Introduction to Fluid Mechanics
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Lecture - 05
Concept of pressure in a fluid

Well let us continue with what we left last time that is we were discussing about the index notations for the stress tensor components and how to relate the stress tensor components with the traction vector. So, this is what we came up with.

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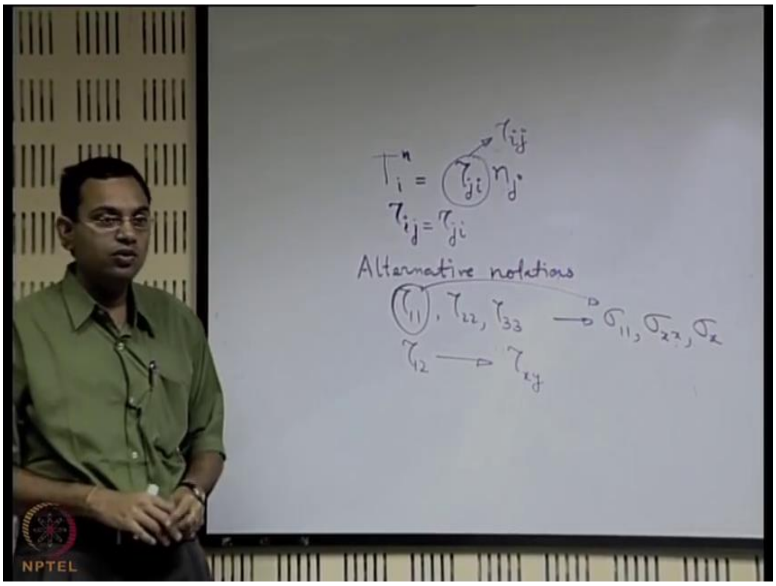

We are not writing the summation notation because as I told that if you are having a repeated index there is an invisible summation over that. So, this i is called a free index because it may be whatever 1 2 3 like that. So, being in the right hand side and in the left hand side in the right hand side only once this is like free, but j is a repeated index and therefore it is a dummy index, in place of j you could write k l m n whatever it makes no difference, but whatever is this i should also be the same.

So, this index and this index they should correspond and we also showed that tau i j equal to tau j i therefore, we can also write this as tau i j n j which is the corresponding form of the cautious theorem by taking the moment balance into account. We will also discuss about some of the alternative notations because in text books different textbooks have different notations and of course, the index notation is the cleanest one and I would say that most convenient one to use, but we will also look into the alternative ones. So, when you have this tau 11, tau 22 or tau 33, these are special components of the stress tensor, what do these represent? Normal components of the stress.

So, in many texts you will see the corresponding symbol as sigma 11 say corresponding to tau 11, sigma 11, sigma xx or sigma x equivalently. So, there should ideally be 2 indices, but since both are repeated sometimes texts omit the repeated 1 and just use 1 x, so sigma x. Perhaps this is the original symbol that you learn for the first time when you are learning the mechanics, but tau or sigma with 2 subscripts would be a more convenient and more fundamental notation.

So, if you have for example, tau 12 then in that x y notation you can also write it as tau x y. So, always remember 1 for x, 2 for y and 3 for z. So, that should correspond to any symbolic notation that you have in the book, whatever book you are following and whatever notation that we are going to use in the class. Of course, we will try to use all sorts of notations in different contexts. So, that you feel comfortable with the general notation that is used in different places.

So, what we sum up from the discussion on the stress tensor is that the stress tensor has different components we can clearly identify which are the normal components, the normal components are the components which are appearing in the diagonal of the corresponding matrix representation there are off diagonal components which are like so called shear components. Now we have to see that how the characteristic of the fluid is related to the components of the stress tensor and we will subsequently see that how they are related to the components of the stress tensor is going to be strongly dependent on one important property of the fluid which is the viscosity of the fluid.

So, we will come in to the properties of the fluid subsequently, but before going into the viscosity we will talk about one of the quantities or one of the properties which has a very important relationship with the stress tensor and that is nothing but the pressure of the fluid. So, we will discuss something about the pressure of the fluid.

