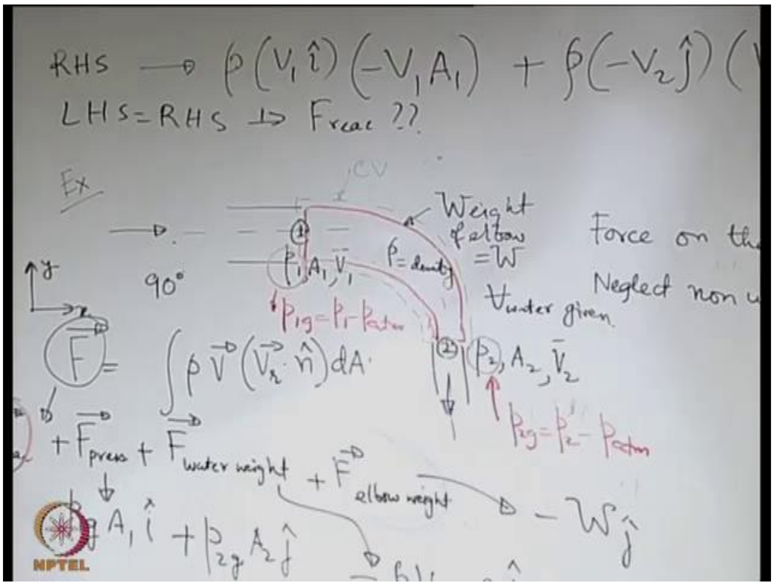
Introduction to Fluid Mechanics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 48
Problems and Solutions

Now, let us look into another problem. Let us say that there is a pipe like this. It has to be fitted with another pipe which is like this. I am trying to give you an industrial perspective of the problem. Rather than just stating the problem as it is.

(Refer Slide Time: 00:36)



So, the problem is you have to connect these pipe lines. It is very common that no matter whatever plant you visit you will see that there are lots of pipelines. And pipelines are not always straight, because they have to connect different systems and there are space constraints. So, the pipeline has to be bent many times. So, there must be some fitting which connects this pipe to this pipe there are 2 things which are happened, one is the direction has changed may be the axis of these are oriented at an angle 90 degree.

It is not required or necessary that it has to be 90 degrees, but let us say that the angle between these 2 is 90 degrees plus there is a reduction in cross sectional area. That also is not a mass sometimes cross sectional area may remain the same or may even may increase, but one has to just have a fitting to fit that. And that fitting in industry is known as an elbow. So, what an elbow does, it basically tries to have a fitting like this, to fit or match with pipelines of different orientations and different sections.

If the angle between these 2 axis is 90 degrees, it is called as a 90 degree elbow like that. So, if you we assume that this angle is 90 degrees we call it 90 degrees. Now this elbow it cannot be free in air because we will see that there is a lot of force that is being exerted on this elbow because of the change of linear momentum of the water that is entering and living. And because of that there is a force on the elbow and it has to be supported. So, it must be supported with a support that provides some necessary reaction force which balances those forces exerted by the water on the elbows. So, that it is in equilibrium.

Otherwise it might have a tendency to move to get deflected from it is equilibrium configuration. And that will disturb the entire stability of the structure. So, when you design a structure you have to be careful of what are the support forces that need to be sustained in by that structure. For that the support force has to basically balance the force exerted by the fluid on the structure. So, we have to know what is the force exerted by the water on the pipe bend. So, that is our objective of solving this problem. That is, we want to know that what is the force exerted on the elbow. What are the data given let us try to list that force on the elbow? Let us say that you have points 1 and 2. Let us call this as sections 1 and 2, so at the section 1. So, there is a equivalent pressure p 1 which is given area of cross section is given. At the section to you have p 2 and A 2, these are given. Let us say that the velocity profile is uniform. If it is not uniform at least you know what is the average velocity. So, you know the average velocity at section 1, you know the average velocity at section 2.

