Introduction to Fluid Mechanics
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Lecture – 43
Problems and Solutions

Last time, we were discussing about the integral forms of the conservation equations. And as an example we looked into the integral form of the mass conservation and its corresponding differential form also we revisited. And we found out that it is possible to convert one form to the other.

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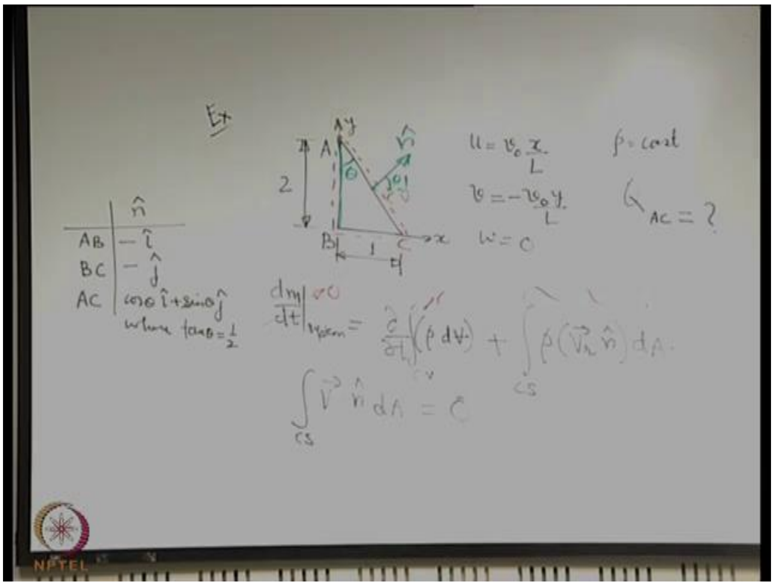

Now, let us look into some more examples of the use of the integral form of the mass conservation equation. Let us say that you have weight shaped element like this with the axis oriented along x and y. And the velocity field is a 2-dimensional velocity field is given by say u is given by this, v is given by this one, and 2-dimensional w is 0. Let us give some names to the faces of the elements here. The objective is to find out that; what is the volume flow rate through AC? The dimensions are given let us say this is 1 meter or just 1 unit, all are given in some consistent units say this is 2; these are given.

So, what we also are assuming that v 0 is a constant, it is not a function of any other variable that we are looking for. So, how should we proceed with this problem? Let us say that we are interested to use the integral form of the mass conservation. And that is one of the natural things that we should use here, because there are the 3 faces across which fluid will enter and leave. So, the net rate of transport should be given by the integral form of the mass conservation that is you have dm dt for the system, plus this for the control volume plus integral of rho. The left hand side is 0 because no matter whatever system you are considering it is by definition of fixed mass. The right hand side, because we are assuming that rho is not changing with time neither the control volume is changing with time. What is our control volume? Let us say that this triangular shaped element is our control volume.

So, whenever we are making a control volume analysis it is important to identify what is the control volume that we are taking, because you may take different control volumes. Of course, for this problem this is an obvious choice of the control volume, but there could be problems where there could be many different choices of the control volume. So, the equation that you are writing should be pertinent to a particular control volume that you are chosen and that should be clearly mentioned. So, this triangular shaped thing is a fixed control volume; the volume of that is not changing with time. So, it is not a deformable control volume neither the density is changing with time, so this term goes to 0.

So, what remains is this net term which is nothing but basically the net rate of out flow minus inflow of mass equal to 0. If rho is a constant; let us make a further simplification that rho is a constant. Then this will boil down to basically integral of v dot n d A over the control surface equal to 0. And when we say v hear the relative velocity and absolute velocity are the same, because it is a stationary control volume, it is not a moving control volume. Now we may break it up into 3 parts, because the control volume has 3 different distinct oriented surfaces.

So, you can write this as the sum of the effects of AB BC and AC. So, for AB when you write what should be this corresponding expression? Let us identify that where is that AB, so you are looking for this face. You are interested to find out what is v dot n integral of that over the entire area. So, one important thing is here n is not a variable it is like a line oriented along y axis, so n is a constant. So, for AB what is n? Let us try to identify what are the normal directions for the different edges. Here the edges are all straight lines so they have unique normal directions.

So, let us say AB, what is the n for that?

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Minus i: then BC: and AC. So, for AC let us say that this is the unit vector normal. So, this will have its components; let us say this angle is theta. So, if the angle between the normal and the horizontal is theta then the angle between the edge and the vertical is also theta. That means, you can say that n is definitely cos theta i plus sin theta j, where you have tan theta equal to 1 by 2. So what will this imply? This implies that like you have identified the direction normals for all the faces, only thing you require is the velocity. So, the velocities for the faces; for AB what is v, let us write the in the same table let us try to write what is the velocity v cap.

