Introduction to Fluid Mechanics
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Lecture – 50
Problems and Solutions

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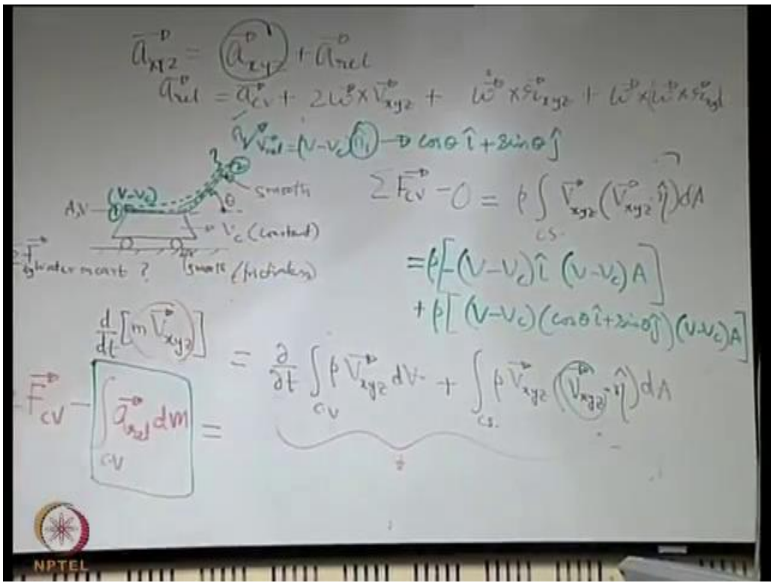

On a frictionless ground and there is a water jet which comes on it and which leaves the cart. The angle change is theta relative to the horizontal and the surface over which it is changing it is direction the water that is considered to be a smooth surface. And this is also considered to be smooth the friction is 0, frictionless.

Now, what we have seen earlier, we have seen that because of the impeachment of the water jet on the system, and because of it is change in direction there will be a force. And because of the force the cart will have a tendency of moving. So, we saw that one way of like restraining it from moving is like we will be making a pulley mass arrangement. So, that it is restraining from it is motion, but in general that will not be the case. So, in general it will be free to move and therefore, it will start moving. So, we will consider a first case, when it is moving, but moving with a uniform velocity. Just as just as a simple case we will then move on to more and more complicated cases when it may move with an acceleration.

So, first let us consider that it is moving with a uniform velocity. So, this cart is moving with a velocity V c towards the right. The water jet comes out with a from an area a with a velocity V and it gets deflected like this. And we are interested to find out what is the resultant force exerted by the water on the cart, by water on the cart what is that that is the question. So, only difference from the previous problem that we considered is that now it is moving, but this is constant. It is not moving with a variable velocity. So, let us try to write it relative to the moving reference frame. So, the left hand side resultant force on the control volume minus the relative acceleration, see what are the terms which are there. First is acceleration of the control volume. Here the control volume we attach the control volume on the cart. So, or that is we attach the sorry we attach the reference frame on the carts small x y z reference frame on the cart.

So, small x y z is moving with a uniform velocity V c. So, it is not having any acceleration linear acceleration. There is no angular motion of small x y z; that means, the a relative term is totally 0. So, that is 0, and the right hand side if you consider it to be a steady situation where the velocity is not changing with time, anyway the density we are considering to be constant. So, this term will go away and you are left with the last term that is rho integral of V small x y z into V small x y z dot eta d a. So, if you consider say the flow bounded is say this is the flow liquid film that we are considering. So, in this liquid film we know what is the velocity here V, because that is what is the velocity on which it is saying pinching from a nasal or someplace. Question is what is the velocity here. Again you may neglect the height here by considering that the kinetic energy effects are much more important.

Question is and a very big question is that if all the assumptions of the Bernoulli's equation are valid still, can you apply the Bernoulli's equation from here to here directly. Remember it is a moving reference frame, yes or no. See more and more these types of questions you answer yourself your fundamental understanding of that is clear. Many times it is it is commonly understood that Bernoulli's equation in is one of the easiest things in fluid mechanics. To me it is one of the toughest things in fluid mechanics. It is very easy formula at the end, but it is restrictions are not well understood many times. So, the question is can you directly use it keeping in view that all the assumptions that we have learnt otherwise are valid except, we are now in a dilemma why we are now in a dilemma because this is itself moving.

See although this is moving we are saved with one important thing this, is moving with a uniform velocity. So, this is still an inertial frame. So, we have seen that in Bernoulli's equation, all the quantities are relative like pressure is relative. You may express it relative to something. Potential energy is relative you may express it relative to some reference or data the height. Similarly, kinetic energy is also relative provided you are still using a inertial reference frame. So, here since this reference frame is inertial, you may have the velocity like the kinetic energy the relative velocity that is what is only of importance. So, what is the relative velocity here? Relative velocity here is V minus V c. And that relative velocity will be preserved because the kinetic energy is preserved even in a reference frame which is moving, but non accelerating.