Where from you know the average velocities experimentally, what you can always find out it what is the flow rate. And if you know the area of cross section say it is a circular one you know the diameters. So, you know the area of cross section. So, A into the average velocity is the flow rate from that you can find out the average velocity. If you neglect the viscous affects then the average velocity is same as the velocity at a point at any point. So, you may neglect that let us say that neglect non uniformity in velocity profile. What else is given that what is the weight of the elbow say, it is equal to W. Weight of the elbow it is a solid. So, it has it is own weight. So, we are considering that weight and we are given the density of the water which is their inside.

Rho is the density. And what else we require? Let us say the angle between this inlet and the outlet. So, the water is entering like this it is living like this, the angle is 90 degrees. If it is not 90 degrees then also like if it is inclined, it will have it is horizontal and vertical components of the flux velocity and so on. Now we are interested to write the expression for the force component along x and force component along y. Sometimes you see that the pressure at 1 is given, but pressure at 2 may not be given. But if you assume an in visit flow and you connect a streamline say from the centre of the section 1 to centre of the section 2, you can use the Bernoulli’s equation to find out what is the pressure, and if you assume that the pressure is uniformly distributed over the section, then that will be the pressure throughout the section 2.

You have to keep in mind that what can create non-uniformity in pressure. So, if you have pressure at the centre line say at p 1 what can make it deviate if you go to a different location in the cross section curvature of the streamlines? So, if streamlines are almost parallel to each other than the change in pressure is very small or negligible. So, then we are assuming that streamline here and here are almost parallel to each other. See if you take that on the bend that is not valid. So, we are considering the section 2 which is which has actually cross the curvature part of the elbow. This is an assumption; in reality the piece may be short. So, that may not be a very good assumption, but this is what we are assuming otherwise you have to also consider a non-uniformity in pressure across the section, which itself adds to the complexity. We are not going in to the complexities, but I am trying to highlight the complexities because these are important this may be important in some realistic conditions.

So, 2 important complexities may be non-uniformity of velocity over each section and non-uniformity of pressure over each section. And when non uniformity of velocity over each section is occurring then; that means, that and it is always there until and unless it is a highly turbulent flow and the velocity profile due to high mixing is almost uniform. Otherwise if there is a velocity profile it gives an important understanding that yes viscous effect is important. And when viscous effects are important you cannot apply Bernoulli’s equation along the streamline between 1 and 2.

Still you can use A 1 V 1 average equal to A 2 V 2 average that is a conservation of mass that does not depend on how viscous forces are occurring or not, but you cannot really relate the pressure at 1 with pressure at 2 using the Bernoulli’s equation. One has to solve the viscous flow equation to find out that. Now when you write the resultant force along x, let us try to write the resultant force in a vector form. So, we are using the Reynolds transport theorem the right hand side the first term due to unsteadiness that is 0. Next term is integral of rho v.

When you are writing this force f, what is this force f? Let us now write what are the constituents of these. One force is the force exerted by the elbow on the water. So, that is the f reaction. Then what other force is there? Force due to pressure is there, plus force due to 2 weights, one is the weight of the elbow itself another is the weight of the water which is instantaneously there within the elbow. So, f due to water weight, and plus f due to elbow weight. So, let us try to write this expression of these forces of course, this is what you are interested to find out every action. So, this is an unknown force due to pressure. How do you find out what is the force due to pressure? What is the force due to pressure resultant force due to pressure on the control volume?

P 1 A 1 for section 1 along x p 2 A 2 for section 2 along y like that. It is not like that. I mentioned it earlier why, when you have a pressure distribution on a surface, you have to consider the force due to gauge pressure only because atmospheric pressure is there from all sides and that is nullifying the total force when it is integrated over a closed contour. So, when you are writing the force due to pressure it should be the net force because of the pressure over and above the atmospheric pressure. So, to calculate the force p 1 has to be converted into the gauge pressure at 1. So, p 1 gauge has to be evaluated that is p 1 minus p atmosphere.

Similarly, this has to be converted into gauge pressure. These are subtle, but very important things these are places where like in most of the cases students will make mistakes of course, if you practice enough problems he will never make such a mistake, but general tendency is like before the exam you just look into worked out examples. So, when then these things are not highlighted, you just look into the gross formula, but these are very important things that you have to keep in mind.

