Introduction to Fluid Mechanics
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Lecture – 44
Problems and Solutions

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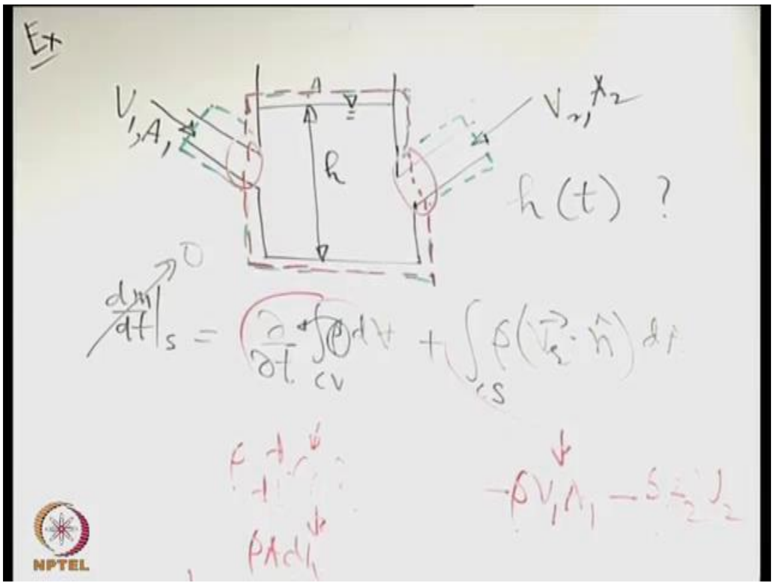

Let us see a similar problem, where you have a tank like this, and there is a free surface here. So, in the previous problem, we assume that there is no change in level of the surface in the tank, but in reality that does not happen, so that is something which is a bit hypothetical. In reality, it may happen that approximately the change in this level is 0, because this is such a large area that no matter whatever is entering and leaving. It is not changing the height these types of tanks are called as constant head tanks. So, they maintain a constant head because with respect to the inflow and outflow, the change in height of this is so small because of maybe this is a large reservoir. So, the area is so large that it does not change any level, but the more realistic version of the previous problem is that is yes the level of the water will also change.

So, let us say that the height of the free surface from the bottom of the tank is h. Just to simplify the situation we would now go back to a case of a uniform velocity profile.

Because we have already seen that if it is non-uniform that it is not a very difficult thing we have to just integrate the velocity profile over the section. So, with that understanding let us say that you have a situation like this. Let us say it is uniform. So, you have a velocity V 1 here and let us say area of cross section A 1 you have a velocity V 2 area of cross section A 2.

When we were talking about the previous problem see we could get rid of the situation of change of height of the tank even in a real case by some approximation that yes the rate at which the water is entering is the same at which the water is leaving. So, it is not changing the height of the level of the tank, but when both are entering that is not the case. So, here the only chance of the height of the tank or the level of the water in the tank not changing with time maybe only for the consideration that the area of cross section of the tank A is so large as compared to the others that the corresponding change in height is very small. Otherwise, here there will always be a change in height smallness or largeness depends on the areas.

In the previous problem, you could cleverly come up with a situation where there is 0 change in height by making sure that the rate at which it enters is exactly the same at which it leaves. So, it appears to be bit hypothetical, but it is not that hypothetically it is really doing that. So, if you have a reservoir and water is entering and leaving at the same rate, why should it change the level of the reservoir it will not, but here both are entering. So, here our objective is to find out how the height is changing with time, given this velocities areas and the information that is velocities are uniform over the area.

So, let us take a control volume again. Write the equation, so d m dt for the system is equal to this one left hand side - 0, right hand side - the first term. Let us assume that the density is a constant let us say that is given. So, if the density is a constant, it will come out of the integral, but the volume is not a constant. So, the volume within the control volume is what, it is A into h, A is a cross sectional area of the tank and h is the height. So, this effectively boils down to what this effectively boils down to d dt of rho into d dt of A h. Area of cross section is a constant, so this is as good as rho A d h d t. See h is a function of time only and nothing else, so this partial derivative becomes an ordinary derivative here. And what about this term, yes?

