Introduction to Fluid Mechanics
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Lecture – 42
Application of RTT: Conservation of mass

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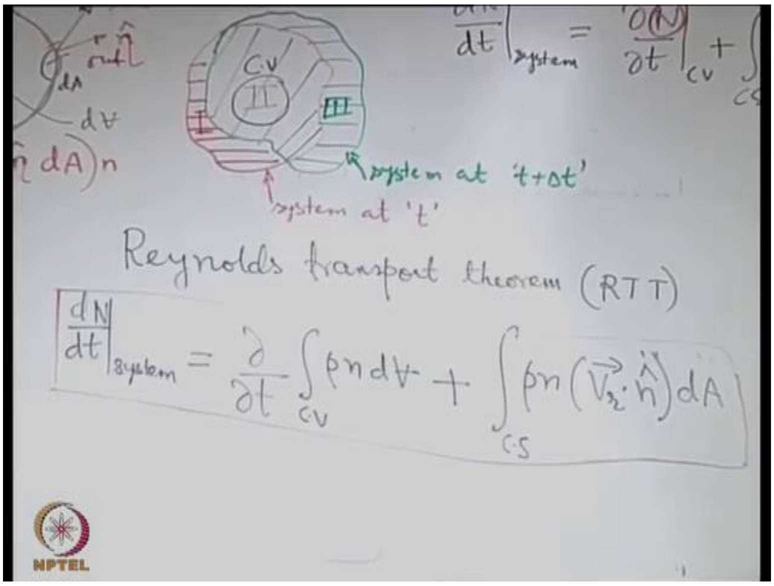

The good thing about this theorem is its generality. Till now we have never committed what is this n, and because we did not have to commit what is this n, we may try to apply it for different cases with n parameterized in different ways physically representing the different principles of conservation. As an example, we will start with the conservation of mass. So, in the fluid mechanics, we will be discussing about three conservation principles in this chapter, conservation of mass, conservation of linear momentum and conservation of angular momentum. So, we will first start with the conservation of mass.

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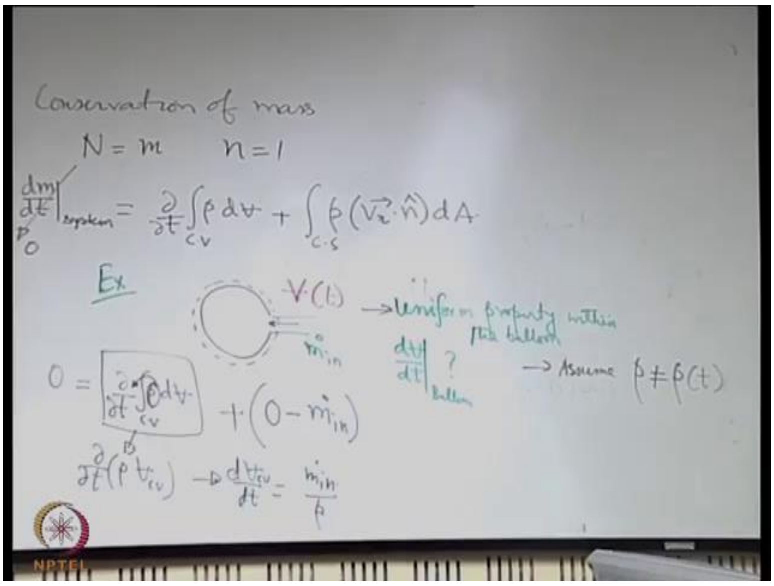

So, conservation of mass, when we talk about the conservation of mass what should be capital N, capital N is what. What is the property that we are conserving here, total mass of the system. So, capital N is equal to m, which is the mass of the system. So, what is small n is capital N per unit mass that is. So, let us substitute that in the Reynolds transport theorem and write d m dt for the system is equal to. Now look into the left hand side, left hand side represents what left hand side represents the time rate of change of mass of a system. What is the definition of a system, a system by definition is of fixed mass and identity.

So, the mass of the system does not change, mass within the control volume changes because something across the control volume is entering and leaving, but when you are considering the system, it is as if it is a conceptual paradigm that you are considering all the mass which comes into your analysis and that does not change. That may occur different positions at different time sometimes that mass is out of the control volume sometimes it is inside the control volume, may be again it is leaving the control volume going somewhere else, you are not keeping track of the mass. But it is a conceptual mass which is the mass that you are considering as conserved, so that does not change with time. So, this is always 0 by the definition of what is a system.

