Introduction to Fluid Mechanics

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Lecture – 55
Pipe Flow-Part-I

In many engineering applications we also have internal flows; that means, flow is there occurring in a confinement. As an example we can refer to the flow through a pipe. So, pipe is a confinement within which the fluid is flowing. And it is unlike the case of a solid surface like a flat plate or a circular cylinder over which the flow is flowing. So, those cases are called as external flows, and the pipe flow is an example of an internal flow.

Now, when you have a flow through a pipe or an internal flow as an example, we have to first understand that what is the basic difference in terms of like - analyzing an internal flow and external flow.

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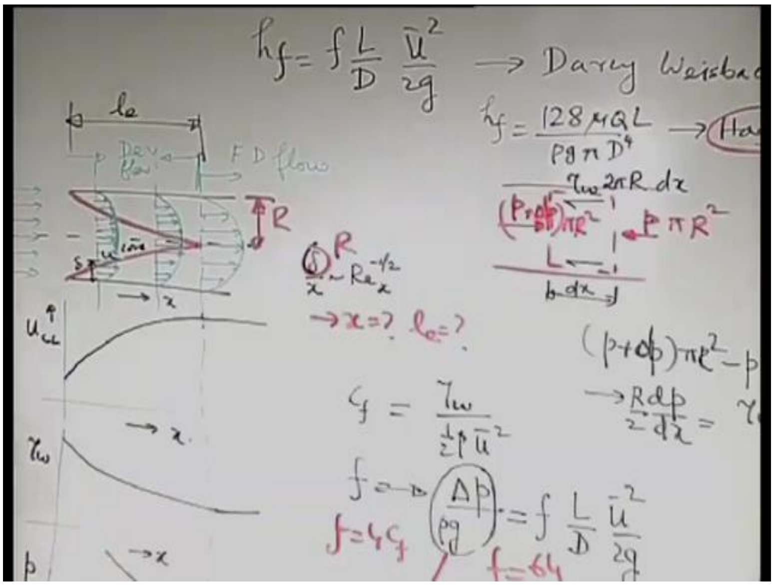

So, if you consider flow through a pipe, we have discussed about some of the elementary aspects of that earlier, and we will just quickly revisit that. So, if you have say some flow entering the pipe. Now just if you consider that the wall is like a solid boundary which is disturbing the flow and trying to retard it. There will be some development of boundary layer. And the boundary layer will be quite thin if the Reynolds number is quite large, but the boundary level will grow, and then because of the symmetry in the geometry and symmetry in the boundary conditions, the boundary layers formed from all sides will merge on the centerline.

Now, what happens to the velocity field in between? So, if we consider a velocity profile within the boundary layer. There are velocity gradients. Then the velocity profile is virtually uniform and then again within the boundary layer it starts having it is gradients. So, the velocity profile is roughly of this sort. Now the big question is that when you have such a velocity profile how does it change as you move along the axis of the pipe. So, if you draw the velocity profile at some other section, you have a greater region over which the fluid is slow down because the boundary layer is now thicker, to compensate for that what you would have is a higher velocity in the core region; that means, fluid in the core region is accelerating. This is something what we discussed earlier that is why I am just going through it in summary.

So, the first observation therefore, is that the velocity in the region outside the boundary layer is not a constant. It is continuously varying one. Why? Because you have a pressure gradient, if you see that for an external flow that thing is also there, but there pressure gradient is imposed by the geometry. And for that is why if you have flow over a flat plate then you do not have any gradient of the free stream velocity. But if you have a surface with the curvature we have seen that how the effect of the curvature induces gradient in the velocity and also the pressure.

So, here it is not the effect of the curvature, it is just the effect of the confinement. That is creating at induced sort of a pressure gradient and that is what is important here. So, the first important observation is, in the core region that is in the region outside the boundary layer the velocity is a varying one. The reason is obvious you have a pressure gradient. Now in this way when you come to this stage the boundary layers have now made at the center line and whatever is the velocity profile that does not change any further. And as we discussed earlier we consider this as a fully developed flow condition. So, this from this onwards it is fully developed flow. And in the other side whatever is there from the entry to that that is known as the developing flow region.

When we were discussing about the exact solution of the naiver stokes equation, we try to get an estimate of the velocity profile and the wall shear stress and friction coefficient in the fully developed region with certain assumptions; obviously, now in the developing region we may also do some analysis, it is possible to have an estimate of similar things either by semi analytical method not fully analytical or may be by numerical method. We will not going to that what we will do is we will try to make a sort of a qualitative sketch or a qualitative assessment of how the central line velocity varies with the axial coordinate, how the pressure varies with the axial coordinate, how the wall shear stress why with the axial coordinate. This type of understanding is very important. Because this gives us an insight of how these parameters may vary without going into the details of the mathematical form. And that is one of the important engineering insights that all of us should have.