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So, it has now effect of the 2 areas 1 and 2. So, because we have seen that formally; what is the consequence of the dot product and all those things we will now try to write it directly without going to that route. So, for the surface 1, what will be minus rho V 1 A 1; and for the surface 2, minus rho A 2 V 2. So, sum of this should be equal to 0. So, from here you will get what is d h dt that is very straight forward. Again you can make an observation that if d h dt equal to 0 that is not a possibility because these 2 terms of the same sign cannot cancel that. And only way it can cancel if V 1 and V 2 are of opposite sense, that is if one is entering the other is leaving, and then again it boils down to A 1 V 1 equal to A 2 V 2, so that is like a previous case that we had considered.

The other important thing that we might discuss in this context is that can we try to choose a different control volume. Let us say that we choose a control volume, which is not the previous one, but say that new dotted line that I am drawing. Fundamentally, there is nothing wrong, but it will not help us solving the problem, why because now the control volume is cutting across some flow surfaces across which you do not know the velocity profile or you do not know how the velocity is. So, how will you find out this integral term, you have to have those locations where at least you have an idea of how the velocity is varying or what is a velocity. So, the choice of the control volume is not that it is only something which is unique and you cannot choose anything else. But if there are many alternatives you have to find out that what are the unknowns and known involved with that alternative. And it is wise to choose a control volume which gives very easy situation by reducing the unknowns, and the initial choice that we made is an obvious choice towards that.

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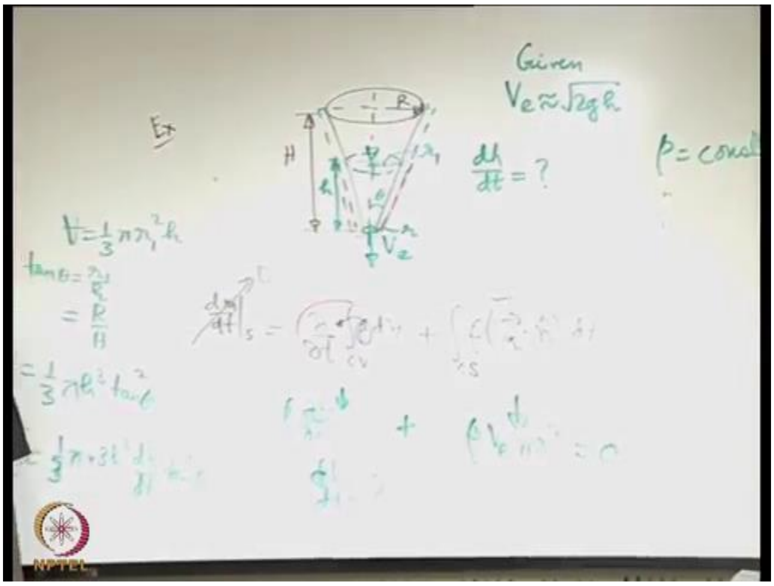

Let us take another example of similar type may be a bit different. Let us say that you have conical tank just for a change, of radius capital R and height capital H. There is a small hole at the bottom of the cone through which water is coming out with a velocity V e, and because of this leaving of the water the height of the water in this conical tank is reducing. So, maybe initially it was the full height capital H, but because water is leaving at some instant of time say the height is like h small h. So, this small h is changing with time, because water is continuously leaving. Let us say that it is leaving through the small hole with radius of small r both like this is a circular hole and cone is a circular cross section. Then you have to find out what is dh dt.

It is given that you may approximate V e by root 2 g small h, it is given. We have see earlier that this is not actually a very correct estimation, but it gives us sort of approximate situation under certain simplified assumptions, and we have discussed those assumptions in details when we had discussing about the Bernoulli’s equation. So, this problem is fundamentally not very much different from the previous one except that the geometry is such that you have a variable cross sectional are nothing bit I mean more complex than that.

So, let us try to use the conservation of mass here. So, the first term what it will be, again assume rho is a constant. Then if rho is a constant, so the bad thing that we have approached is like this that we have tried to solve the problem without identifying the control volume. I am trying to do it in the same way in which you are habituated to do just like straight way going to an question and solving a problem, now it is see it is ridiculous. We are trying to apply an equation for a control volume, but we do not know, what is the control volume? And that perhaps we are trying to do.