So, the integral form of the conservation of mass fundamentally is this one. This is very, very general and it all depends on that the special cases depend on that what further considerations that we make. See regarding the choice of the control volume, we have again remain very, very general. We have never committed that the control volume is stationary or fixed that is why in fact, this v relative we have introduced so that even if the control volume is moving, it has no consequence, it can still be applied. The other thing is we have not committed that the control volume is non-deforming that means, what is example of a deforming control volume.

Let us take an example let us say that you have a balloon. Initially the balloon is very small, but you are pumping air into the balloon. So, it is getting inflated. So, if you consider the control volume like this, which is like, which is encompassing whatever air is there within the volume then that volume is changing with time. So, this is if you consider that air within the volume as the air within the balloon as the constituent of the control volume and the balloon is being inflated then the volume of the control volume is a function of time. So, here you have this V as a function of time, this is an example of a deformable control volume. So, if you have a deformable control volume that is also not ruled out here because we have not committed here that v is non deformable

So, in the most general case, v may be moving and v may be deformable. Moving and deformable are two different things when it is when we say it is moving it did not be deformable it is like it may be moving like a rigid body. And what is deformable it might be locally stationary, but deforming; and when it is moving and deformable it is the most general case that is it might move as well as continuously deform. So, all those possibilities are there.

And let us try to see that in such a simple case how we make an analysis let us say that we are interested to find out let us say that we are given let us consider maybe a simple problem related to this. Let us take an example that you have a balloon like this there is a rate of mass flow of air into the balloon, which is being supplied by the pressurizing mechanism. So, some air at a given rate of m dot in is entering the balloon. The state within the balloon is such that you have a density it is considered that the property within the balloon is uniform. So, uniform property within the balloon is an assumption. Our object is to find out what is the rate of change of volume of the balloon with respect to time of the balloon.

So, how we do that, we use this integral form of the conservation, left hand side becomes equal to 0. What happens to the right hand side, look into this term integral of rho dv. When we are calling that or when we are stating that its uniform property within a balloon; that means what the density is uniform, but it may change with time we have earlier seen that uniformity does not ensure steadiness. So, rho in general could be a function of time, but at a given time rho is uniform everywhere. So, if at a given time rho is uniform everywhere, it may be possible to take the rho out of the volume integral because this the integral is testing the rate of change within the volume.

So, if you take rho out of the integral then this term this is only the first term this becomes rho into the volume of the control volume, because if you take rho out of the integral the integral of dv becomes the total volume of the control volume. And what is the remaining term plus. so what does it represent, it is the net rate of flow of what here the quantity is mass. So, it is net rate of outflow minus inflow of mass. So, outflow is 0, there is nothing which is coming out. And what is coming in, so outflow is 0 and inflow is m dot in.

So, you have the differential equation relating the volume, density and m dot in this way. Now, you may simplify further if you assume that the density is not changing with time; otherwise, you have to know that how the density is changing with time and it is it is not a trivial situation density is definitely expected to change with time. So, if you consider the density is invariant with time, it is not a very practical assumption, but at the same time you have to keep in mind that it is not a rigid tank. If it was a rigid tank and if you are supplying the air what will happen the density will increase with time always, but because it is a flexible balloon then whatever mass is coming in this balloon is getting adjusted to that.

So, it might be possible that the density is changing, but changing only slightly. If it was rigid that assumption would have been a very bad assumption, but because it is flexible maybe it has capability enough to adjust to that. Whether it has capability enough to adjust to that it depends on many parameters it is actually one of the very toughest problems in mechanics because it needs the understanding of what is the elasticity of the balloon material. So, based on that how it adjust to that changed in terms of the new density within inside is something that may need to be looked into more carefully.

But if you make an approximation that as if it is not only uniform, but also another assumption that rho does not change with time rho is not a function of time. Then it is possible to find out what is the rate of change of the volume of the control volume with respect to time that is m dot in divided by the rho. So, at least we have got a fair idea that they it could be a deformable control volume; and if it is a deformable control volume it is not a very trivial thing to deal with. So, this the first assumption may not be bad because if it is pumped well enough, it will assume a homogeneous distribution of the density quite quickly, but this is perhaps not a very correct assumption it may be approximate, but it is not so good. So, we will leave apart the deformable control volume and what we will try to do is we will try to now considered a special case of a non-deformable control volume.