So, if you apply the Bernoulli's equation here, then that V should be replaced by V minus V c because you are writing it relative to the reference frame. So, the thing is that when you write these expressions here, what will come out. So, there are 2 boundaries flow boundaries 1 and 2 first let us write for 1. So, for the flow boundary 1 minus rho first let us write plus minus sign will we will see inside, rho then what is V small x y z here let us write it in a vector form. So, that is V minus V c i cap, what is the remaining term again what assumption we are making. That it is a uniform velocity profile over the thickness that we are considering. So, this is as good as some relative velocity into area with a proper sign.

So; that means, relative velocity is V minus V c, the area is A the proper sign is negative. So, we put a minus here then for the out flow boundary, for the out flow boundary what you have? V small x y z it is V minus V c magnitude because kinetic energy only bothers about the magnitude. So, that is preserved means V minus V c is preserved, but it has changed it is direction. So, when it is coming out the relative velocity is V minus V c, with this normal direction say n 1. And what is this n 1, cos theta i, plus sin theta j. So, this is cos theta i plus sin theta j. So, keeping that in mind it will become V minus V c cos theta i, plus sin theta j. And the what about the V relative into A, that term positive A into V minus V c. Remaining what is easy you can say for it the x and y components by taking the coefficients of the i cap and j cap, and write what are the components of the forces. These are forces exerted by the cart on the fluid.

By Newton's third law the force exerted by fluid on the cart will be equal in magnitude of this one, but opposite in sense right. So, we have seen an example where the control volume is moving. It will not make us very happy because the expression that we have derived we are not able to exploit that fully. The entire relative acceleration term vanishes. So, let us look into some example where it does not get entirely vanished and then what happens.

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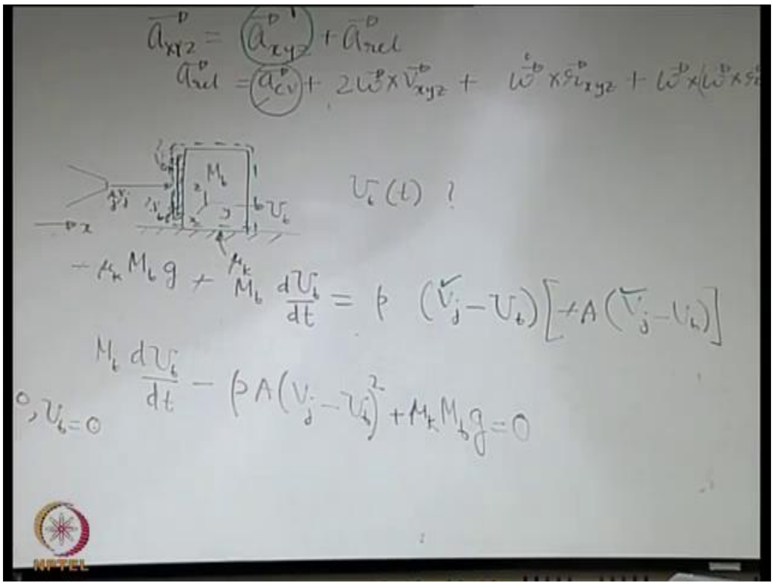

Let us say that, there is a block which is moving on a surface. Which may move on a surface from a nasal a water jet comes, strikes this one and gets deflected in the 2 sides. The area of the velocity jet area of the jet that comes out is given the velocity of the jet that comes out V j, that is also given A j V j, area of the jet and velocity of the jet.

The coefficient of kinetic friction between this surface and the block that is given and our objective is to find out an expression of if let us say U is the velocity of this block. Or let us give it a different name may be U b, for the U block then how this U block is changing with time. The mass of the block is given let us say M b is the mass of the block. Let us try to look into this problem with a framework of an arbitrarily moving reference frame. Now this is also a simple form of arbitrarily moving reference frame because these cannot having angular motion. I mean if this may have angular motion again that becomes a very interesting problem.

If you consider extend the dimensionalities of these and consider the top link possibilities. Because of the force which is acting on it, but if you just idealize it like a point mass type of then it is having just a translation, but it may have a variable transitional velocity. In general, it should be like that. Because before the jet is striking on it, it is stationary. When the jet has jet is striking on it, it will try to make it move. So, its motion from 0 velocity will come to a non 0 velocity over a finite time. So, there will be a finite acceleration.

So, it is expected in reality to be a accelerating reference frame if you choose your reference frame attached with the block that is small x y z attached with the block. Now let us consider a control volume may be let us say that we have the control volume, like this which contains the block and whatever water is adhering to the to this vertical plate.

One important thing we should understand, what is that important thing the thing is that, now let us say we now at least we are confident in one thing that we should be bothered about the relative velocity, when we are writing with respect to the reference frame small x y z say which is moving.

So, all the expressions that we write in the Reynolds transport theorem should correspond to the relative velocity that much we have seen. So, when we are bothered about the relative velocity. Now if you say that the velocity here is V j, there should be some relative velocity here as it is a water jet deflected into 2 parts, one will go to the top let us say it goes with the top as V t V top and another goes to the bottom with a velocity V bottom. Just 2 parts of the jet question is, is it, that V top in a relative sense is V j minus U b. Assuming that other assumptions other restrictions of the Bernoulli's equation are somehow satisfied. Answer is no because it is now an accelerating reference frame.

