Introduction to Fluid Mechanics
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Lecture – 45
Application of RTT: Conservation of linear momentum

In this lecture, we will look into the integral form of the conservation of linear momentum. To do that we will start with the Reynolds transport theorem general expression, which is valid for any conservation?

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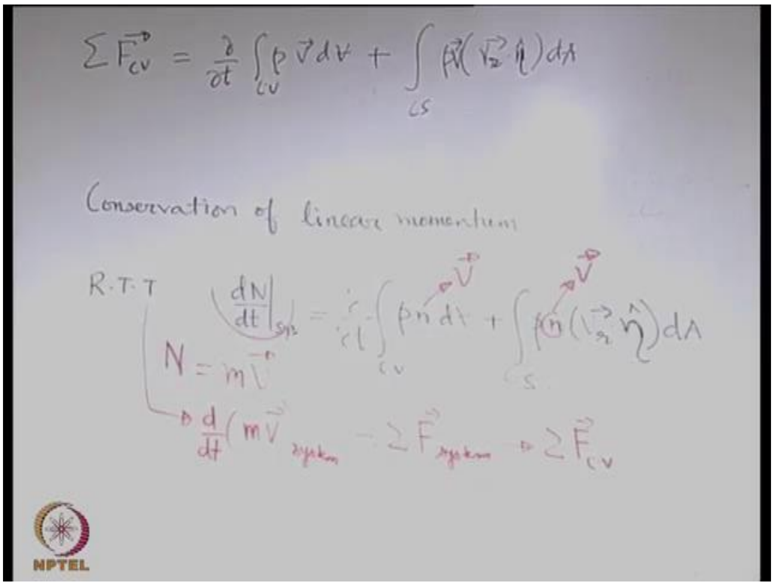

Now, if we want to conserve the linear momentum then what should be this N? Capital N is mass into the velocity that is the linear momentum. So, we can write the left hand side as d dt of m V of the system. So, when we write d dt of m V for the system by Newton's second law of motion this is the resultant force which is acting on the system.

We have to keep in mind that we have derived this expression with the limit as delta t tends to 0, in the limit as delta t tends to 0 system tends to the control volume that we have to keep in mind and therefore, in that limit this also tends to the resultant force acting on the control volume. So, the left hand side essentially becomes the resultant force which is acting on the control volume and the right hand side we can express by writing what is small n? Small n is capital N per unit mass. So, this is V and this is also V. Very important observation this V in a reference frame which is stationary we have considered a stationary reference frame here we will see later on that it is not necessary to have a stationary reference frame we may have a moving reference frame not only that a reference moving arbitrarily with arbitrary rotation as well.

So, a non inertial reference frame what happens to the statement of the Reynolds transport theorem in a non inertial reference frame that we will also see, but here we are considering inertial and a special case of inertial which is stationary.

So, here this V is the absolute velocity, but even in this particular term we are not trying to disturb this particular term because this particular term is going to be always this irrespective of the control volume moving or stationary or whatever because this is something which is giving rise to a net flow of mass across the control surface and that depends only on the relative velocity not the absolute velocity. So, here it will be absolute velocity here it will be relative velocity and that we will keep in mind, but when it is a moving reference frame this will also change to relative velocity that we will see.

So, the statement of the Newton’s second law here boils down to a like expression which is the resultant force acting on the control volume and see the power of the Reynolds transport theorem we never committed that n should be a scalar vector or whatever. So, we have applied it for a vector and there is no restriction towards that. With this understanding on the fall of the conservation of linear momentum for a stationery reference frame let us try to work out a problem.

