Introduction to Fluid Mechanics

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Lecture - 40
Application of Bernoulli's equation-Part-III

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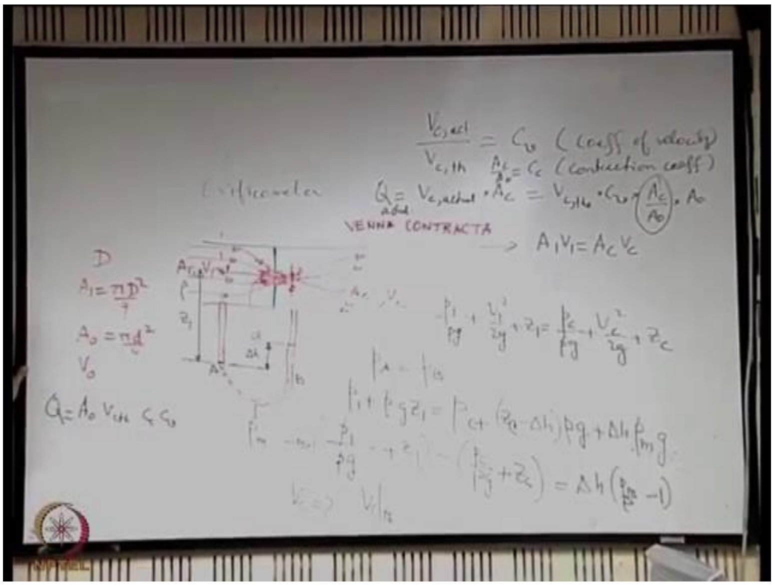

So, orifice meter is another application of the Bernoulli's equation. So, orifice meter is something like again the purpose is the same, that you have a pipe line you want to measure the flow rate through the pipe line. So, what you are doing here you are putting an obstruction in the form of a plate. So, this is like a circular plate, if it is a circular pipe it may be a circular plate with a hole at the centre which is called as orifice.

So, here also what happens here if you consider the stream lines the stream lines were originally say parallel to each other, not that they always have to be, but just as an example. Now you know that because of this constriction the stream lines have to be force to flow through this small section. So, stream lines will converge like this, and then when the stream lines pass through this so, there is also stream line at the centre.

So, when the stream lines pass through this constriction after that what happens, that is very interesting. So, after this stream lines pass not that they become, because of the inertial effects the stream lines go on tending to converge. So, there is not that after coming out of this they become parallel. So, they go on converging till the stream lines come to a condition where the distance between the extreme stream lines is a minimum, and then the stream lines tend to diverge again from that and the divergence is again to match with the pipe contour.

So, that type of stream line behaviour is there; qualitatively it is important to first appreciate this because from this we will get an apparent similarity with the venturimeter. What is that in the venturimeter? You try to have a deduction in the area available for flow, here also you are having the same thing, but what is the difference. Difference is in the venturimeter, you had a gradual transition from the bigger area to a smaller area, and here you are trying to have a more abrupt transition. And abrupt transition is something which is not so good, because the flow does not get enough opportunity to be adjusted to that abrupt change, and that may create additional losses; not only that the there are more uncertainties in the measurement.

Why there are more uncertainties in the measurement let us try to see. Again our policy will be that we will try to measure the pressure difference or to be more fundamental the piezometric state difference between 2 points. What are the points that we should choose? See when we are choosing a particular point we are making a tapping in the wall of the pipe right. So, as if we are making a hole and fitting a manometer, that is the arrangement; the arrangement does not change here the philosophy also does not change, but implementation becomes more difficult why.

See here you have taken it at a distance substantial enough from here, so that this effect in the curvature of the stream lines is not important. You are interested about the pressure at these points actually not; actually you are interested about the pressure at a point which is at the central line, but at the same section. There may be a difference in the in this pressures if the stream lines are curved.

But if the stream lines are parallel they that will be not. So, the pressure rate here and the pressure here will; obviously, mean almost the same effect of stream line curvature will not change anything. Here also if you want to utilize the same principle, you should come to a location where the there is negligible stream line curvature and that is there only at the place this has come to a minimum. So, if you consider a curve which has come to a minimum that tangent is parallel to the axis. So, at this location where the distance between the constituting stream lines or the extreme stream lines is a minimum, here almost stream lines are parallel to each other. So, there is negligible error because of neglecting the curvature of the stream lines at that location, and this location where the distance between extreme stream lines is a minimum is known as a Vena Contracta.

So, that is the name Vena Contracta and that is located somewhat away from the orifice, it is not exactly located at the orifice. So, if you connect this leem of the manometer at the position of the Vena Contracta then your analysis is quite good; question is how you will know where the Vena Contracta is located. One has to do a lot of experiment to figure it out and it is it depends on the flow conditions. So, it is not like a universal location where it will always be located. So, it is not that trivial to put the manometer location correctly, that is one of the big errors because we are assuming that the manometer leem is being put at the section of the Vena Contracta and we are writing our equations accordingly, but actually it may not be.

