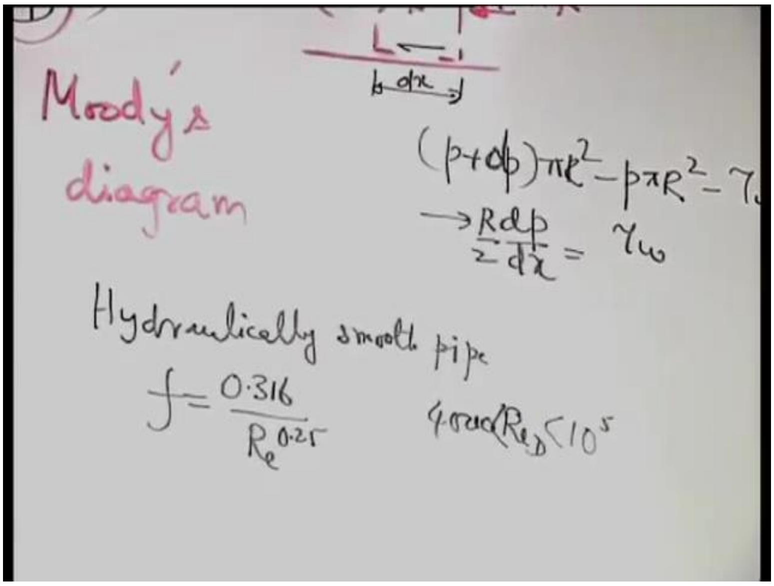

So, Moody's diagram is a chart which summarizes the friction factor as a function of Reynolds number and epsilon by D for wide range of Reynolds number and we will now see that how the Moody's chart looks like.

So, if you look into this chart let us try to understand it bit carefully. So, if you look at this chat you see that on one side of the, first of all in the horizontal scale you have the Reynolds number.

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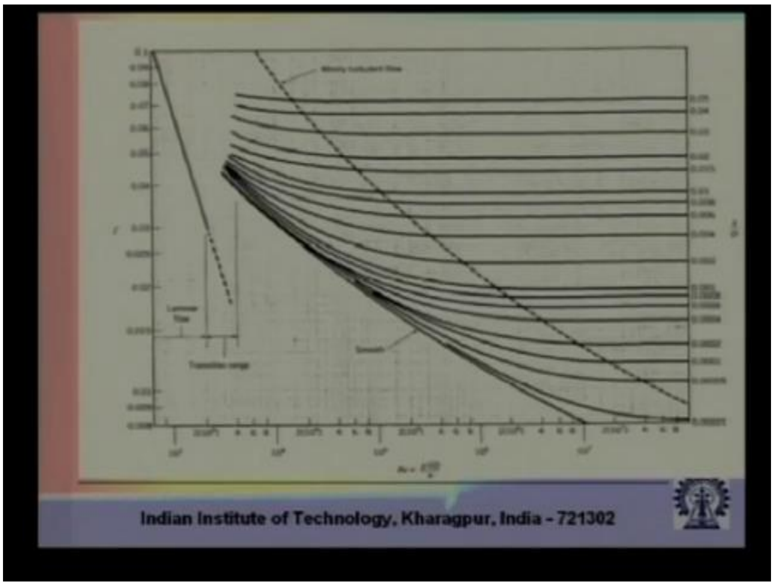

So, rho u average that is here it is written as v into D by mu. So, that is the Reynolds number and because Reynolds numbers is the sort of the independent variable here it is plotted in a logarithmic scale because you want to cover a very wide range of Reynolds numbers in a single diagram. So, only the log scale can compress it because if you see the Reynolds number range from a very low Reynolds number to a very high Reynolds number it is covered.

Then what is the other important independent variable or rather input variable need not be independent on that is the average surface roughness divided by the diameter. So, that plotted in the right hand side. So, the right hand side all the value values you see are epsilon by D. So, where from you get epsilon by D if there is a manufacturer of a pipe then depending on the material and the manufacturing process the manufacture of the pipe has a quote for the epsilon by D. So, for a given pipe you know what is the epsilon by D, but of course with warranty and that may change, but at least initially whatever the manufacturer codes is a reliable one.