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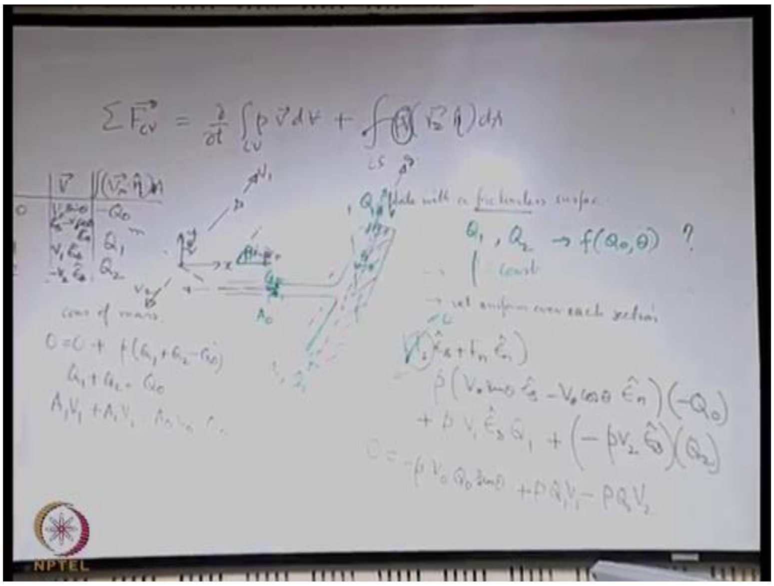

Let us say that you have a water jet which comes out of a tube or maybe a small pipe and say there is a plate which is frictionless and oriented incline to the jet. So, this is a plate with a frictionless surface. So, what happens to the water? The water extreme streamlines come like that and water leaves across the ends of the plate in this way. We have to find out if the flow rate volume flow rate here is Q 0 and here the volume flow rate is Q 1 and if the volume flow rate is Q 2 and let us say that the angle made by the plate with the vertical is theta which is the angle between the plate and the vertical.

When we say vertical let us neglect the change in height between various points, just to simplify the situation. Let us say that it is like vertical means a vertical line drawn in this plane the entire thing may be located in a horizontal arrangement or even if it a vertical arrangement the plate height is so small that the change in elevation, that the change in potential energy between various points is insignificant as compared to the other forms of energy like the kinetic energy. Now let us say that this area of cross section of this extreme streamlines, so these are like stream tubes these are called. So, if you see that this is not a pipe basically, this is a free water jet exposed to the atmosphere, but the water jet takes a form like a tube its outer periphery takes the form of a tube and outer ones are the extreme streamline.

So, the figure that is drawn here the lines represent the envelope of all the streamlines. So, this is like this is called as a stream tube which engulfs all the streamlines the same is there for the exit ones. So, what are the consideration that you now could apply for finding out? So, what is our objective? The objective is to find out Q 1 and Q 2 as a function of Q naught and theta, that is our objective. To do that, one important thing that we should not forget that mass conservation is always applicable and we should try to apply that, not that whenever we are trying to apply a momentum conservation mass conservation is insignificant not like that.

So, no matter whatever conservation we are applying again first identify a control volume. Let us say that this is the boundary or the surface of the control volume that we are looking for, you will see that like whenever we draw the boundary of a control volume or control surface we just draw an arbitrary dotted line which is like which is not coincident with the surface, this is just what a clarity in representation this is very very symbolic. It is not that if I have drawn it a bit far; that means, the air here is involved as a part of the control volume. So, do not take it literally. Now when this control volume is drawn we are interested to find out write an expression first for the mass balance, conservation of mass.

Assume that the density of the fluid is a constant, let us say that the areas of flow here are like here it is a 2 and here it is a 1. These are the parameters which we do not know because these are not within our direct control; what is in our direct control maybe this area because this depends on the injected jet from the tube or the pipe whatever nozzle, but when it comes on the plate and moves these depends on the many other things, so directly we cannot say, but one thing we can say is that the mass will be conserved. So, if you write the conservation of mass till by this time you have got habituated in writing for different cases. So, we will not write the full Reynolds transport theorem form we know which terms are important and which terms or not, left hand side first term is 0 always because the system has a fixed mass, the second term if you have a control volume which is of a fixed volume and density as a constant that will be 0. So, it will be only the rate of mass out flow minus inflow. So, what is the rate of mass outflow?

Student: (Refer Time: 11:02).

So, that is rho into Q 1 plus Q 2 is outflow minus Q 0. So, that is the outflow minus inflow. So, you can straight away write Q 1 plus Q 2 is equal to Q 0. Let us make another assumption that the velocity is uniform over each section. So, you can write this as good as A 1 V 1 plus A 2 V 2 is equal to A 0 V 0 or Q 0 whatever equal to Q 0. Now there is a relationship between V 1, V 2 and V 0 it is possible to have a relationship between V 1, V 2 and V 0, but before coming into that relationship let us complete the exercise of looking into the linear momentum conservation of what linear momentum conservation gives us.