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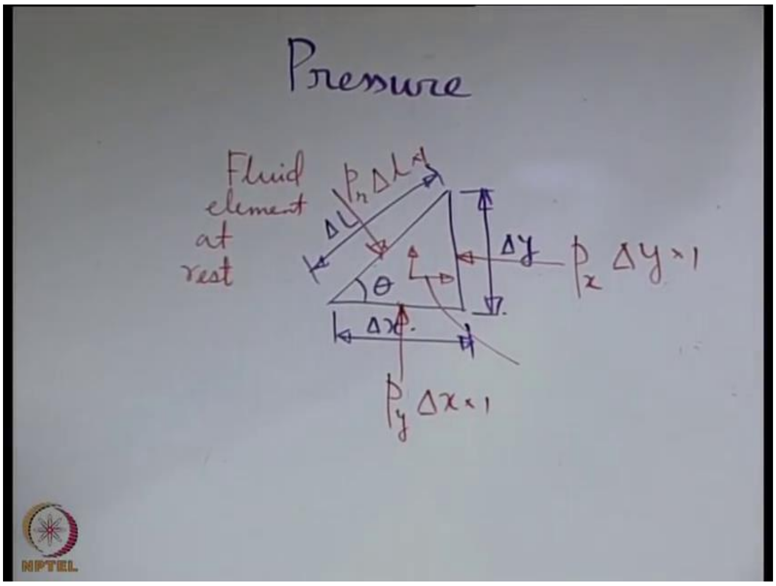

Whenever we start discussing about pressure of a fluid we generally start discussing about fluids at rest this somehow gives a misunderstanding to people that pressures are quantity is very relevant only to fluids at rest it is not so, as we discussed earlier that when a fluid is at rest there is no shear that is acting on it. So, only the normal component of force is acting on it and therefore, pressure becomes the only relevant surface force in that context and that is why it is easy to isolate all other effects and just focus on pressure if we are discussing about fluids at rest. If we are discussing about fluids in motion; obviously, pressure does not get irrelevant it may even get more and more relevant, but to begin with we will consider that we are discussing about say an element let us say which type element like this - again I mean you may consider it 3D version by considering uniform width perpendicular to the plane of the board, but let us just consider that it is a section of the wedge in the plane.

So, we are in; what we are interested to see is that what are the forces which may act on this weight shift fluid element when the fluid element is at rest. So, we are giving some names of these dimensions let us say this is delta L and maybe this angle is theta. Just like what we did earlier for a general fluid element which may be subjected to normal and shear component, here we are going to discuss about a fluid element where there is only normal component of force on the surface. The reason is quite clear we are assuming a fluid element at rest. So, there is no shear component. Keeping that in mind, this let us keep in mind this is a fluid element at rest.

We will later on see that even if the fluid element is not at rest, but internal deformation is negligible, but the fluid element is moving like a rigid body then also similar considerations may be valid, but for the time being we consider it at rest. So, we are identifying various forces which are acting on the surfaces and the volume the surface force and the body force. So, fundamentally we will treat any mechanics problem continuum mechanics problem, in terms of the forces as a collection of body forces and surface forces which will try to keep it in either static or dynamic equilibrium.

So, for surface forces we will first consider say the force on the surface. So, a normal force and pressure by nature is acting normally inward like the town pressure by that we qualitatively understand, intuitively understand it something which tries to compress the element. So, it tries to act inward to the surface. So, whatever is the outer fluid element that is trying to apply a normal reaction it is like a normal reaction, but the normal reaction is inward always. So, it is trying to pressing it, trying to press it so to say.

So, what will be this? This will be let us say that we do not know whether pressure will be different along x y or z. So, this is a pressure along y. So, we call it say p y, when we call it p y this is the force per unit area all of you know about that and this multiply by say delta x into with say 1 is the total force. Similarly and for this let us say that this surface has an orientation such that the direction normal is n, so we call it p with subscript n. Why we are keeping these subscripts? We are still not sure that pressure should be varying as we change the orientation. Fundamentally we should be unsure to begin with because we have seen that any force acting on the surface is likely to be strongly dependent on the orientation of the phase that is chosen. So, there is no reason to believe that pressure should not be where whether it should be or should not be that is what we are going to derive.

Now, this will be this into delta L into 1, there will be no shear component because it is a fluid at rest, there will be a body force which is acting on this. So, let us write the body force components say body force along x and body force along y. So, what will be the body force along x?