So, let us identify a control volume we will identify one type of control volume, and I will leave it on you as an exercise to identify a different type of control volume and come to the same answer at the end, and that will give you a good idea. Let us say that we identify a control volume like this. Let us say with respect to that control volume, we are writing this term. So, with respect to this control volume, when we are writing the term, let us say that we neglect the density of the air, which is there at the top of the water in comparison to that of the water. It is not a very bad engineering assumption because the reason is that air is much, much lighter than that of water right like typically 1 by 1000. In fact, that is what we did also in our previous problems where if you have a tank maybe in the tank some part is water, but the remaining is air. But when we wrote this term we did not write the term corresponding to air that was an inherent assumption that we were making keeping in mind, but not explicitly stating.

So, then like if you take rho as a constant and out of the integral what will become this integral of dv, so that this term will become basically rho derivative of V with respect to time. Now, the volume is the volume of the water. So, at this instant that we are considering the volume of the free surface is located here. So, the volume of the water is one-third pi local R square. So, let us say let us give it a name say R 1 one-third pi R 1 square H. And from the semi vertical angle of the cone let us say theta you have tan theta is equal to R 1 by H which is same as capital R by capital H. So, it is possible to write the whole volume in terms of small h. So, the whole volume becomes one-third pi in place of R 1 it is H tan theta, so H cube tan square theta.

Then the other term what will be this one, how many flow boundaries are there in a control volume, only one flow boundary only one exit boundary. So, what is that? So, this boundary V n the normal vector are located oriented similarly, so the dot product will give a positive term. So, it will be rho then what, so V dot n, V is uniform over the area let us assume that. So, V dot n will come out of the integral, rho will come out of the integral and integral of dA will become a. So, it will become like rho V e into A e, so it will become rho V e in to pi R square.

So, then it is very straight forward these 2 together is 0. And you can differentiate V with respect to time. It is like an ordinary derivative again because h is just a function of time. So, one-third pi into 3 h square dh dt into tan square theta. So, if you substitute that here, you can find out that what is dh dt. At a given instant, when the height is h, small h. Now, I will leave on you solve solution of the same problem, but with a control volume choice like this say you have a control volume which is erupting itself with a movement of the water. So, this control volume is a moving order deformable control volume in a way that if the water level is coming down this is also coming down with it. So, this type of control volume and then with respect to that type of control volume, you make the same analysis and try to come up with the same answer.

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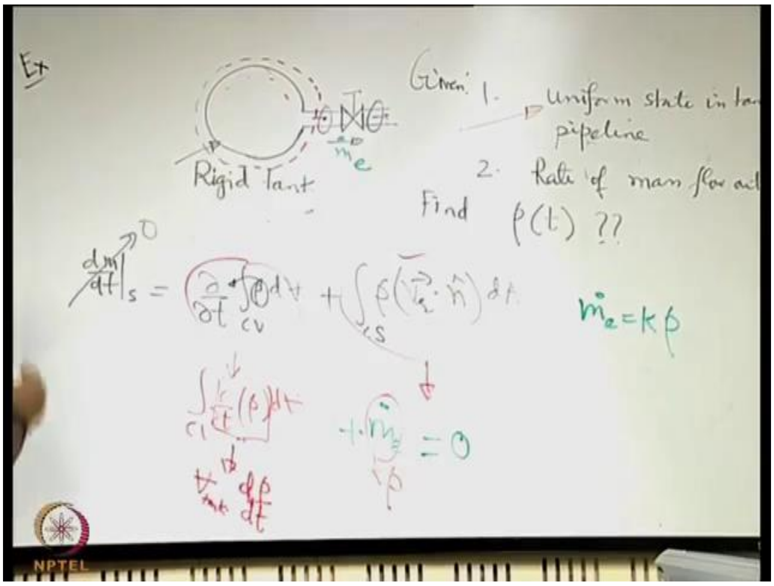

Let us work out another problem. Let us say that we have a rigid tank spherical tank, which contains air. And originally there was a valve located here, which was preventing the air inside to go outside. Now, the valve is opened, and once the valve is opened, the air will go out of this pipeline, which is connected to the tank. It is given that number one the state is uniform within the tank, in tank and pipe line that means, the properties are the same at a given instant of time everywhere that we are considering. And number 2 is that the rate of mass flow out is proportional to the density at that instant, this is given. You have to find out that how the density is changing with time. It is expected that the density will change with time because it is a rigid tank. Because it is a rigid tank, if you take away air from it its density will fall, because now less mass is occupying the same volume, so that you have to find out that how that density is changing with time.