But let us say this is this is put in the section of Vena Contracta let us say that area of cross section of this is AC and the velocity of flow through this section entire section is uniform and is equal to V C; again we are assuming uniform velocity profiles which is a deviation from the reality and with such a kind of abrupt chain the deviation of from the reality is more severe.

Now, here also let us say we consider this as section one or may be a point one on the section one, but if it is a uniform velocity profile we consider we want to be same throughout the section, let us consider a one as the area of cross section which is basically if capital D is the diameter of the pipe, then A 1 is pi capital D square by 4. Let us say that small d is the diameter of the orifice, and let us utilize the subscript o to indicate the orifice. So, let us say A o is the area of cross section of the orifice, which is pi small d square by 4; where small d is the diameter of this orifice and let us say that V o is the velocity through the orifice again we consider it is a uniform, otherwise there is no meaning of the term velocity though the orifice it will vary across the section.

So, if you write the Bernoulli's equation between say 2 points, let us mark 2 points let us say we have a point 1 and a point C, point C is located on the same stream line as that of one, but in the Vena Contracta section. So, we are writing the Bernoulli's equation between points 1 and C along the stream line which is identified by this black colour.

So, what is the equation P1 by rho g, plus V 1 square by 2 g, plus V 1 is equal to p c by rho g, plus V C square by 2 g plus b 2. So, again the questions comes that how will you find out the difference between P 1 by rho g plus V 1 and P 2, p c by rho g plus b c that is by using the manometer principle. So, let us say that you have the depth of the leem as marked in the figure, and let us say that delta h is the difference in the reading of the 2 leems of the manometer. So, utilizing the principle of manometry you can write that you have A and B as these 2 points, you have pressure at A is equal to pressure at B.

So, pressure at A is nothing but pressure at 1 plus, if rho is the density of the fluid that is flowing through the pipe plus rho g let us say that this is the datum also with respect to which we measure the height. So, rho g, g 1 is equal to pressure at b is equal to pressure at c plus g 1 minus delta h, rho g plus delta h into rho of the manometric fluid into g where rho m is the density of the manometric fluid. This is say like the same equation what we had for the venturimeter there is no difference. So, from here you will be getting difference between p 1 by rho g plus g 1, minus P c by rho g plus sorry this is b c right plus b c; what is that that is equal to delta h into by rho m by rho minus 1 into g, g is already there so only this.

So, that you can substitute in this expression and you can write A 1 V 1 is equal to A c V C right. So, you can eliminate V 1 by expressing it in terms of V c. So, from this expression what we will eventually get, you will get V C by combining this manometric equation and a one V 1 equal to A c V C, but when you get this V C let us call it say V C theoretical because again we are used the theoretical equation, this assumes that you know the area of cross section of the Vena Contracta which you actually do not know. Now the actual velocity V C actual by V C theoretical this is not equal to one because of certain non idealities which have not been considered in this equation, just like the volume flow rates are also not same. The velocities calculated the actual and this theoretical configuration they are not going to be the same. So, this is again considered to be a coefficient C v this is called as coefficient of velocity.

Coefficient of velocity you have to keep in mind it is also a coefficient of ignorance used by the engineers, because actually we do not know what is the velocity we can only estimate from our reading the some kind of theoretical velocity, but these there is a difference between these 2 and because of losses the actual will be less that theoretical. So, this will be less than one.

Now, if you want to find out the flow rate Q, Q is the actual Q it is V c actual into the area of cross section of the Vena Contracta right this is Q actual. V c actual you can express in terms of V c theoretical. So, V c theoretical into C v into; now see area of cross section of the Venna Contract you cannot really measure when you are doing experiments what area of cross section you know with more confidence you know area of orifice; because that is like it is it is usually given the geometrical construction and everything the manufacturer knows exactly what it is. So, you can change the basis from A c by writing this as A c by area of cross section of the orifice into area of the cross section of the orifice, by changing the basis from A c to A o.

So, this is again another coefficient which is a coefficient of ignorance; we do not know, but we expect that the manufacturer has done a lot of experiment to figure it out, and this is again not a constant it depends on many things that what is the ratio of the big diameter to the small diameter what is the velocity of flow. So, it depends on many things, but if the manufacture has done lots of experiments and has calibrated the device again something more standard, then the manufacturer can give a data on that.

So, this we call A c by A naught as another coefficient, A c by A naught this we call as C c which is called as contraction coefficient. So, we can say that the final expression is we have Q is equal to A naught into V c theoretical both of which we have determined A naught you know area of the orifice, you know the V c theoretical from this simple analysis multiplied by C c into C v; and this we call as the C d here coefficient of this term. Because this is the sort of ideal flow rate, but the thing is this is a different this is bit different from the previous case, because in this case the areas and velocities are referred to 2 different sections.

