Introduction to Fluid Mechanics
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Lecture – 47
Problems and Solutions

Last time we were discussing about the linear momentum conservation and we were looking into some examples to illustrate the use of the Reynolds transport theorem for working out problems related to that.

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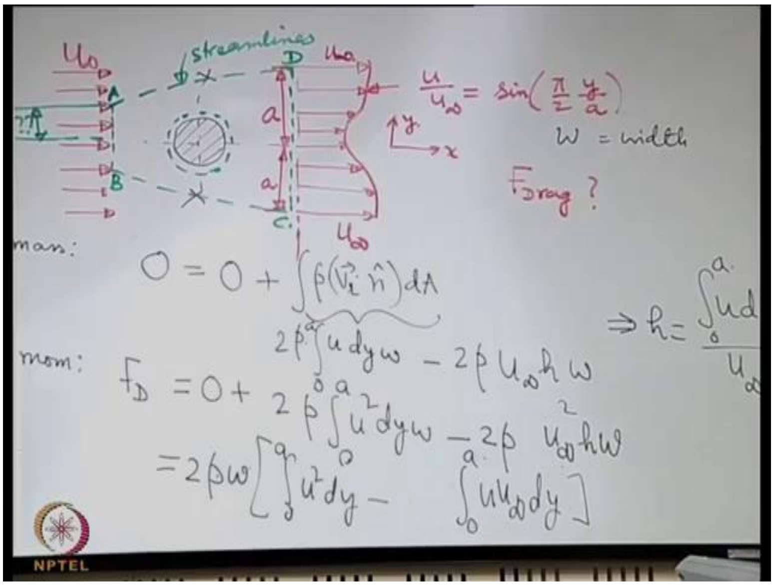

Now, we will continue with some more examples. Let us take an example where let us say you have a body. Say a solid body of whatever shape may be circular shape, if you want it to be. And fluid is flowing it is coming from a free stream, with a velocity uniform velocity say u infinity. And because of the presence of the solid the velocity is disturbed, and if you go a little bit away from the solid and if you draw the velocity profile, say the velocity profile is obtained something like, let us make a sketch of how the velocity profile is there.

Let us say that the velocity profile varies in this way. In one of later chapters we will see that what are the factors that will determine that what should be this velocity profile. Or what should be the nature of variation of this velocity profile, but for the time being let us say that this is the qualitative sketch of how the velocity profile varies. Assume that it is totally symmetric with respect to the centre line, and the velocity profile is such that like at the middle, that if you draw it totally in a symmetric manner, at the middle it is like a minimum, and then it increases in both sides comes to almost u infinity at a given height.

Let us say that this height at which it comes to almost u infinity is a. And let us say this velocity profile is given in terms of the x and the y coordinates. Let us say that x is the axial direction and y is the transverse direction. And the velocity profile say is given by u by u infinity, say is equal to sin. This is given, not that it has to be like this. This is just an example. We are trying to satisfy the condition that when y equal to a u equal to u infinity that is how this velocity profile is there. So, the question is that what is the total drag force on the solid body exerted by the water or the fluid.

Let us assume that the density of the fluid is rho, and we have to find out based on these dimensions. So, how do we go about this? We walked out of a very similar problem, when we were considering flow over a flat plate. This is not flow over a flat plate, this is flow past some body of a arbitrary contour, but the policy or the philosophy remains the same. So, we have to basically find out or we have to identify a control volume and see what is the net force on the control volume. So, to identify a control volume, see what part of the control volume will have some inlet and some outlet.

So, one inlet is this one which is straight forward that the flow is entering. Outlet this is straight forward and you can see that outlet is interesting only up to y equal to a or y equal to minus a because beyond that the velocity is uniform. So, if you take a control volume, say something like this, where we consider one inflow boundary one out flow boundary and across other boundaries we do not want any flow. So, what should be the edges of the other boundaries? There should be streamlines. So, that there is no flow across those. So, let us say that we consider a streamline like this, I mean these are not horizontal lines these are inclined ones. So, let us magnify those a little bit to represent that. Let us say that this is one extreme streamline this is another extreme streamline. So, these are streamlines.

Keep in mind that these it is not necessary to choose a control volume which contains streamlines, but only elegance it gives to us is that we do not have to bother about the cross flows, but if we just take say some horizontal lines at the top and the bottom and constitute say a rectangular piece at the control volume then there will flow across that and one has to make calculations related to that. So, it is just a matter of convenience for choosing the control volume.

Maybe you consider only the water in the control volume. So, you exclude the solid part. Now let us write the expressions for the mass and the linear momentum conservation for the control volume. So, one thing remains unknown is that what is this height h because we have constructed a streamline from the right from the edge of this layer where u becomes u infinity. This is not a boundary layer we will discuss later what is the difference between this and the boundary layer as such. So, when we have the streamline starting from the edge, streamline will end up here at some arbitrary point which is not known to us. So, we have to find that out.