So, let us say you want to plot the center line velocity as a function of x. Let us say this is the x coordinate. So, how it should look like? So, we have a fully developed flow at this location that we have to remember. So, the center line velocity what it will do? It is gradually increasing right. So, it is just an increasing function. Then it comes to the fully developed state beyond that it remains the same right. So, this is the fully developed center line velocity which may be easily obtained by referring to the parabolic velocity profile that we have derived earlier.

Let us say you want to plot the wall shear stress versus x. So, if you want to plot the wall shear stress versus x, what should be the variation? So, wall shear stress is dependent on what, it is dependent on the velocity gradient at the wall. So, it is roughly like mu into the velocity gradient is what, velocity gradient is u at the core region. So, this is the core region divided by the local delta. That is the order of magnitude of the velocity gradient. So, this is the velocity of the core fluid and this is the delta. So, roughly the gradient is of this order. As you move along x, what you expect? As you move along x mu you consider that it is a fluid of constant property. The centerline velocity is increasing delta is also increasing. So, the key is which one is increasing at a faster rate? Because you are interested about the ratio, right.

So, what can you say about this? Remember one thing. You see is a change from uniform state to somewhat enhance value as the fluid enters the pipe or the channel. Change in delta is very abrupt, because it is limitingly tending to 0 when it is it is just entering the pipe, and it is suddenly nonzero when it has just traversed the little distance inside the pipe. So that means, if you have a slight change in u c, for that you have a large changing delta right.

So, delta was initially tending to 0, and u c was something finite. So, this was very large. Suddenly delta comes to something which is more finite. So, and u c have not changed that much as delta has changed by say fall by traversing a very little distance. So, delta has got increased to a greater portion then the change in u c. So, what will be the effect in the ratio? It should decrease. So, the wall shear stress should decrease as you are moving along this one, moving along the axis of the pipe. What happens to the wall shear stress when you come to the fully developed state, it becomes a constant because the velocity profile is a constant. So, velocity gradient at the wall is also constant it does not change with x.

So, if you want to plot that, you may have say constant tau all like this and maybe something like this, in the developing region, where you have the maximum here and it is decreasing. Now if you want to plot pressure versus x. Again you have to keep one thing in mind, that like the wall shear stress and the pressure they are somewhat related. Or the wall shear stress on the pressure gradient they are somewhat related. Because in such a case you have the pressure gradient overcoming the resistance effect of the wall shear stress.

So, what happens in the fully developed state? D p d x is the constant right so; that means, p versus x is straight line, decreasing, and in the developing region there is something which should match with this in terms of the gradient and it will go like this. Of course, I am not drawing it completely, but like it would it would have roughly similar physical behavior as exhibited by the wall shear stress. So, just for the continuity act the fully developed condition you must have the tangent to this line same as this one. So, this is a qualitative variation. This type of qualitative variation is important because it gives as a physical inside of how these quantities are varying, but more important consideration is what happens in the fully developed state, because usually in industries one is having pipe lines of large lengths and a major portion of that is fully developed.

How can you estimate that what should be this length that which it becomes fully developed? This is called an entrance length; entrance length is the length beyond which the flow becomes fully developed. You may have a very rough estimation which is in an exact sense not very correct, but it will give you an idea. Let us say that you consider that this 2 are like flat plates. So, you know the delta by x roughly will scale with Reynolds number to the power minus half. This Reynolds number is not the Reynolds number for the pipe flow, but Reynolds number based on the local length x that you have to remember.

For the pipe flow the Reynolds number is based on the diameter of the pipe, but this is just we are considering as if it is flow between 2 parallel plates and each on each plate there is a growth of the boundary length. So, we can find out that what should be the x for which delta is equal to let us say that this is R delta becomes equal to R. So, what should be the corresponding x that is the entrance length?

For parallel plate channel it is not very inaccurate. For pipe it is more in accurate because this type of estimation does not consider the curvature of the wall of the pipes. So, it is just considering it like a flat plate, but even this estimation is not very correct even if you consider it as a parallel plate channel. The reason is this was derived with an understanding of a constant u infinity. Whereas, here the, so called u infinity is the velocity in the core region which is continuously changing with x. So, there is; obviously, and error. So, whenever we do something, and it is very approximate it may be erroneous, it is important for us to at least appreciate that it is erroneous the whole idea is not to come up the correct estimate, but an estimate of the like the order of the length.