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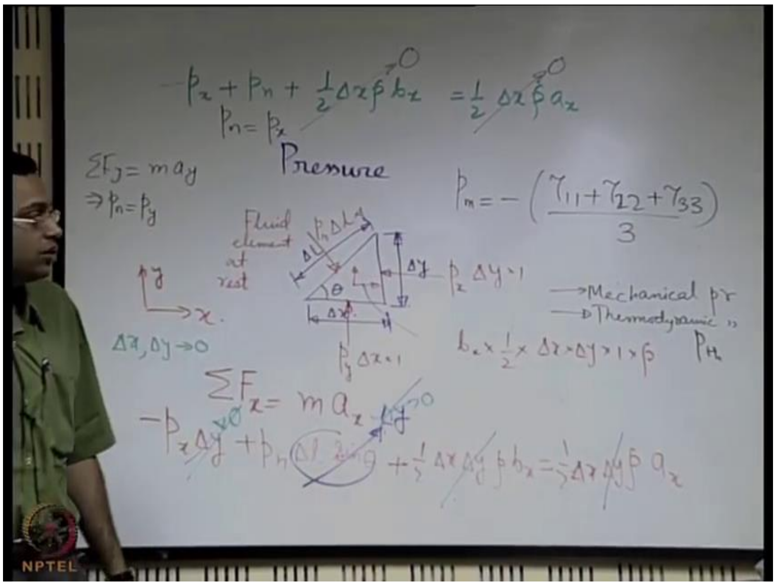

Let us again say that b 1 is the body force or say b x is the body force per unit mass acting along x. So, this multiplied by the mass of this. So, what is the mass of this? So, this is like a triangle, so half base into altitude that is half into delta x into delta y into the width that is the volume that multiplied by the density is the mass then the b x is the body force per unit mass. So, this is the total body force which acts along x.

Similarly, you have total body force that acts along y, I am not repeatedly writing it just to save time. So, we have written all the forces the resultant force, if we write the force equilibrium resultant force along x is equal to the mass of the fluid element times acceleration along x. Well, when we say acceleration along x your general idea would be that we are considering a fluid element at rest. So, fine, let us first consider fluid element at rest.

So, if we consider a fluid element at rest the right hand side is 0 obviously, but we will see that whether the effect of right hand side is there at all or not, for that first let us write that if left hand side. So, when you write the x component you will have minus p x in to delta y then the component of this p n in the direction of x. So, you have x axis like this, y axis like this. So, what will be the component of this p n along?

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So, p n into delta l into sin theta then plus there is a body force component along x, so half delta x delta y rho b x that is equal to mass of the fluid element. So delta x delta y half delta x delta y rho times acceleration along x. Of course, you can write delta l sin theta as delta y right. So, we will make this simplification in place of this we will write delta y.

Remember just like we in the last class try to find out expression for the stress tensor components and traction vector in terms of stress tensor components at a point. So, here also we are interested to do that. So, that we want to shrink the size of these as in the limit as delta x delta y all tending to 0. So, we will see what is the consequence of delta x delta y all tending to 0. So, if you take that limit then what happens, let us see that limit.

So, if you take that limit as delta y tending to 0 this will be 0. So, our limit is delta x delta y tending to 0, this will be 0 tending to 0, these are tending to 0. So, you can cancel delta y in all sides; that means, this you cancel, this you cancel, this you cancel, this you cancel. So, what is left? You have minus p x plus p n plus half delta x rho b x is equal to half delta x rho a x right. So, when you take the limit as delta x tends to 0 then; obviously, this term goes away and this term goes away. Therefore, you get p n equal to p x irrespective of whether this is accelerating or not this is a very very important concept because in your high school physics you perhaps have done the same thing, but assuming that it is at rest and that might create a misconception that this will not work if it is moving like a rigid body with an acceleration and if you see it does not matter even if it has a body force. So, even if there is a body force that is acting still this equality is valid.

Similarly, by considering the force equilibrium along the y direction it will follow that p n equal to p y. So, as a conclusion we can say that p n is equal to p x equal to p y which means that we are talking about a quantity which does not sense the direction. So, it is insensitive to the direction and it is acting always normal to the surface on which it is being evaluated. So, this quantity we call as pressure. So, since it is insensitive or de action insensitive we can just call it with without any index and therefore, unlike the general stress tensor it is not a tensor because if a second order tensor requires 2 indices for its specification you should remember, whereas this requires no index for its specification. So, of course, it is a tensor, but tensor of order 0. So, it is more easily termed as a scalar.