So, now what is there in the vertical access in the left is the friction factor that is your output from this diagram which you want to utilize for estimating the head loss. So, the friction factor if you see for lower Reynolds number you have to keep in mind that f equal to 64 by Reynolds numbers. So, if you make a log then in the log plain it is like a straight line coming downwards. So, that is what you see for the laminar flow. Now suddenly beyond the critical Reynolds numbers. So, for a pipe flow rapidly close to 2000 or so, you see that the friction factor jumps up that is the first observation.

So, why does the friction factor jump up? See pipe flow is not a very special chapter that we are studying, it is like a discussion based on whatever fundamentals that we studied in all over earlier chapters, directed a focus towards an engineering analysis. So, now, from that can you tell that why should you have a jump of the friction coefficient, friction factor is an indicator of the what? Wall shear stress, wall shear stress for a given flow rate is better for what laminar flow or turbulent flow?

Student: Turbulent flow.

Because you have almost uniform velocity profile in a turbulent flow, so you have very high velocity gradient close to the wall. So, for a given flow rate the wall shear stress is higher for the turbulent flow and that is why the friction factor suddenly jumps from low value to a high value then you see that the friction factor it becomes a function of the epsilon by D. So, for different epsilon by D and if Reynolds numbers these are all experimental data and what you can see that all these curves are (Refer Time: 08:16) or like enveloped by something which is written as smooth in the diagram that is called as a hydraulically smooth pipe.

So, for a hydraulically smooth pipe the friction factor is sort of like it is dependent not only one the Reynolds number because it is hydraulically smooth it is not a function of the wall roughness. So, then for such cases, I like for a range of Reynolds number between say 4000 to of the order of 10 to the power 5 this friction factor for hydraulically smooth pipe is given by 0.316 by Reynolds number to the power 0.25. So, this is they have to remember that this is like valid roughly between the Reynolds number of 4000 to may be 10 to the power 5 of that order.

So, that is represented by the line by the smooth line that you are seeing in the diagram. Then you can see that as you increase the Reynolds number there is a very interesting behavior that you see for very high Reynolds number what you see for very high Reynolds number the friction factor is almost independent of Reynolds number, why? Because if you see the lines which are there towards the end you see that you those a parallel lines; that means, only if you change the epsilon by D the friction factor is changing, but with respect to the Reynolds number variation it is a horizontal line. So, it does not change with Reynolds number. So, you see, one of the important observations is at very high Reynolds number the friction factor sort of trends to become independent of the Reynolds number and this independence starts more quickly for more rough pipes.

So, if you go towards the top of the figure you will see that this inception of the horizontal portion it starts very quickly with a relatively lower Reynolds number for what for? For very high wall roughness. So, we have to understand that in that horizontal location what is the effect of the wall roughness that is creating such a behavior.

So, if you have first of all very high Reynolds number flow then what happens? Then if you have a turbulent flow, but a very high Reynolds number or whatever Reynolds number it is adjacent to the wall the flow is laminar and that layer where it is laminar is known as the viscous sub layer. So, the viscous sub layer becomes thinner and thinner as you increase the Reynolds number. So, then what happens? The viscous sub layer becomes thinner and thinner than the outer flow is expose directly to the wall roughness elements if the viscous layer is quite thick it cushions the wall roughness elements. So, it covers the entire wall roughness element, but if the viscous varies thin now the wall roughness elements ported out and they interact with the outer turbulent, outer flow and that will actually induce a lot of form drag or pressure drag because local roughness elements are like flow pass blow bodies. So, if you have local rough walls and around which the fluid is flowing, so it is flow pass blow bodies where the form drag pressure drag that becomes very important.

So, when the form drag becomes very very important we have seen that under certain case when the Reynolds number is very large and the form drag is very important when both of these are satisfied the friction factor is almost independent of the Reynolds number because the form drag is what it dominates not the skin fraction drag and that is what happens for the horizontal portion in the diagram that you see. So, the entire understanding physical understanding of the diagram should be very very clear that why we are having different types of variations in the different parts of this diagram.