Again let us start with the choice of a control volume on which we want to have our analysis. So, the control volume let us say that we choose a control volume like this. If you choose a control volume like this, it does not matter whether the valve is open or closed, there will always be a flow across it. So, now if you are ask that what happens after the valve is open, you should take it like I mean of course, the pipeline does not end here. So, the pipeline continues beyond the valve. So, you should take it like where the effect will be apparent when the valve is open. But again in this particular case, only for objective is to look for a mass conservation it makes no difference as such because even if you take your control volume like the surface which is to the right of the valve and to the left right. At any instant whatever is the mass flow rate here the same is the mass flow rate along this pipeline in terms of the mass flow rate.

But that may not be the case if the density in the pipeline itself is a function of time. So, if the density in the pipeline is a function of time, and if there is a possibility of like see variable density cases are very typical cases. So, for variable density cases it is not so trivial to say that like mass flow rate is the same in all cases, but here no matter how the density varies you can say that these 2 mass flow rates will be the same. Why, yes?

Student: (Refer Time: 23:05).

Professor: Area of cross section is same.

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Professor: So, density is like I mean whatever it is like even if you forget about the density like just fundamentally think, if you have a mass flow rate that goes out of this. If the same mass flow rate does not go out of this, where does that mass go, is any mass like accumulated in between, no, velocity may be different if the densities are different, but if the density is uniform then that may be ruled out provided the time dependence of the density is not creating any big change. So, if you see that like what happens within this pipeline in terms of the velocity or density something we do not have enough information really to talk about that.

But one important information we have that whatever happens individually to velocity, density, but the mass rate what goes to this is same as mass rate that goes out because where otherwise it will go, it cannot sit on the valve that mass. So, we are assuming that there is no accumulation; it is a steady flow system. So, if it is a steady flow like whatever enters here the same leaves here then like it does not matter really whether you are control surface goes through this or this or whatever.

Now, let us look into the different terms in this expression. So, if you look into the first term, so here it is a rigid tank. In the previous class, we try to solve the similar problem, which is like a flexible balloon. So, there the volume could change with time that is maybe, if it were a flexible balloon you could have expected that by taking away air out of it the balloon will try to shrink, but here it is a rigid tank. So, it cannot respond to that, it can only respond to that change by having a change in density inside, but not its change of its own volume. So, the volume of the tank is a the density of the fluid in the tank is definitely varying with respect to time, but the volume is not varying with time, so that means, the first thing that you can do is you can take the time derivative inside because the volume of the control volume is not a function of time anymore.

And the next thing is that how this density changes with respect to time that does not change from one point to the other within the control volume. Why, because it is given that is uniform state in the tank and in the pipeline, that means, the density may change with time, but at a given instant of time the density is same everywhere within the control volume, so that means, this is like an isolated term which does not depend on the volume. So, this you can take out of the integral so that means, eventually it will become the volume of the tank into the rate of change of density with respect to time in the tank.

So, it is not so difficult to come up with this expression, but we have to keep in mind what are the important assumptions that are leading to this type of a simple proposition. If the density was varying within the tank itself then we could not write it, then we had to integrate it by keeping in mind that rho is a function of both position and time. Here the dependence on position we are chosen, we have assumed that that is not there. Then for the next term, so if you see that this now you have only one out flow boundary for the control surface when the valve is opened, the air is leaving here. So, you have an m dot exit, which is the rate of mass flow rate out.

So, the rate of see this is eventually giving what this is giving a rate of mass flow rate, outflow minus in inflow. There is no inflow, so only outflow, so this is as good as plus m dot e this is equal to 0. What is this m dot e, m dot e is proportional to the density. So, let us say that m dot e is equal to some k in to rho. So, this rho is an instantaneous density that at the time, whatever is the density, in the system proportional to that the mass is coming out. So, this you can write some k into rho, where k is a constant.