Do not just take it as a formula keep in mind that why it should be. So, that why you have to take the gauge pressure for evaluation of the force. So, force due to the pressure, what should be the corresponding expression? P 1 gauge into A 1 that is the net force due to pressure at section 1. So, the in a vector notation we give call it this i cap, then plus p 2 gauge into A 2 j cap. You have to keep in mind that pressure is always into the surface, in whatever face you are considering pressure is always acting towards that.

Then force due to the water weight. What is that? It is not impossible to calculate what is the volume of this given this contour. Let us say the volume of the water is given, that is the volume of the elbow basically. So, if the if the volume of the water is given then, it is the what is the mass rho into volume of the water that into g is the weight, g is acting along negative y then this is minus of this j. And the elbow weight what is the elbow weight? Minus W j and the right hand side the integral of when you are considering this integral, first is what are the surfaces across which fluid is flowing 1 and 2. What is the control volume that we have taken? Since we have represented the elbow weight we have considered the elbow also as a part of the control volume. So, the control volume let us draw the control volume. So, till you express explicitly that what is the force what is the resultant force that you are having? It may not be so, straight forward to say that what is the resultant like what is the control volume that you have taken so.

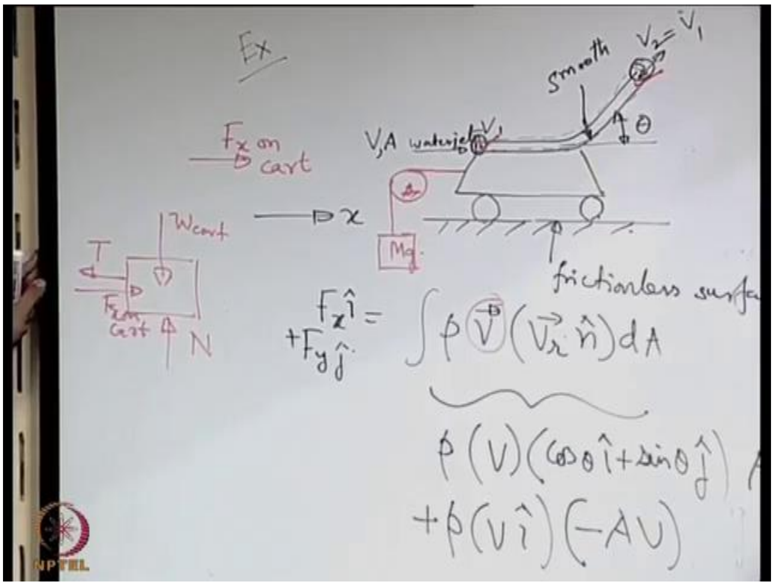
If you say, if you take this as a control volume; that means, this excludes the elbow and then if elbow weight is not there, but ones if elbow weight is there you have basically taken including the elbow. So, it is elbow plus the water that you have taken the control volume. So, the question is then when you have taken this as the control volume the outer one as the control volume say, which includes both the elbow and the water the question is, then what is this force f reaction. This is provided by it is now not provided by elbow by the water.

So, what is this? So, it considers there is a support which is their outside which exerts the force on this elbow plus water system. So, there is some support which is their which is not drawn in the figure, but it is highlighting that support. So, now, for that particular control volume, we are having how many inlet us and how many outlet us we have one inlet and outlet and let us write that. So, the right hand side first let us write for the section 1. For the section 1 if you assume a uniform velocity profile then like this entire integral will be based on the velocity say V 1, which is over uniform over the section 1. So, rho then for V 1 it is V 1 i cap, and it and then then into V 1 A 1. So, with what sign plus or minus right. You can clearly see that if there is a velocity variation along y then this expression is not valid. Then you have to integrate the velocity profile and that would give a net momentum flux, this is this is like a momentum flux. So, the net momentum flux will be different and one if one assumes a uniform velocity profile and it is really not. So, then that is an error and want us to adjust the error with some momentum correction factor maybe.