So, kinetic energy in an accelerating reference frame cannot be in a straightforward way brought out from the Bernoulli's equation as preserved. So, we do not know what are these. In the previous problem we could exploit the situation that the control volume was moving with a uniform velocity. Now it is not moving with a uniform velocity. So, we do not know actually what these, are even in a relative sense, but the good thing is that since these are vertical these will not have any consequence on the horizontal momentum transfer. So, our ignorance or lack of knowledge on these will not be magnified. This is a good thing for solving the problem, but it is important to know that it is very important ignorance, it is not so easy to tell what are these velocities because this is an accelerating reference frame.

Now, let us try to apply these equations for the resultant force on the control volume. So, for the resultant force on the control volume, let us write it only the x component because that will give rise to the variation of velocity because we do not expect that this block will be lifted from the ground because of this. So, now, if you write the equation the force, then what are the forces which are acting on these? So, if you draw the free body diagram of the control volume or may be free body diagram of the block plus water whatever is content there, you have one frictional force. So, in the vertical direction there is equilibrium of forces, because the block does not get out of the surface. So, you have n equal to M b into g.

Now, you have to keep in mind that you are not considering it just the block. Block plus some water, but the water mass may be neglected; it is very small as compared to the block mass. So, whatever mass of water is just like flowing in contact with this plate that water mass it is so, small as compared to this capital M b, that for weight or mass calculation aspect we will not consider the water weight. That is an assumption that we have to keep in mind. Otherwise whenever we write the total weight it should be the weight of the water in the control volume plus the weight of the weight of the block which is there, but we are neglecting that weight of the water in the control volume, which is a very valid assumption because it is a very small amount of water which is there that is very practical.

So, then the normal reaction will be capital M b into g because of the equilibrium along the y. So, the friction force will be minus mu k into M b into g. Then there is no other force along x. Then what is this correction term? This correction term here is important because you do not have the omega related things, but you have acceleration of the control volume. And here this integral will just become a relative into the mass because it is just moving like a rigid body. So, all points have same acceleration. So, this will be minus mass of the block times acceleration is d U block d t right.

So, it is clear why we could take this a relative out of the integral, because all points on the block are moving with same acceleration. And we could have been in trouble if there was lot of water in the control volume, because then for water we cannot have such a rigid body type of assumption. Then at each and every point the acceleration would be different, but we are neglecting the mass of water present in the control volume for writing this expression. The right hand side again there is a very important thing. The unsteady term is fundamentally not 0, but we may approximate it by 0 because again if you see the volume of the water that is present in the control volume is small. So, it is not because that V small x y z is small, but because of the volume of water which is there in the control volume is small, there is time rate of change with respect to time is also very small

So, these are certain important things. See we drop this term for solving the problem in exam fine. When you drop the term and solve the problem in the exam if you solve it correctly we have no way to deny you from the credit, but your conceptual understanding is not satisfied there. You must be convinced that why this term is dropped despite the fact it is an unsteady flow. It is not because that this velocity is 0 that is why you are doing it. Clearly it is coming out with some velocity, but just because you neglect the effect of that volume of water which is there inside and the time dependence associated. And the final term will be what? See here only the x direction is important. Because in the y direction whatever it goes the out flux there are outflow boundaries, but those are not along x.

So, this velocity if it has component along x then only that will remain for force calculation along x. So, that will be rho a into what? You can straight away write it what is V small x y z. So, V j minus U b, and then minus V j or if you want to put the minus sign properly then this times minus of A into that is the flow rate V j minus U b. So, once you have that expression, now it is quite convenient because it is a simple differential equation with variation of U b as a function of time. What are the constants in this expression the constants in this expression are this V j? So, may be let us write one more step. So, what you have here is M b d U b d t.

So, if you cancel all the minus signs in all both sides, minus rho a V j minus U b square, plus mu k M g, that is equal to 0. Keeping in mind that at time equal to 0 you have U b equal to 0. So, it is a straightforward differential equation. Even if you do not solve it you can find out that if when it attains a steady state what should be U b. When it attains a steady state see the initially the velocity is 0. Now because of the striking of the jet, it will be pushed to a velocity and it will come to a standstill it will come to a uniform velocity that is then it that change in velocity will not be there any more, when the d U b d t term will be 0. So, it will come to an to a limiting uniform velocity which then will be considered by equating these 2 terms.

So, it is possible to find out that what should be that equilibrium velocity. And that equilibrium velocity is quite obvious. It is the situation that occurs when the driving force on the block is just sufficient to overcome the kinetic friction. And then it is in equilibrium, but it takes some time for the driving force to be in equilibrium, with the kinetic friction till that time the velocity will be changing with time. And that how it changes with time is governed by this equation. So, when we have considered this problem one important thing that when we have considered this theory, as such one important assumption that we have made is that the fluid in the control volume is of constant mass, but it might. So, happen that the mass itself is a variable within the control volume. So, what we will do if the control volume is accelerating plus the mass within the control volume is a variable it is not a constant. So, we will see that in the next class next class we will take up the cases of variable mass in the control volume.

Thank you.