So, maybe it is it is roughly like 60 times the diameter of the pipe or something like that. At which classically it may be found that the flow becomes fully developed. Now the length of the pipe lines being so long fully developed flow is supposed to prevail in most of the part of the pipeline, and that might dictate the fictional characteristics. Therefore, it is very important to understand for engineers that what is the consequence of the fully developed flow condition on the frictional resistance. Because for engineers for designing systems if the frictional resistance that is important. Because more the frictional resistance more the pumping power you have to employee to overcome that. That is in terms of an engineering perspective.

So, if you now recall that we introduce certain terminologies. So, we introduce something called as the friction coefficient c f. So, that was what the wall by half rho u average square. Here for an internal flow the reference velocity scale is the average velocity, not u infinity because here there is no physical significance of u infinity. Because it when it enters the system the so, called u infinity or the core region velocity changes continuously, but the average velocity for a given flow rate and a given cross sectional area will always remain uniform. Therefore, it remains as a standard basis. And in another way we also defined a friction factor f, in terms of the following equation, that you have delta p by L and if you just, So, this is the pressure drop over a given length of a L.

So, if you consider an axial length of L this if we want to characterize in terms of the kinetic energy here. So, you may write it in a non-dimensional way. So, this in a non-dimensional way is delta p by rho g L. So, that is writ10 as f L by d into this one. So, we have again we had earlier introduced this type of relationship when we were talking about like the flow in parallel plate channels or pipes as we were deriving the velocity profiles and the wall shear stress expressions from the exact solutions of the naiver stokes equation.

So, this is just another way of writing it and these 2 must have a relationship. The reason is that because of the wall shear stress only you require a pumping power and corresponding pressure drop that is taking place. So, the pressure drop is something which is a sort of equivalently, it is a driving force it is giving rise to a driving force that overcomes the wall shear stress. So, these 2 are sort of correlated and it is very easy to find that out.

So, if you take care if you consider a small element of the pipe and say if you consider some fluid element here. So, you have say hear a pressure p plus delta p, and say here you have a pressure of p if you consider what are all the forces which are acting, you also have the wall shear stress. So, wall shear stress is according in this direction. So, that is the tau all times let us say it is the, if R is the radius of the pipe then 2 pi R into this length. Let us say it is d x that is the shear force between the fluid and the solid. And the force because of the pressure is this pressure times the area that is pi R square here also this into pi R square.

Fully developed flow is a very interesting thing. Why it is interesting if you consider the naiver stokes equation the left hand side of the naiver stokes equation becomes identically equal to 0 for fully developed flow. Left hand side represents acceleration; that means, fully developed flow is a non-accelerating flow. So, when fully develop flow is the non-accelerating flow; that means, the forces are sort of been equilibrium. So, forces are in equilibrium means you must have p plus delta p into pi R square, minus p into pi R square minus tau all into 2 pi R d x. So, if you have a very small differential length then the pressure change will also be differential. So, in place of delta p you can write it as d p, because you have considered a differential length d x. So, over which the pressure drop will only be differential.

So, from here you can find out what is d p d x, that is tau all by. So, R by 2 into d p d x equal to tau all right, so you can clearly see that these 2 are related and related because of this. So, as if the pressure gradient is a driving force wall shear stress is the resisting force and they are sort of equilibrium. So, this is the force picture in a gross sense for a fully developed flow. When it is an inclined pipe you replace this pressure with a piezo metric pressure that takes into account the height effect as well.

So, since those 2 a related these 2 are also related. And we have derived some expression that c f is nothing, but equal to 4 f. And we also derived that for a fully developed flow through a circular pipe f equal to 64 by Reynolds number. Or it might be the other a f equal to 4 c f right f equal to 4 c f right. Now and these have certain name. So, this c f is known as the fanning friction coefficient. And this f is known as Darcy's friction coefficient or friction factor. And of course, since they are related one may use either of these 2 either c f or 4 c f that is f. This Reynolds number is based on the diameter of the pipe and we have to remember that what are the conditions under which this is valid. So, this is valid with what, this is valid under the conditions of fully developed and laminar flows steady fully developed laminar Newtonian flow through a circular pipe.

So, these are the important assumptions that go behind. And the corresponding expression actually this delta p by rho g sorry I think I have used one extra L here sorry please correct it. Because this is expressed in terms of a head right this is this is a unit of length this is a unit of length. So, it should match f is a dimensionless quantity. So, there was one extra L please correct it. So, it is delta p rho g right. This is a unit of this is called as what head. So, this is sort of a head which represents a loss to overcome the frictional effect. This is also called as head loss or h f to overcome the friction the symbol that is commonly used in text is h f.