But here because of uniform profile assumption that such a correction is not necessary. Otherwise if the velocity profile is given to you can integrate it to get this expression. Then there is no correction factor necessary. Then for the surface 2, plus rho what is the velocity at section at 2 let us say V 2. It is directed along which direction minus y. So, minus V 2 j into this is plus. So, the left hand side is equal to the right hand side. And that will give you what are the components of the reaction force. This is the force exerted by the support on this system. So, it should provide an equal and opposite force on the support. And the support must be good enough to sustain that force.

So, if it is unsupported then because of some resultant force it with the elbow may start moving.

(Refer Slide Time: 20:02)



Let us say there is a cart like this. And water jet striking on the cart and it is changing it is direction. Let us say this angle is theta. Let us say that the velocity of the water jet is V. And the corresponding area over with the jet is moving here is A cross section area and let us assume that this is smooth. So, if this is smooth; that means, there is no friction that the fluid is encountering as it is moving along the cart, only it is direction is getting changed. The first question that we will like to answer is that will it be possible to keep the cart stationery, if such a water jet falls on it and changes it is direction. For simplicity let us assume that this is a frictionless surface. And maybe assume that the cart is having a particular weight, but that is not of great concern for us, because we are interested to considered the motion along x whether there will be any motion along x or not. First of all, let us say that what do you expect to be the velocity at which the water leaves the cart.

Say it enters the cart at one and leaves the cart at 2. What is a velocity that you expect? If the velocity here is say V 1 which is equal to V, what is the velocity at 2? There what are the; first of all if you consider a streamline that connects some point at the inlet with some point at the outlet, then could you apply the Bernoulli's equation along that streamline? If it is the first question is if it is in visit flow, then like that that is the first question that you would like to ask. So, assume that it is an in visit flow. If it is an in visit flow yes, provided other conditions are satisfied. What are those? You have density as constant.

Student: Steady flow.

Steady flow; obviously, although unsteady Watson or Bernoulli's equation is also there, but let us assume that it is a steady flow. So, if this is a smooth one. We and the water this is this is quite thin and this this because of the smoothness, there is no there is no such wall roughness effect that is propagated into the fluid.

So, it is as if like a frictionless flow. Although you this is this is a great idealization. In reality the effect of the solid boundary will always be propagated into the fluid, and in in all cases it is it is likely to give a viscous resistance. But here we are just idealizing it by too much and assuming that that effect is not there. If that effect is not there if the velocity here is V 1 the velocity V 2 should be equal to V 1, provided that the difference in height between 1 and 2 is neglected. So, we are neglecting the Z 2 minus Z 1 that is neglected. It is really a very small height and the corresponding potential energy change is insignificant as compared to the kinetic energies of the jets, see in engineering when we say that we are neglecting something, there is a very important thing that we should keep in mind we are not actually neglecting potential energy. We are neglecting the change in potential energy, and that change itself may not be negligible in an absolute sense. What we are banking on is that the jet is falling on with a very high kinetic energy, with respect to those kinetic energies the potential energy effect is negligible.

Not that it is always in an absolute sense negligible. And regarding the pressure both are exposed to atmospheric equation. So, the pressure is like p 1 and p 2 are same. So, if the Z 2 minus Z 1 is neglected and if we may apply the Bernoulli's equation with all the assumption satisfied then you have V 1 equal to V 2. In general, if there is a friction here V 2 will be somewhat less than V 1, but because of the frictionless nature V 2 is V 1. And then you can apply the continuity equation then A 2 also must be same as A 1.

Now let us say that we are interested to find out what is the resultant force along x, because that is what is going to make it move maybe. So, the resultant force on x resultant force along x what is that. So, you have 2 sections basically. So, you have one section like this, where the fluid is entering and you have another section at 2 where the fluid is leaving. These are only the 2 flow sections. Where do you choose your control volume, see seems there is no friction on the ground it will not be any difference any different if you include that like all the structural part of the cart and exclude the structural part of the cart for obtaining the force along x, definitely for force along y it will be mattering, but not force along x. So, for force along x, the right hand side first the unsteady term is 0, and then integral of rho V V r dot n d a. So, this is like f x i plus f y j. Because this is in a vector form, now let us write try to write it in terms of a scalar components. At the section 2 what is V? V has a magnitude V what is the direction, cos theta i plus sin theta j right.