Area is referred to orifice, but velocity is referred to Vena Contracta. So, that is the basic difference, but otherwise notionally it is like a sort of ideal velocity, and this is an actual velocity. So, by the definition of the coefficient of discharge it is like Q actual y q ideal. So, you can say that C d is equal to C c into C v; because these 2 combined non idealities are there in the calculation the C d is much less than what you get in a venturimeter. So, here the C d in such a device may be say 0.7, 0.6, 5.7 like that it is not as close to one as that for a venturimeter, that makes it a more inaccurate device than the venturimeter.

But the advantage is that is much cheaper than the venturimeter, you just have to put a plate with a hole in the pipe and put the manometer tapping's properly. Classically if capital D is the diameter of the pipe, this manometer tapping is kept at a distance of roughly say capital D from the plate and this is roughly like capital D by 2. This is one of the standard engineering practices of putting these tappings, it is not necessary always that one has to put that, but with a lot of experiments that has been found at then these 2 represent the proper sections with the kind of consideration that we are looking for.

So,. So, this type of device is known as orifice meter, and this plate with the hole is called as orifice plate. The whole objective is to reduce the cross section area, so that the velocity is increased and the piezometric head is reduced, and reduction in piezometric head is measured through a manometer. So, same principle as that of the venturimeter, but much less accurate one, in reality there is a there is a there is some device which is in between that is called as a flow nozzle.

So, what is the flow nozzle; we will not going to the detail construction of a flow nozzle we will just try to go through the philosophy, because it is something in between the venturimeter and orifice meter, it is the cost is in between the accuracy is also in between. So, what it does is instead of putting a sharp orifice with an abrupt chain, it puts a kind of nozzle at the wall, to have a more gradual change of cross section of the area. It does not make it as good as the venturimeter, but sort of compromise between the venturimeter and the orifice meter. So, that is the flow nozzle its performance is also a compromise.

So, with this kind of flow through the orifice let us consider a very simple example to illustrate it that in what other conditions these types of concepts of Vena Contracta also come into the picture, and one such example is something which you have encountered many times that if you have tank and if you have a hole through the tank, there is a water jet that goes out.

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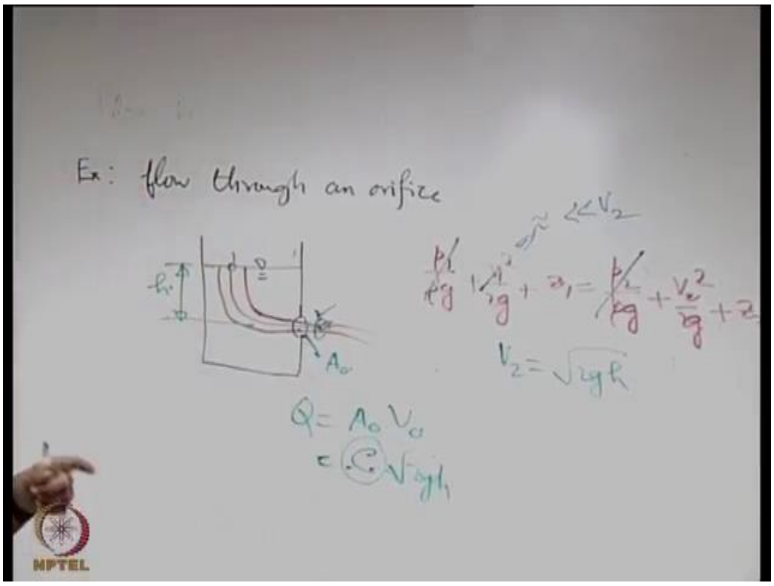

So, example of flow through an orifice; let us see have a tank like this and there is a hole, through this hole some water jet comes out, let us try to draw a stream line say. So, let us say you are coming from the free surface the stream line gets bent or curved to accommodate this one, all the stream lines which are there they are getting bent or curved, and just like the previous case the stream lines come to or converge to a location of minimum distance of separation between these 2 before they diverge, and then maybe the water is falling like this. So, the location where the extreme stream lines come to a minimum distance of separation is somewhere here, which is the Vena Contracta here, but not at the orifice ok.

So, this is the place what we are looking for; and let us say we identify a stream line from going like this, and we want to apply a Bernoulli's equation from in between the points 1 and 2. Along the stream line assuming it to be ideal, and let us make certain approximations so that it matches with the high school thing that you have learned. So, what are the approximation we will make we will make that we will we will assume that it is a study flow that is number one.