So, the right hand side you see L by d is a dimensionless number, these square by 2 g is a unit of length. So, f is a dimensionless coefficient which agrees with whatever developments that we had. So, this equation which relates that h f, with u is h f into h is equal to f into L by d into u average square by 2 g. This is known as Darcy Welsbach equation. You have to keep in mind that is f need not be a constant just for the example for fully developed flow through a pipe, f is itself a function of the Reynolds number. So, f is itself a function of the velocity. So, for the fully developed flow actually we derived the f equal to 64 by R e from the consideration that h f is 128 mu q L into rho g pi d to the power 4. This also be derived this known as the Hagen poiseuillis equation right.

Since we had deriving all those thing earlier I am just recapitulating instead of going through the derivations again. And keep in mind that q is the average velocity into pi d square by 4. So, it is one of the illusions that the h f for a fully developed laminar flow the head loss is proportional to what, it appears as a bit proportional to the square of the average velocity, but it is not true because there is also a velocity in f, and there is a one by average velocity dependence there. So, they get cancelled out with one u average remaining. So, actually h f of is proportional to the just the velocity not the square of the average velocity for fully developed laminar flow.

So, whenever we learn on equation. We should keep in mind. What are the important assumptions? So, what are the assumptions of the Hagen poiseuille equation you tell, fully developed steady laminar flow of a Newtonian fluid through a circular pipe. So, these are the important assumption. So, if it is a turbulent flow definitely these not valid. Now the other important couple of things that we would referred here, that regarding the diameter see all pipes need not be of circular section. So, pipes or channels maybe of different sections.

So, for that if you if you want to utilize some of this relationships approximately, for other sections you have to then replace this diameter with some equivalent diameter and that may not be very exact or accurate, but if you find out that if you have a section which is non circular in nature, but an equivalent diameter type of dimension for that section such that it is almost like an equivalent diameter of section which is circular section replacing that non circular section. Then that diameter is known as hydraulic diameter.

So, let us see what is the hydraulic diameter. See keep in mind that these are not very basic scientific entities. These are entities introduced by engineers for having some simplified analysis. So, the whole idea is see, what is the diameter if you considered for a circular section.

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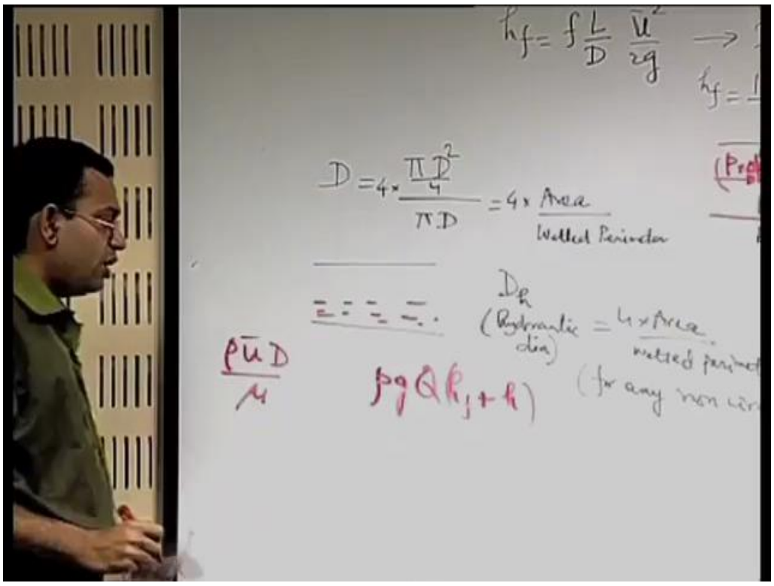

So, the one of the important things that the diameter is the function of the cross sectional area and the perimeter, so what is the cross sectional area, pi d square by 4. And the perimeter is pi d. So, you can see that we have to adjust it with a 4 to get back the diameter.

So, you can write this has 4 into area divided by not just a simple perimeter; we call it as weighted perimeter. Why weighted perimeter? Because it may be also possibility that we have a channel in which the liquid is not totally occupying the channel it is may be only partially occupying the channel. So, the total perimeter of section is not weighted by the liquid. Very classical example is if you have flow in can also or say these, a typically like open flows, where you have a channel let us say river bed. May be it is only partially being occupied by the liquid, or may be any other channel I mean through. So, a channel may be. So, you do not have always the full height occupied by the liquid.