See one very important thing is it is considered to be a frictionless surface; that means, there is no shear force between the fluid and the plate in that direction tangential to the plate. So, if you write a linear momentum conservation with the components say along x, x like say tangential to the plate or maybe let us call it s, to indicate that it is a tangential to the plate and maybe a direction n which is like normal to the plate, but we are bothered about for the tangential to the plate. So, tangential to the plate what is the situation? The situation is that there should not be any force on the water which is there in the control because it is a frictionless thing.

No matter what, whether it is tangential or normal. So, let us write this f for the control volume as, let us write it in the general vector form. So, let us write that conservation of linear momentum F as F s into unit vector along s plus F n into unit vector along n this is the resultant force on the control volume. General form with we know that F s is 0 from the physical considerations of a frictionless plate, this is equal to. For the first term we are assuming that it is a steady flow velocity is not a function of time we are assuming a constant density, density is not a function of time and the volume of the control volume is not also a function of time. So, all these 3 considerations lead to the fact that this term has to be 0.

So, when this term is 0 then you come to the second term. So, what will be the second term? So, for the second time you have, so the integral is not important because the velocities are uniform over each areas. So, we have to write this for the 3 surfaces 0 one and 2 just like that individually. Integral will eventually not have to be evaluated because of the uniformity of the velocity.

So, let us write this for the section 0. So, for the section 0 you see what is V? So, may be let us make a chart what we did for solving one of our earlier problems just for your convenience to begin with. So, we have a section say 0 at which we are interested to find out what is V and what is the normal vector. The normal vector will be insignificant we will see because eventually we are dealing with the flow rates. So, if you consider the V to be uniform and rho to be uniform, you see rho into V we can take out of the integral; if you take rho into V out of the integral then what remains within the integral is integral of V r dot n dA that is what? That is the total volume flow rate over the section that you are considering. So, it is as good as rho into velocity vector into the volume flow rate over the section with a proper sign plus or minus.

So, keeping that in mind we will not write this vector not that it is not existing, but it is not important for solving our problem, it will just give a redundant exercise we will put too much of a effort which is not necessary, but the velocity vector is important. So, the velocity vector for this one like, let us say that we have our original axis like this is x and this is y and the x axis is like this which makes an angle theta with y, maybe you can also choose a n axis which is normal to that.

So, now what is V for this one? V is a V naught I cap, maybe instead of writing this normal vector let us write what is V r dot n A or more fundamentally integral of V r dot n dA. So, what is there that for the surface 0? So, this is V r and or V n, this unit vector they are oriented in the same sense or in the opposite sense?

Student: Same sense.

This is the direction of V; this is the direction of the unit vector of this surface. So, this will be what?

Student: Minus of Q 0.

Minus of Q 0, then for the surface 1, so you are having a flow for the surface one in this direction which is having a velocity of V 1. So, that may be resolved along x and y. So, V 1 will have components what V 1 sin theta I plus V 1 cos theta j right. What is V dot n dA here? It is Q 1 because the flow direction and the unit vector out of the area they are oriented in the same sense. For the surface 2, what is v? Its magnitude is V 2. So, the velocity is like this now, this is V 2. So, it will have some component along x and some components along y, what is the component along x? Minus V 2 sin theta I and minus V 2 cos theta j, what is this one? Q 2.

See this is very straight forward, but I want to emphasize on this because this; the place where students make mistake many times because by default it is a common thing that the outflow boundary give raise to a positive flow rate, but the velocity that comes with it, it has nothing to do with the flow rate. So, that you have to write in proper vector sense. So, this term plus or minus has no conflict with the flow rate term that you should keep in mind otherwise like just by inertia of writing you might write this in a similar sense as we are written this. So, let us now substitute that.

So, if you substitute that let us say that we are interested about writing the expression in terms of the tangential and the normal components why you want to do that because we are interested about equating this tangential force to 0, so that we want to exploit. So, in there in that case this coordinates are not good coordinates for us, this I just wrote for a practice of showing that how to write different terms, but more convenient will be to use the s and the n coordinate.