So, for AB what is v cap? For AB x is 0, so u is not there, there is some v. So, it is minus v 0 y by l j. For BC, similarly you have y equal to 0, so it is v 0 x by l i. And for AC it is the real sum of the 2 components, because here x and y are both non 0. So, we are not writing it because it is like, let us just write it as in general u i plus v j. Now to get the integral you have to keep in mind that if the velocity varies along that length, then you have to integrate it over the length to get the total flow rate. When you come to the; so let us start with AB, you can of course try to evaluate this, but let us not do that bull work. You see that the velocity along AB is like it is oriented along AB, so there is no normal component of that. So, there is no net flux or influx or out flux of flow across AB, because there is no normal component. So, if you make a dot product of this that you can clearly make out. And that will not give rise to any net flow.

Same might be true for BC right. So, the choice of the axis here has been such that if this the velocity field then that is the case, but the same is not the case for AC. So, for AC how will you find out what is the total or the net rate of flow? Yes.

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So, if something in there; there is nothing that enters here, there is nothing that enters or leaves here you are expecting that there is nothing enters or leaves here. How could you verify by doing the integral that it should be 0?

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No, that is what; like this is q AB this is q BC this is q AC like if you evaluate by doing the dot products. Will the dot product automatically give it?

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It is expected that the dot product to integral over that should automatically give it, so that you should check. So, that it gives you a confidence of how to calculate that, because here I have given a special type of velocity field. So, that actually this problem solution is not necessary, I mean I am trying to go through this through a formal route to give an idea of what should be done in a case when the problem solution deserves that, but here actually does not deserve. So, this is more intuitive case where nothing is entering and nothing therefore is expected to leave.

Now, if by chance you get something which is leaving through this and nothing is entering, so that will really violate the law of mass conservation. So, that one has to be careful of. Now let us look into some other problem which is not as trivial as this one; so another example.

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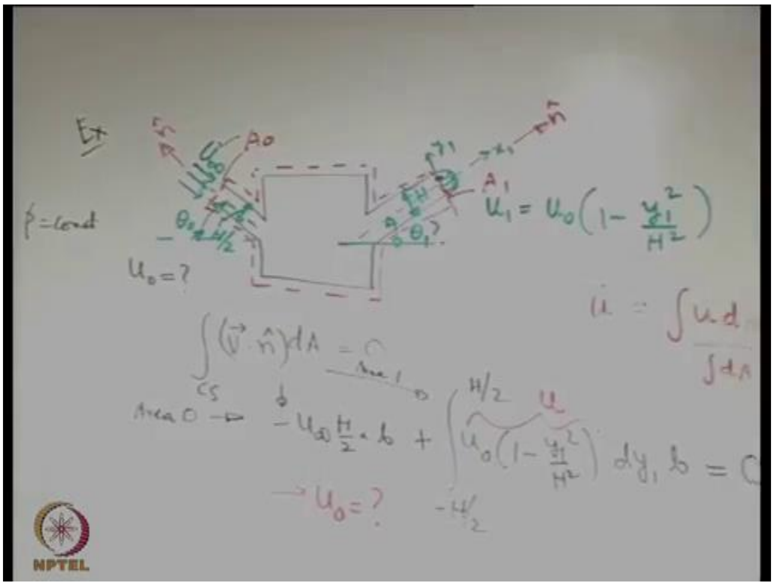

Let us say there is a tank like this, the velocity profile at the exit of the tank is through this pipe is given by this one and in terms of a local coordinate system this; let us say that the local coordinate system is x 1 y 1. It is given as u 1 is equal to some u 0 in to 1 minus y 1 by y 1 square by h square. Where y 1 is the transverse coordinate and h is this height. And the fluid entering here, here again in terms of the local coordinates you can specify it but that specification may not be necessary, because it is given that it is a uniform velocity profile here. With a velocity u infinity which is uniform, and let us say this is h by 2.

The directions of the axis are not given; that means it is not given that what is this angle theta. So, theta 1 it is not given that what is this angle say theta 0; these are not given. What is given? It is given that the density is a constant. We have to find out what is u naught? Given the width of this figure perpendicular to the plane where it is drawn is uniform. So, u infinity is given h is given these 2 things are given. How will you go about it? Again looks like a situation where mass conservation should be applied. And if you want to apply the integral form of the mass conservation; let us say that we consider a control volume.

So, what should be a good choice of the control volume? So, a good choice of the control volume is something where across the surface we are totally confident about the velocity field. So, let us say that we make a choice of the control volume something like this. So, with this choice of the control volume you can write the law of mass conservation. If you want to write that then like it is because, rho is a constant again it will boil down to a case very similar to the previous one where eventually it will be integral of v dot n d A over the control surface equal to 0. The remaining terms will not be relevant, so this is the only term that is relevant.