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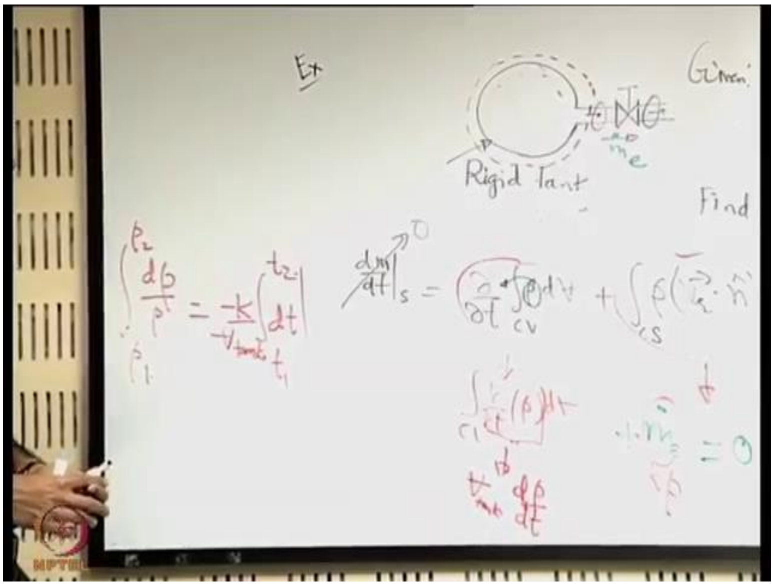

So, you can the next work is very easy, all of you enjoy doing it, very simple integration.

So, d rho by rho is equal to minus k by V tank dt. So, you can integrate it, from say time equal to t 1 to time equal to t 2 say the density changes from rho equal to rho 1 to rho equal to rho 2. So, you can find out how the density changes with time. So, important is not the solution of the problem of course, the solution is quite easy, but solution is definitely having some importance, but to my understanding the greater importance is what are the assumption that are leading to the solution, because in reality the problems are not so simple as many of these one. So, this might not be very, very common as our analysis equations, because if you see this equation that we have written these are really on the basis of such simplified conditions or assumptions which might not be prevail in practice.

So, the final answer maybe like interesting in terms of solving a particular problem, but the reliability of the answer may not be so strong because you might have a strong variation of density within the tank itself. But at least by having a simplified assumption it is giving us a fair idea of like how the analysis should involve the conservation of mass. So, when we have been discussing about the conservation of mass, we have discussed certain types of problems, what types of problems we have discussed, we have discussed about one case when it is totally steady. That is you have like the density first of all the density is not changing with time, volume of the control volume is also not changing with time, so that this term is not there. So, it is just a balance between the rate of outflow and inflow and physically that represents a condition where rate of outflow is equal to rate of in flow.

For that we have to keep 2 things in mind one is that what is the sense of the velocity, velocity component normal to the area, is it opposite to the area vector or is it along the area vector. The second point is, is the velocity uniform over the cross section or is it non uniform accordingly we might need to integrate or not. And when it comes to us unsteady case there are 2 types of possibilities one is the density is changing with time, another is the volume of the control volume is changing with time. And maybe a third case when both are changing with time, but like we have perhaps not considered that case, but that is like it is a combination of the cases that we have considered. So, all these cases have given us a form footing or understanding of how to use the integral forms of conservation equations.

What is the advantage of use of the integral form of conservation equation? See, when you are using an integral form the important advantage is that you are being abstracted from how things vary within the control volume, you are only bothered about a gross manifestation in terms of what is entering, what is leaving. So, what is happening inside, you are just representing it in an integral sense or a overall sense, you are not really representing it a point by point variation. So, we have to keep in mind here a very important thing what is the difference between an integral equation and a differential equation physically. Differential equation gives you the variation at a point, whereas integral equation gives you the variation over a domain. Of course, the domain is constituted of many such points, but the differential equation is valid only at an identified point in the domain.

So, the integral form when you are writing, you may get back the differential form from that, but at the same time you are not forced to track or to keep in mind that what is happening as a point by point variation, so that differential nature of variation you may not be interested in. So, in such cases, where you are not really interested in that it may be convenient to use the integral form, so one has to keep in that mind, when should we use the integral form and when should we use the differential form, it depends on the physical sense of the problem that we are trying to solve.