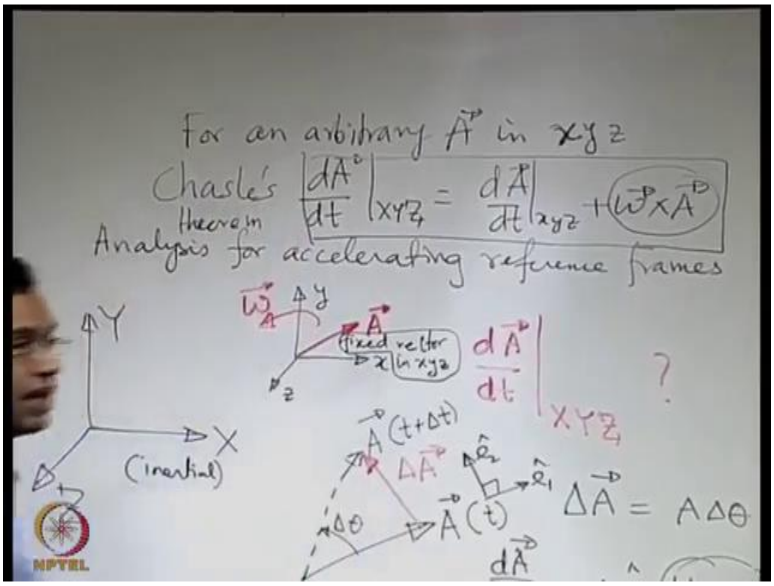
So, that is the v in the vector form and then the remaining is A into V, that is integral of V dot n d A what for the section 1 what is the velocity V i and minus A V. So, what is the force along x, you can find out only the x component of that that is rho V rho A V square into cos theta minus 1. This is the force exerted by the solid structure on the fluid right. So, if you consider say a control volume like this. We just encompass the fluid jet. So, this is the force exerted by the cart on the fluid. The fluid exerts an equal and opposite force on the cart. So, this force is positive or negative. This is negative this is along minus x. So, force exerted by water on the cart is along plus x. So, you have a plus f x that is there on the cart. And that is quite obvious even if you do not go to go through the mathematics; if there is a jet striking like this it should exert a force along the x direction. So, there is a f x on the cart. So, the cart under this force may try to move and if this is a frictionless surface it will move always. If there is a friction the static friction may just balance it and keep it in equilibrium, but if it is not then it has to move.

If it moves, the question is that then is this consideration valid that is like here we are having to use the relative velocity, but we do not know what is a velocity of the cart. So, how you should go about it, that is a first question. Second is whether this velocity then we have to use the absolute velocity or the relative velocity. So, these are the questions that we will like to address in in a subsequent theoretical development, where we consider also the moving reference frames. Till now we have considered only the stationary reference frames, but in the jurisdiction of stationary reference frame, if you have to consider it you have to consider somehow that this is stationary. Now how can you design a system such that this remains stationary? There could be many ways. Let us say let me give an alter one alternative when you say that whether it is good alternative or not. Let us say we have a pulley like this. And let us say there is a weight M g which is there this, pulley is in supported like this. Is it acceptable?

Will it work? No or yes? It depends on what is this weight. And that you can design exactly because you know what is the exact magnitude of the force. So, if you draw the free body diagram of the cart, what are the forces that you will see? You will see a tension in the string and you will see a force f x exerted by the water on cart. So, when you have these 2 of course, the other y component is there. So, y component you have the weight of the cart, then you have a normal reaction like that, but for us interesting is the x component and if you want to keep it in equilibrium you have to balance t with f x on the cart. And if you consider it to be all those idealistic situations, that it is a frictionless pulley and then what you get is that you get this tension same as the M g. So, this in turn from the mass pulley system is equal to the M g. So, you know that what has to balance what. So, you can put the correct mass here to keep it in equilibrium.