Now out of the surfaces that you have, only you have one inflow and one outflow surface across the other surfaces fluid is not flowing. So, those surfaces are not relevant. So, you may break it up into integrals: one for this inflow and another for the outflow. The first one is a very straight forward let us do that. So, when you have the v dot n d A; see this v is uniform over the area over which it is flowing for the inflow. So, you can take away this v out of the integral not only that the dot product also because n is also a constant here. So, v dot n the entire thing you can take out of the integral. What it will become? It will become v dot n into area of the face over which it is coming. So, let us call that this area of the face is a here A naught and here the area of the face is say A 1. So, when you write v dot n you have to keep in mind that what is the direction of n in along this surface; so the direction of n is opposite to v. So, v dot n will give the minus of the magnitude of v. So, for the area 0 or area o it will become; this term will become what? Minus u infinity into h by 2 into the width, let us say b is the width perpendicular to the plane of the figure.

Then the other area for A 1 we cannot have the same consideration because the velocity varies over A 1. So, to see how the velocity varies along A 1 we are already given that with respect to the local transfers coordinate how it varies. But n is a; but you have to keep in mind that forget about that functional dependence on y 1, u 1 is going out of the area, and what is n? N is also oriented out of the area. This means that if you just take the vector sense v dot n that will give you the magnitude of v into 1; because the dot products of 2 vectors in the same sense that will be leading to that conclusion.

Because it is varying with y 1, now you have to really do the integration. So, when you first have v dot n that will become; u 0 into 1 minus y 1 square by h square that will be v dot n. And d A; d A is what? You take a small element on the axis y 1. So, this is the small element of the area. So, what is that small element of the area, say at a height y 1 from the centre line see the coordinate system is from the central line. So, at a height y 1 from the centre line say you have taken a small area of width d y 1. So, the elemental area which is like the d A given symbolically it is d y 1 into b. If you integrate that from minus h by 2 to 2 h by 2 then that will represent that what happens for the area 1.

So, sum of these 2 should be equal to 0. Clearly, from the minus sign of the first term you can make that it is inflow and the plus sign of the second term means that it is outflow. And it is possible to complete disintegration in a very simple way; we are not going into that just to save some time. But important thing is that see this is the variation of u over the section. So, you can write it equivalently as integral of u d A in the fundamental form, in a scalar form. So, this is not like a vector form, it is like just like; because eventually with the dot product it has become a scalar. You have to keep in mind this is this u is nothing but the component of velocity which is normal to the area. Here fortunately all components is normal to the area, there is no component which is cross that.

So, this you may express through some quantity which is called as average velocity. So, average velocity is this divided by the area. That means, if this velocity was uniform, but the same flow rate was there. See, if it was uniform then that uniform velocity times the area will give you the flow rate; that is what the first term has told us. And if it is not uniform obviously we have to integrate it to get the flow. So, if the flow rates vary uniform in a hypothetical case sorry; if the velocity profile was uniform in a hypothetical case, but the flow rate becoming still the same as it is in the real case then if you equate those 2 flow rates then that equivalent hypothetical uniform velocity this is called as average velocity.

So, it is like a equivalent uniform velocity that would have prevailed across the section satisfying the same volume flow rate as it is there in the real case. Therefore, this is like as good as some u 1 average into A 1, and this is just like u naught into A naught. So, it is it is just like A 1 v 1 equal to a 2 v 2. We have to keep in mind again that what is v 1 and what is v 2 these are very important. Again I am repeating these are fundamentally the average velocities over the sections 1 and 2. See when we are writing here 1 and 2 or maybe 0 and 1 or whatever subscripts there is a fundamental difference from what we wrote for using the Bernoulli's equation. Those we wrote for points 1 and 2, now we are writing for sections 1 and 2. So, maybe subscript wise they look very similar, but meaning is entirely different.

When it is uniform; it does not matter it is as good as writing for a point, because velocity does not vary from one point to the other, but you do not have anything called as area at a point. It is basically when you write A 1, no matter in the context of what we have written earlier for 2 points when you use the Bernoulli's equation. We should keep in mind that there also A 1 was for the section that contained the point one. So, it is not that area of a point one or something like that. But there, we use velocity at the point one; the reason is that in the Bernoulli's equation we use velocity at a point. So, we had to link it with velocity at a point. Now it is like we are linking things through velocity over an area.

So, if you complete this problem you will get what is u naught, because all other things are known. Now let us look into some other examples where maybe we look into different case, maybe unsteady case.