Now what we will do we will try to see that how to make use of this diagram. So, we will look into certain numerical examples to illustrate that how we make use of this diagram.

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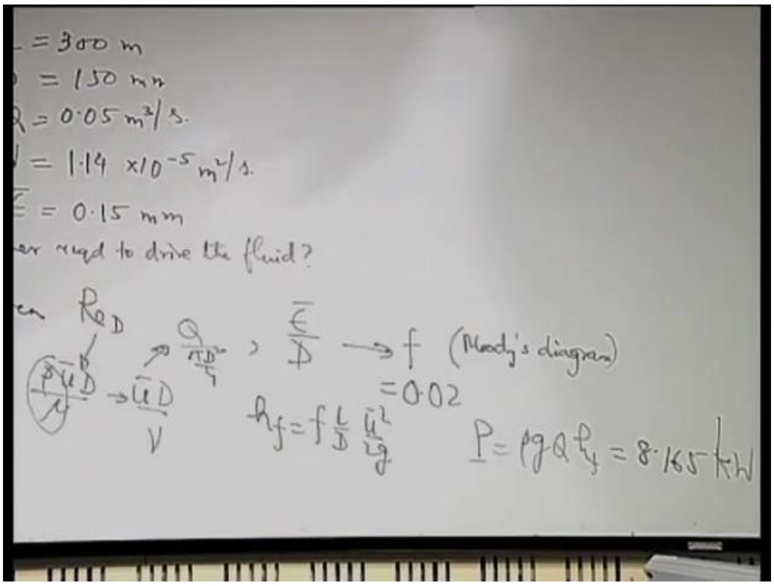

So, first example let us say that you have length of a pipe as 300 meter diameter of the pipe as 150 millimeter flow rate is 0.05 meter cube per second, the kinematic viscosity of the water at that prevailing condition is 1.14 in to 10 to the power 5 meter square per second the average surface roughness is 0.15 millimeter.

So, you have to find out that what is the power required to drive the fluid. Of course, we will not work out the problems in with full numerical details it will take a long time we will just outline the procedure and you can always look in to the Moody's diagram and find out they exact values. So, let us refer to the Moody's diagram and see that how we can utilize these diagrams. So, what is given to us what is given is first of all what is given is the length of the pipe, but more importantly for the use of the Reynolds; for the use of the Moody's diagram the diameter is given; average velocity you can find out by Q divided by pi D square by 4.

So, the average velocity we can find out the density of the water at that particular condition say it is known and the velocity is also there. So, from that you can find out what is the Reynolds number. Epsilon is given and D is given so that means, you can refer to. So, the problem works down to that given Reynolds number that is rho u average into D by mu and basically the kinematic viscosity is given. So, that is basically u average D by nu that you can calculate average is Q by pi D square by four. So, the Reynolds number is known and epsilon by D is known.

So, by referring to the diagram for from the Reynolds number and epsilon by D you can get f from the Moody's diagram. So, I am just giving you what is the value of f that should come out so that you can have a practice of looking in to the diagram later on and check with the value. So, this is the value of f that you get and once you get the value of f then; obviously, we can calculate h of as f L by D into u average square by 2 g and the power you can find out rho g Q into h f. Here the power required the question is just the power required to overcome this friction nothing more than that. So, the answer to that is 8.165 kilo watt that you can check later on.

Let us look into a second example. So, in this example you have the kinematic viscosity 10 to the power minus 5 beta square per second. The diameter of the pipe 100 millimeter average roughness is 0.25 millimeter, the head loss is given as five meter of oil, this is the oil which is flowing the length is 120 meter the question is what is the flow rate. So, we have to again work out as strategy for this. See what are the things that we know; first we have to figure out that whatever information is there with us is it sufficient enough to read from the Moody's diagram the relationship between friction factor Reynolds number and epsilon by D.