So, then the weighted perimeter has to be taken as that part of the perimeter, which is weighted by the liquid. For the flow though circular pipe where the entire pipe section is occupied by the liquid the weighted perimeter and the actual perimeter at the same. Now form this we get a clue, what we get a clue, that if you have anything which is our non-circular section we can still define something called as a hydraulic diameter. Because for any section you have an area and you have an weighted perimeter. This is for any non-circular section. It is important to keep in mind that the whole idea of coming up with this is to draw an equivalence with the circular section nothing more than that.

So, if you have a non-circular section you just find out as some equivalent diameter, so that you can substitute that in this type of formula to get an estimation of the head loss. It is important to get such an estimation that estimation maybe erroneous. In fact, it will be erroneous because this is derived exactly for a circular cross section. So, for a non-circular cross section the deviation from that we will be will be quite significant. Still how engineers get read of that in accuracy, the thing is very simple. So, if you have their head loss. What do you do in the head loss? So, if you have a head loss you have supply that equivalent head and that equivalent head say supplied by a pump.

So, how do you calculate what is the power required by the pump. So, if rho is the density of the fluid, g is the acceleration due to gravity, q is the flow rate and h f is the head loss, the total head that the pump should provide to overcome this head loss the pump is not just overcome the head losses also he has to overcome a static elevation difference, say from ground floor it is translating water to the topmost floor. So, that head also has to be added, that head plus the head loss, so this head loss plus some extra head that is necessary for the lift. So, this is the total head that the pump should supply. This, the flow rate that the pump should supply and the rho into g. So, this gives the total power that the pump should have. So, from that now the pumps have certain efficiencies and motors have certain transmission efficiency is from the electrical form of energy to a mechanical form of energy. We look in to that in more details when we talk about the fluid machines.

So, this is the power that you require at the end that is the output power and if that is related to the input power with certain losses. So, you know that the efficiency is related to those losses. So, you can calculate the input power and based on that input power you may select the pump from the available once in the market. And you may always select something which is just having the power which is greater than what is coming out of these and that anyway is one of the possibilities that you have, because say this power comes out as 23.149 kilowatt.

Now, you will not get a readymade sort of a motor which will which will have that rating. So; obviously, whatever rating is there close to that and available in the market fitting your other requirements that type of arrangement you should have. So, you take something which is greater than this one, and which is available in the market. And when you get do that automatically that extra head that you consider it nullifies your ignorance of the actual head loss.

So, these what is practical engineering. It is sometimes primitive personally, I do not like that approach so much what is in this way it works in practice in many of the industries. So, we should not have any complaints on that. Now that is the first thing about the concept of the hydraulic diameter. So, to have it for a system where the cross section it is not exactly circular. The other important thing is the sanctity of the Reynolds number. See Reynolds number how did we define inertia force by viscous force right.

Now, you tell for a fully developed flow through a pipe what is inertia force. Inertia force is 0 because acceleration is 0 left hand side of the naiver stokes equation. So, inertia force is 0. Does it mean that the Reynolds number is 0? Because Reynolds number you are eventually writing in terms of rho v rho u average into d by mu. So, all of these have their existences right. You are giving an interpretation as inertia force by viscous force that is something else, but. So, this does not become 0. You have a diameter of the pipe you have a density of the fluid viscosity and average velocity none of this are tending to either 0 or infinity.

So, the question is then what is the sanctity of the definition of inertia force by viscous force. And that is where actually one has to understand that the understanding of inertia force by viscous force that an interpretation of Reynolds number is the very loose understanding. It is only not a bad understanding for like a beginner, but one has to understand that in a literal sense you need not always considered that interpretations. So, you may say that the fluid has some energy here, but that energy is utilize to overcome the friction, it is not utilized to accelerate the fluid, but if the same energy would have been utilized by the fluid to accelerate it, then whatever would have been that equivalent inertia force divided by viscous forces this one.

So, it is an equivalent situation where whatever energy the fluid is possessing, if it would have utilized it. So, only to accelerate the flow, then, that equivalent inertia force by viscous force not in a real, always in the actual case the inertia force by viscous force. So, that we have to understand. But at the same time just bit of quotients whenever you are interviewed by a component never say that Reynolds number is anything beyond inertia force by viscous force because everything is not for everybody. So, you for them it is good enough to say inertia force by viscous force, and it that ensures a good job for you should not worry about disturbing them with saying that well for fully developed flow the inertia force is 0 and so on.