So, you can see that although force and sorry stress and pressure both are expressed in terms of force per unit area mathematically and fundamentally their characteristics are somewhat different, and that we have to clearly remember whenever we are discussing about these 2 related terms. There are more inward concepts on pressure which we will come across subsequently one important concept is that there are 2 terminologies involved - one is called as mechanical pressure another is called as thermodynamic pressure.

So, first we start with the mechanical pressure, we briefly discuss about the concept we do not elaborate, but the certain concept we will try to understand. So, when we say mechanical pressure what we mean? What we mean is that see if you are thinking about the normal stress components of the stress tensor you have tau 11, tau 22, tau 33, and if you feel that this p is a representative of the normal stress states of the element that is chosen because we are not considering the shear stress effect of normal stress we are representing by p; that means, somehow there is likely to be a relationship between these tau 11, tau 22, tau 33, and p. The relationship may not be straight forward, but since p is same in all directions we can say that there may be a component of or a part of this tau 11, tau 22, tau 33, which is like direction insensitive and that we may say just for the sake of definition of mechanical pressure. So, it is a basic definition that the mechanical pressure is defined as the average arithmetic average of the 3 normal components of stresses with a minus sign, minus sign is straight forward to understand because the positive sign convention of these were outwards from the surface whereas, the pressure by nature is inverse to the surface, so to adjust that this minus sign is there.

So, this is also called as something like a hydrostatic component of stress; that means, you are assuming that it is like it is representative of a state of stress where it is represented by a quantity pressure which acts equally from all direction. So, it is as if like a fluid under rest that we are considering. So, any state of stress which deviates from this hydrostatic part is known as deviatoric part. So, that is what is something which deviates from a hydrostatic state of stress. We will come in to the details of these concepts later on, whenever we are going to discuss about the equations of motion for viscous flows, but this is just an elementary definition of mechanical pressure.

Now, when we talk about pressure actually we are not really referring to this mechanical pressure always fundamentally because whenever we talk about pressure think about, so you are talking about pressure for gases you always relate pressure with density and temperature through equation of state. So, pressure from a thermodynamic point of view is something which satisfies the equation of state through the density and temperature, for an ideal gas it is very simple, for non ideal gases it may be a more complicated equation of state, but still equation of state is something which relates pressure density and temperature in some mathematical form.

So, thermodynamic pressure is that pressure p which will satisfy the equation of state. Now the question is, is the mechanical pressure going to be equal to thermodynamic pressure or not. So, what is the fundamental mechanism that will dictate that whether they are equal or not, say there is a bubble - inside the bubble there is a particular pressure density and temperature, now you are making the bubble to fluctuate its frequency of formation that is the bubble is changing its state very fast. So, what will happen? There will be a particular thermodynamic pressure density temperature; suddenly you are changing it state to a new state, new pressure density temperature. So, in that way say the bubble is suddenly expanding and contracting, expanding and contracting like that and it is doing it very fast.

Once it is doing very fast the change is not so easily adjusted, so the system requires at least a threshold time to adapt itself to the change and make sure that it has its mechanical pressure which is like a kind of pressure that acts equally from all directions same as what is dictated by thermodynamic state. So, thermodynamic state is a change that imposes a kind of disturbance to the system, system requires a time to attain equilibrium. So, that it will eventually have mechanical pressure equal to thermodynamic pressure and therefore, we generally do not distinguish between mechanical and thermodynamic pressure we say that is just a pressure.

But if the change is so fast that the system at intermediate states does not get enough opportunity to attain equilibrium, so that whatever change is in the thermodynamic state the system does not get enough opportunity to adjust to those subsequent changes and then in such cases you will not have mechanical pressure equal to thermodynamic pressure, but those are very rare cases. So, for most of the practical engineering applications the changes are such that those changes will be adjusted or adapted to by the system in a way that you will have mechanical pressure equal to thermodynamic pressure and therefore, whenever we will be talking about pressure we will not be distinguishing the mechanical and the thermodynamic pressure, we will be just calling it as pressure. So, that is how we will be going about it.