But you can clearly see that this is a force full arrangement to keep it in equilibrium, but in general because of this forces it will not be in equilibrium. And when it is not in equilibrium with this forces it might have a velocity that velocity itself might change with time. So, it might have a situation when the reference frame which may be attached to the cart itself, is moving and moving with arbitrary velocity or arbitrary acceleration. So, we have to also be equipped with an analytical ability by which we can encounter such situations, that is situations where you can encounter accelerating reference frames in general. When we say an accelerating reference frame, we mean accelerating frame reference frame in all respects; that means, it could be linearly accelerating it might have an angular velocity, because of which it has its original acceleration. So, we have to next go for an analysis for accelerating reference frames.

(Refer Slide Time: 34:02)



So, we will use certain nomenclature. We will consider an axis say capital X capital Y capital Z for an inertial reference frame. And small x small y small z, reference frame as an arbitrary, it maybe inertial maybe non inertial, but it is a moving reference frame. If it is moving with an acceleration, then it is non inertial. If it is moving with a uniform velocity, it is still inertial.

What we are interested to find out is that if we have a vector say A here in this reference frame. And let us say that this reference frame is moving with an arbitrary angular velocity omega, then what is the derivative of this vector with respect to the inertial reference frame? Can you show it if I ask you to show it? How do you show it? Say you have a vector A which is there in a reference frame that is rotating with an angular velocity omega. Let us say that the angular velocity is such that the rotation is taking place in the plane of the board. The rotation will take place in some plane, what is that plane? The plane is perpendicular to the axis of rotation. It might not be x y plane or y z plane like that, but it is some plane. So, in that plane this vector a is rotating. So, when it is rotating, it comes to what state? It comes to a new location say this is at time t, it is at time t plus delta t.

What we are keeping in mind? We are keeping in mind this is it is a fixed vector in a moving reference frame. So, this is a fixed vector in x y z. That we have to keep in mind, it is not any arbitrary vector. That means, if you were sitting on this you do not see any change in the at least in the length. And if you are outside although it is same in length, but because the reference frame is rotating this also rotates. Let us say that it traverses an angle delta theta over this time delta t. So, what is the change in the vector? The change in the vector is this delta A right. So, what is this delta A? For small delta t the delta theta is small. So, this is just like an arc of a circle. So, delta A in terms of magnitude is what? A delta theta in terms of A vector you have to give it a proper direction and sense.

So, if let us say that this is a is in a direction of the e 1, then it should be a direction which is normal to e 1 say e 2. If you want to find out what is d A d t, we if we are not mentioning any subscript capital X Y Z; that means, it we are talking about inertial, then it is basically we are dividing this by delta t and taking the limit as delta t tends to 0. So, a e 2 into limit as delta t tends to 0, delta theta by delta t which is nothing, but the magnitude of the angular velocity. And what is omega cross A, omega is what is a omega vector, it is omega scalar times a unit vector e 3 which is perpendicular to the plane of the board. So, you may take e 1 like x e 2 like y and e 3 like Z, just like that. This cross A is a e 1. So, it is omega A e 2, e 1, e 2, e 3 form orthogonal basis just like x i j k. So, you can write that d A d t, capital X Y Z is equal to omega cross A, but this is only for a vector A which is fixed in that small x y z reference frame.

If it is moving in a small x y z reference frame, then that velocity also has to be added with this. So, in general for a for an arbitrary vector A in small x y z, you have d A d t capital X Y Z is equal to d a d t small x y z plus omega cross A. So, this change is failed even if A is fixed, but if A is moving relative to small x y z this is an additional change. So, that is the total change. And this you know from your earlier studies that is known as chasles theorem. So, we will take up from this and try to write an equation of linear momentum conservation for a control volume which is having arbitrary motion. It may have angular motion it may have linear motion it may be a non-accelerating reference. It may be accelerating reference frame in general. So, we will take that up in the next class.

Thank you.