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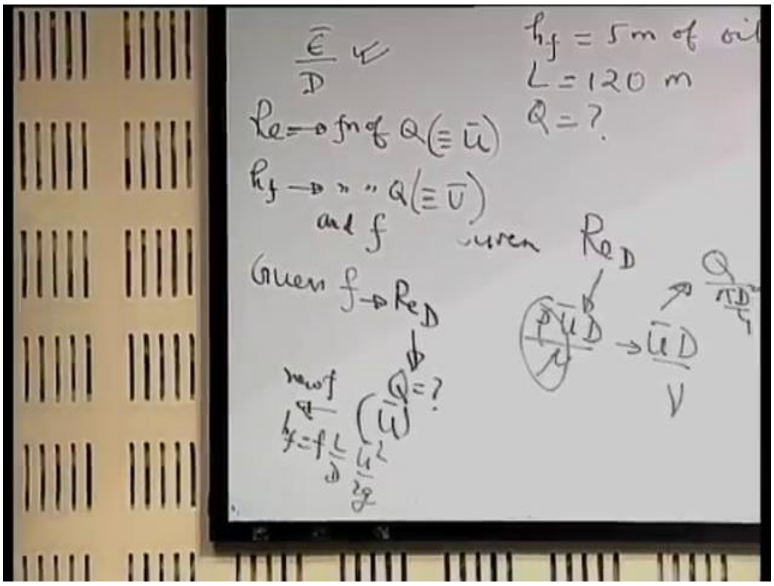

One thing is we know what that is epsilon by D that is given, then you do not know the flow velocity because you do not know the flow rate. So, you do not know the Reynolds number.

So, the Reynolds number; obviously, you could calculate and it could be a function of Q the remaining things are known and h f that also you can write as a function of Q and f. So, when you write this as a function of Q equivalently you can write this as the function of u average also, either function of u average or function of Q because diameter is given you can express either in terms of Q or u average.

Now, you see here that given the epsilon by D what prohibits you from knowing what is the friction factor is you know h f, but you do not know what is the average velocity. So, one of the ways in which this case may be tackled is by a trial and error method. So, you start with a guess value of f guess some value of f. So, once you have guessed a value of f then from that you can find out what is the Reynolds number or when you have a f and one when you have a epsilon by D from that what you can find out.

Student: (Refer Time: 18:56).

You can find out the Reynolds number that which will give the Reynolds number. From that Reynolds number what you can calculate you can calculate the Q right or the average velocity when you have calculated the average velocity then what how would you improve your guess.

Student: (Refer Time: 19:19).

So, this u you have h f. So, from this you can find out a new f from what h f is equal to f L by D into u average square by 2 g right. So, you started with a guess f that has as Reynolds number that Reynolds number gives a Q or u average from that using the non head loss you can calculate a new f. If this new f fortunately matches with the old f then; obviously, your iteration has convert that once, but nobody will be that lucky until analysis you know the answer. So, you again go to a new with this new f and repeat this cycle till you get the convergence and answer to this problem is f is equal 0.0318 that is the friction factor that you get and from that you can this is converged f and from that you can calculate what is the Q.

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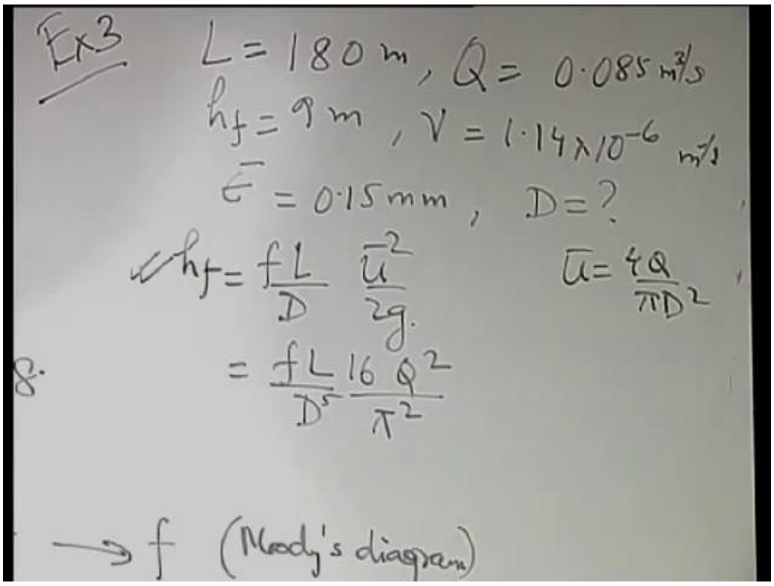