So, in terms of the s and n coordinate if you want to write the good thing is the flow it does not change it is independent of the coordinate type, but what changes is now the expressions for the velocity vectors that you are looking for. So, let us write these expressions now in terms of the s and n vector which will be useful for solving this problem. So, in terms of the velocity at section 0, so you have a velocity at section 0 like this you have V 0, you have a coordinate s like this and a coordinate perpendicular to that as n and with this one it makes an angle 90 degree minus theta. So, you can resolve V 0 into components along s and n. Typically, how the resolution will look like? Maybe you have this along s this along n, so the resultant is this one. So, what will be the component along s?

Student: (Refer Time: 22:00).

So, V naught sin theta then minus V naught cos theta epsilon n, this is for the surface 0 then for the surface one V 1 epsilon s, for the surface 2 minus V 2 epsilon s. Let us put that here. So, if you put that then it will become first we put for the surface one. So, rho into V, V is V naught sin theta epsilon s minus V naught cos theta epsilon n into minus Q naught, then for the surface one plus rho V one epsilon s into plus Q 1, for the next one minus rho V 2 epsilon s multiplied with plus Q 2.

So, if you extract the tangential component and keep in mind that because of the friction less case this is equal to 0 you are left with that. So, if you equate the epsilon s components you will get 0 equal to rho minus rho V 0 Q 0 sin theta plus rho Q 1 V 1 minus rho Q 2 V 2 right. See again see the deceiving thing as if this minus sign will is giving inflow what this is a minus, but this is a outflow. So, the origin of the plus and minus has to be clear it is not because of outflow or inflow the plus or minus that has come as a combination of out flow and the direction of the velocity vector. So, rho being a constant you can write like from that equation Q 1 V 1 minus Q 2 V 2 is equal to Q 0 V 0 sin theta. Now the question is what is the relationship between say V 0, V 1 and V 2 let us somehow you could measure both A 1 and A 2, say somehow you could measure, now let us try to see that how that measurement could give us a good picture.

Let us say you are you know that what is V 0 you are interested to find out what is V one. So, what you will be tempted to do. If you know, what is V 0, your objective is to know what is V 1 from that, what intuitively you try to use.

Student: Sir.

Yes. Let us consider a streamline that goes from the section 0 to the section 1 because it is a uniform velocity like if you take a 0 here and a point 1 here these are representatives of the velocities as good as at section entire section 0 and entire section 1 because it is a uniform velocity at each section that we assume. Now if we apply the Bernoulli’s equation between any 2 points on the streamline. So, let us try to apply that and see that what new information we get. So, the first information that we get is, first of all let us write it and the information will be clear.

We assume that both 0 and one are atmospheric and it is true it is not an assumption because it is a free jet in the atmosphere, this is also a free liquid filled being passed on to the atmosphere. So, in both places sections 0 and 1 it is exposed to atmospheric pressure and because the height difference is not that large atmospheric pressure will not change substantially. So, these 2 are equal and because the height difference is not appreciable as we assumed earlier so the z naught and z 1 are approximately the same or their difference is negligible. So, from this we can conclude that at least in an approximate sense V naught is same a V 1 and with a similar consideration you can say that V naught similarly you can say V naught is approximately same as V 2, so in this expression where you have V 1, V 2 and V naught you can cancel that from both sides.

So, what you get? You get Q 1 minus Q 2 is equal to Q not sin theta. So, let us see we get 2 expressions involving Q 1 and Q 2 in terms of Q naught and theta. So, you can solve for Q 1 and Q 2 individually as a function of theta from these 2 very simple. And important thing is it does not require any information on what are the areas of these liquid films at 1 and 2. So, it does not require what is a 1 and what is a 2. There could be a very interesting observation now you find out what is the normal component of the force. Normal component of the force definitely is not 0 and the normal component of force will come out if you see that because of the term rho V naught, Q naught, cos theta.