So, let us say that we mark the edges as A B C D. And to try to identify that what are the expressions for the conservation of mass. So, if you write the conservation of mass, you have the d m d t for the system, I am just writing it straight away without going for the explanation of the different terms because we have already encountered that. So, d m d t for the system in left hand side is 0, then we assume that it is a steady flow not changing with times. So, the right hand side you have first term 0 and then the term integral of row v r dot n d a. So, that term we have to basically write. So, what will be the corresponding expression here? So, you have one out flow and one inflow. So, there is no flow across these 2 that is the advantage of taking the streamline. So, what will be this term for the out flow? Say rho what is d a d a you can take as d y into the width.

Let us say w is the width perpendicular to the plane of the figure. That is the width of this body. So, rho into u d y into w, that is the total when integrated from say y equal minus a to plus a; that means, 2 into y equal to 0 to a, that is the out flow.

And then inflow that is there, that is with a minus sign because velocity is along the positive x outward normal is along the negative x. So, what will be that? Minus again 2 rho u infinity into h into w right, from here you get what is h, because that is there that unknown that you have find out mass conservation tells us how to find out that. So, h equals to integral u d y 0 to a divided by u infinity. Next, conservation of linear momentum, conservation of linear momentum what it will tell us, the resultant force which is acting on the control volume.

So, what is the resultant force that acts on the control volume? There is a pressure distribution, but because it is there it is open to the ambient the pressure is same on all around. So, net effect of pressure distribution may be 0, but in reality it may be. So, that here the pressure is not same as the atmospheric pressure, but something which is different from that, but let us assume that there is a uniform pressure distribution, just for simplicity. Then the only force that remains along x. So, let us say that we are bothered about the linear momentum transport along x. The only force that remains important is the drag force, is equal to again the unsteady term is not there in the right hand side first and then, basically these things will be multiplied with another u. So, it will be plus 2 rho integral of u square d y w then minus.

So, you can substitute the value of h here, sorry this is u infinity square, 2 rho we have already taken as common. So, that will be the corresponding expression right. So, what we have done is, we have replaced h with this expression. Now what does this force represent this is the force exerted by what on what.

Student: Solid cylinder.

This is the force exerted by the solid body on the fluid control volume. And you can easily see say you do not know it, but the mathematical sign will tell you. See u square is less that u u infinity. So, this integral when it is evaluated it will be negative. So; that means, you have a negative force; that means, force which is along the negative x direction. So, what force is there along the x direction? It is definitely force exerted by the solid on the fluid because it is trying to resist the motion of the fluid. On the other hand, the force exerted by the fluid on the solid is equal in magnitude to this, but opposite in sense. So, that is along the positive x direction that we all that we also call as drag force on the solid object. There are certain interesting things that we can observe from this problem. One interesting thing is it appears as if this force does not depend on the shape of this object, but that is an illusion. Why where the effect of the shape of the object comes into the picture.

Student: Velocity profile.

The velocity profile, so the velocity profile will very much depend on what is the shape of the object. So, we have assumed a velocity profile, but this is like it, it does not come just arbitrarily. This, whatever is the velocity profile that velocity profile should come from the shape of the body. And therefore, that is where the shape of the body becomes critical. Now the other thing is that, it will also appear as if the force does not depend on the viscosity of the fluid it appears. So it is a kind of like not an intuitive thing that is expect the force due to viscosity, because if there is no viscous effect perhaps there would be no drag, but there is no there is no viscosity here. There is no presence of the parameter viscosity here. So, what is the viscosity doing here?

Or the question may be posed in this way that is it always necessary that viscosity will directly come into the picture for the drag force calculation. See viscous effect is there. There are 2 important effects which are prevalent here. One is the viscous effect another is the contour of the body. As the fluid is flowing over the contour of the body, there is a change in pressure. So, one is the geometrical effect another is the viscous effect and this velocity profile is a combined consequence of what has taken place.

So, once you are given a velocity profile you may be you are abstracted of anything else, but where from the velocity profile is originated, for that viscosity may be important, but once you get a velocity profile. So, that is what the integral balance is giving you integral balance is the net effect. It is not microscopically looking into what has happened at individual points, but it has got a gross consequence. What is the consequence some velocity profile had the outlet. And this kind of the gross consequences is important because you can measure it experimentally. Experimentally point to point measurement is difficult it is not impossible, but it is always more expensive to do that.

But experimentally you can at least find out velocity profile on a given section. So, you can have different probes all set may not be as simple as peter tube probe, but you may have any velocity measurement along this section that is not difficult. And uniform velocity which is the free stream velocity that you know. So, from the experimental understanding of what is the velocity profile at the inlet and the outlet you may be in the position experimentally calculate or rather to calculate from the experimental data what is the drag force. And the limitation of that is it does not pinpoint that how the flow field varied from one point to another point to give raise to the drag force, but it gives the total effect in a integral sense. Now, how it varies from one point to another point for that we have to look into the corresponding differential equation for viscous flows that we will do in our next chapter.