Let us look into a third example of similar type. So, you have the length of the pipe as 180 meter, the flow rate is 0.085 meter cube per second, the head loss is given as 9 meter, kinematic viscosity 1.14 into 10 to the power minus 6 meter square per second, epsilon average is 0.15 millimeter. So, now, the question is what is the diameter of the pipe. So, you can see that like it depends on what is the known what is unknown and the basis is the same you have to use the Moody’s diagram, but the strategy maybe a bit different depending on what is known and what is unknown.

So, here you do not know epsilon by D itself, but what you know even do not know what is Reynolds number. So, you have h f is equal to f L by D into u average square by 2 g since you know Q better write Q u in terms of Q and D. So, you right u average is equal to 4 by pi D square. So, f L by D u average square means 16 Q square by becomes 5 square and this becomes D to the power 5. So, this is given h f is given. So, when h f is given then what is the next thing that you should do. So, if you know Q and if you have an estimate of what is h f which is already there you can find what is the friction factor or if you have a guess of what is the friction factor you can find out the Q.

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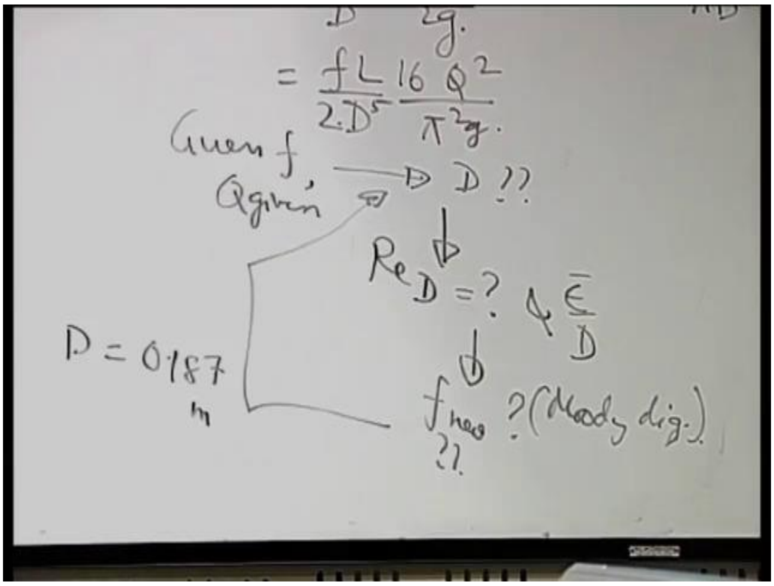

So, let us say you guess a friction factor if you get a friction factor then from this expression you can find out what is Q up I mean guess.

Student: Sir, g (Refer Time: 22:36).

Or there is g here ok.

Student: (Refer Time: 22:40).

Oh sorry. So, you have a guess value of f and you have Q given. So, from there you can find out what is an estimate of D, but it should satisfy their relationship between f epsilon by D and Reynolds number. So, you have to calculate that what is the Reynolds number based on these D because you already know Q. So, you can find out the Reynolds number on the basis of this. So, with this Reynolds number and epsilon by D because you know now D you also can find epsilon by D you find out what is the f from the Moody's diagram and is it same as the guess f, if not you again repeat this cycle.

So, let me give you the answer to this problem, the answer to this problem is the diameter equal to 0.187 meter. So, this examples just illustrate that how you can make use of the Moody's diagram for working out different simple problems and we will continue with that in the next class.

Thank you.