So, because of the normal component of the force this is the force on what? This we have to keep in mind let us write it. So, F n is minus rho V naught sorry not minus plus V naught Q naught cos theta right, Q naught you can write in terms of a naught and V naught, so it is like rho a naught V naught square cos theta. Question is this is the force on what, these are very subtle points see always it is not here the answer ends we have found out the normal component of force, we must ask ourselves that is force on what force exerted by whom on whom, if we are not clear about that then what is a good in finding out a mathematical expression.

So, this is we have to keep in mind that this is the force on the control volume. So, if you had drawn a free body diagram of the control volume if you would have done then what would have been the forces acting on the control volume that you would have shown in the free body diagram. Let us say that this could be your control volume. So, you could have a force in the normal direction say F s you could have a force sorry in the tangential direction F s and normal direction F n, assume that the same thing is occurring almost in a horizontal plane, the weight you are not considering in display not that weight is not there, but in this plane of forces weight is not important. Then any other force on this? Yes.

Student: (Refer Time: 31:39).

Think of the fundamental consideration of forces in continuum mechanics that we discussed, you have surface force and body force. So, body force we have considered the weight just not shown in this plane because it is not significantly there in this plane. Surface force pressure distribution is there. So, there is a surface force from all sides normal to the areas whatever, have we considered the contribution of this? We have not considered.

But what magically we are expecting that it is giving us the correct information on forces and this magical thing is possible because if you have a closed contour over which you have a uniform pressure distribution that irrespective of the shape of the contour the resultant force is 0. Because of a very special nature that pressure local is always normal to the direction on the surface. So, no matter how bad the shape of the contour of the surface is, so long as it is close and if you have a uniform pressure throughout then the resultant force due to that will be 0. This is the very important thing and you should be able to drive it by yourself, if you are not able to do that you let me know later on I will try to help you out, but you should be able to do it.

So, if that is the case then if there is a variation of pressure only that will matter to a resultant force, but if throughout it is atmospheric then it does not matter therefore, our negligence of the pressure in this case has not mattered because it is uniform pressure and that is why you will see that what is important for us is not what is the atmospheric pressure, but the deviation from the atmospheric pressure that only can give rise to a local force. So, always when we considered forces here due to pressure we considered the gauge pressure at different sections because the deviation from atmospheric pressure is what is interesting for us, atmospheric pressure if it was the soul feature it would have existed throughout equally and it would have given rise to a 0 net force. So, in terms of giving rise to a net force any deviation from atmospheric pressure is important. So, the gauge pressure is the important quantity.

Now here of course, the pressure contributions does not come, so F s and F n these 2 are there question is that what should be F s and sorry on which this F s is acting and it is exerted by whom on whom and on which F n is acting.

F s is 0 we have seen and that gives a clue that it is between whom and whom right, when we said F s is equal to 0 we were thinking about an interaction between the plate on the water. So, F n also should be a consequence of interaction between the plate and the water. So, it has 2 possibilities, what? Force exerted by plate on the water or exerted by water on the plate. Yes.

Student: (Refer Time: 35:06).

They should be same, but same in magnitude not same in sense. So, whatever answer we have got here this has a particular sense. So, it represents only a unique thing. So, when we have said F n epsilon a it has a proper sense of the vector epsilon n. So, this is force exerted by water on the plate or plate on the water.

Student: (Refer Time: 35:31).

It is force exerted by plate on the water because it is force exerted on the control volume and what it is there in the control is not plate, but water. So, this is force exerted by the plate on the water present in the control volume. By Newton’s third law you will be having the force exerted by the water on the plate as minus of this. So, the force exerted by water on the plate will be opposite to the positive indirection. So, force exerted by water on the plate will be in this direction, this is you see this is in the positive indirection that we have considered. So, opposite to that is the force exerted by water on the plate and that is quite obvious like logically if you see if the jet strikes on the plate like this it should have a force in this direction not the opposite direction. So, this there is a resultant force which is exerted on the plate in its normal detection. So, this force might be good enough to make the plate move and the plate should be adequately supported to prevent such moment.

In general if that jet is falling on the plate, the plate will also move because of this normal force and later on we will encounter such problems we will try to see that if the object on which the jet is striking or which is interacting with the jet in any form if that itself is moving then how we adopt our analysis of momentum conservation. So, this is like a very simple problem, but it gives a lot of important concepts that we should keep in mind.