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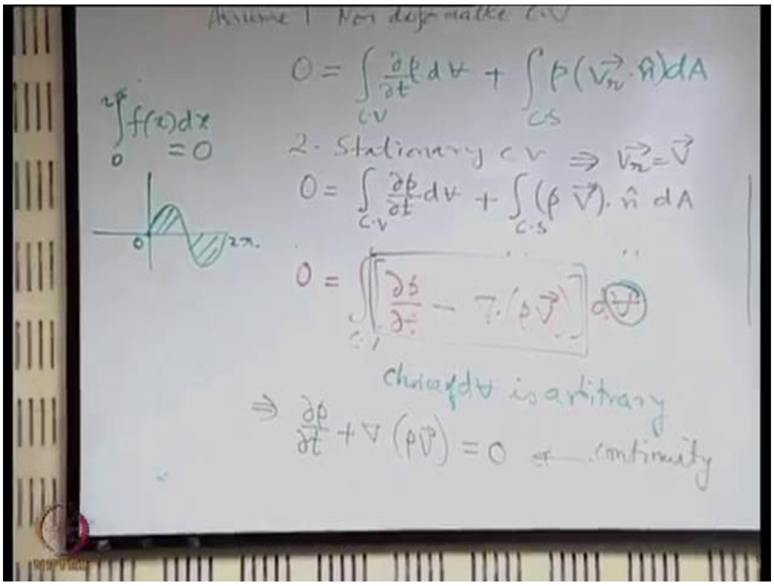

So, we will make two important assumptions first assumption is non-deformable control volume to further simplify the equation. So, if you have a non-deformable control volume then what happens? See, now we are looking into a very important aspect of mathematics, you have the derivative outside the integral. Question is can you take easily the derivative inside the integral that is, is it permissible that you take this partial derivative may be partial or ordinary whatever it is this derivative within the integral sign yes or no? In general, no, because if this v is a function of this time then in general no, but if this v is not a function of time, you can take this derivative within the integral. If this v is a function of time, then you can take that, but with an adjustment of certain terms and that is given by the Leibniz rule for differentiation under integral sign. But when you are considering a non-deformable control volume, we are easily able to take this the time derivative within the integral. So, then our form becomes 0 equal to.

Next, we will make another assumption. The assumption is that the control volume is stationary or fixed stationary control volume. So, if you have a stationary control volume then what is the consequence, consequence is V relative is equal to V, the relative and absolute velocity are the same if the control volume is not moving. So, then the equation becomes plus integral of rho V dot n dA. We will simplify this further by what understanding by a very important, but straightforward understanding that it is possible to express this area integral in terms of a volume integral by using the divergence theorem. So, the divergence theorem what does it state F dot with n dA over the control surface is equal to the divergence of F dV. So, this volume the surface has to the area has to be an area which is completely bounding the control volume that is a only important assumption, F is any general vector field.

So, here what is the F here rho in to V is the F here. So, in this particular example, F is rho in to V. So, the next step is 0 is equal to integral of now you write it, both are now volume integrals. So, this 1 plus. So, the next term has also become a volume integral. So, you are now left with what term integral of something dV equal to 0. The big question is does it mean that the term in the square bracket which is the integral, it has to be 0 yes or no? In a special case maybe yes.

So, let us say that let us think of say a function this a volume integral is a bit more general let us consider a one-dimensional case say you have a integral of f x dx. If integral of f x d x equal to zero can you say that f x equal to 0 in general; obviously, it it may not be in a special case like if you have say sine x type of thing. So, let us say you have integral from 0 to 2 pi. So, this area and this are same and opposite. So, the integral will vanish, but the function f x is not vanishing at all points within the interval. What we have to keep in mind that that is not for any arbitrary domain of the integral domain of the integration that is only for a particular limit.

Here the choice of the volume elemental volume is not based on a particular domain it is absolutely an arbitrary choice of an element within the total volume chunk. So, if you have to keep in mind that the difference between this case and this case is one very important thing. Here this dx is really restricted with the limits maybe 0 to 2 pi, but here this choice of dV is arbitrary that is a very important thing. Since, the choice of the dV is arbitrary then where does it land up if that is arbitrary then if this has to be 0 then the integrant should definitely be 0, because it is for each and every arbitrary choice of the elemental control volume and that means, that you must have the integrand equal to 0. So, it is not for a generally like interpretation for any case, but it is an interpretation based on the arbitrariness in choosing the control volume.

So, with that understanding, now we get this form of the equation, you can recognise that this is the continuity equation that we derived in a different way earlier. So, this gives you not only the continuity equation in a differential form as a special example it gives you a kind of mathematical skill or understanding of how to convert an integral form in to a differential form. So, we started with the Reynolds transport theorem integral form and we could show that we can come up with the differential form of the same conservation.

So, this is again representing the mass conservation principle, but in a differential form. So, it is possible to convert one form to the other keeping the physical meaning intact. So, that is one of the important strengths of the Reynolds transport theorem that it is possible to derive almost all conservation equations in whatever form you like by starting with the most general integral form of the Reynolds transport theorem. So, we will stop here today, and we will continue with more examples on this conservation in the next class.