Number 2 we will assume that its friction less flow, and then we will also we will also assume that the area of the thickness of the orifice is such that it is much less than the area of cross section of the main tank. So, if you have that then you neglect V 1 as compact to V 2. So, if you write say P 1 by rho g, plus V 1 square by 2 g, plus z 1 is equal to p 2 by rho g, plus V 2 square by 2 g plus Z 2. So, once you have that what happens? See 1 and 2 we are assuming both are at atmospheric pressure. So, you cancel the 2 pressures V 1 you neglect as compared to V 2 and Z 1 minus Z 2 let us say that is equal to h. Which is the function of time may be, but at a particular instant therefore you can write V 2 is equal to root 2 g h a very famous formula known as Torricelli's formula, because Torricelli's has derived it this you know from high school physics.

Question is other than the approximation that we made one very important thing deviation that we have made from the high school physics what. We have not considered the area 2 to be at the tank orifice why? Because we have considered the pressure at 2 to be p atmospheric; if there is a serious stream line curvature then there is no guarantee that throughout 2 pressure is p atmospheric, because of the stream line curvature there will be a difference in pressure as you go across it. Only where it is a Vena Contracta that is true, because stream lines are parallel so there is no normal gradient of pressure across the stream line.

So, whenever you have called it that same p atmosphere we have to take this section to a Vena Contracta. So, that is; that means, this is not the velocity at the exit. So, if you want to find out the flow rate if you write the flow rate it should be A naught into the V 0, where o is 0 or o is the orifice, but this is actually the velocity at the Vena Contracta. So, you must compensate for this you can write this also in terms of the coefficient Cc, C v like that. So, your V 0 is not same as root 2 g h, and you can therefore write this Q in terms of the coefficients some coefficient C times root 2 g h, where this coefficient C takes into account these deviation there is not actually at this section that you are considering, but at a section which is located at the Vena Contracta that you have to keep in mind, so that is how.

So, this is like a coefficient of velocity times the area of cross section times root 2 g h, where this is like a coefficient that takes care of that non-ideality. Now finally, we will come into one example where we show the use of unsteady Bernoulli's equation for a practical device. So, let us consider that we have.

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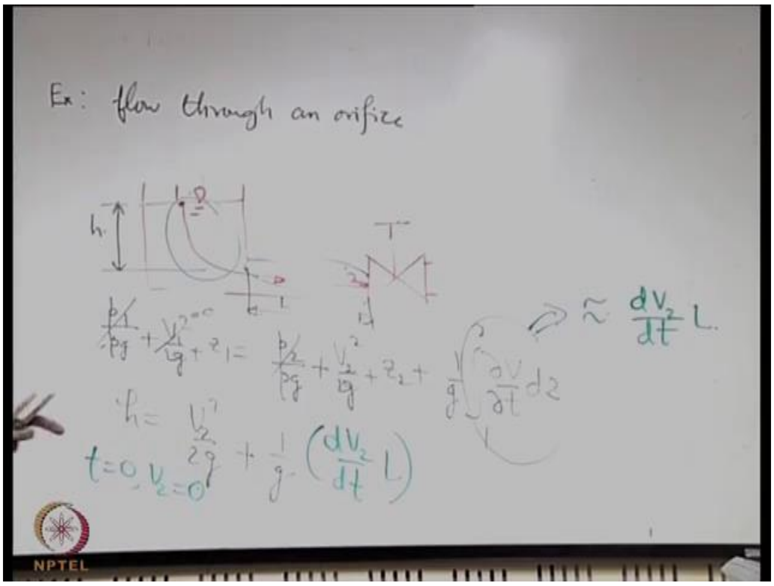

Similar arrangement like a tank with a pipe line, in this pipe line there is a wall. So, we have a valve this valve when it is fully closed it does not allow the water to be discharged through this pipe line. Now suddenly this valve is made open and water is allowed to flow. So, you have to find out that how the velocity changes with time, assuming the flow velocities to be uniform over each section.

So, then if we consider a stream line between the say points 1 and 2, and if you write the Bernoulli's equation here the velocity is clearly a function of time. So, you have to write P 1 by rho g, plus V 1 square by 2 g, plus Z 1 is equal to P 2 by rho g plus V 2 square by 2 g plus Z 2 plus integral of 1 by g that is extra term that you get because of the unsteadiness P 1 is like p atmosphere and when the valve is opened it is also released to the atmosphere. So, this is when the valve is suddenly made open that is what we are trying to analyse. So, then these 2 pressure are the same because then this is atmosphere when this valve is totally open that is exposed to atmosphere.

Let us say L is this length of this pipe. So, you can the and you may neglect V 1 as compared to V 2 if the area of the cross section of the tank is much larger than that of the pipe. So, let us say that you neglect that V 1 minus V 2 is like h let us say. So, you have h is equal to V 2 square by 2 g, plus 1 by g now you have to approximate this term. So, this really has 2 parts: one part is like you may consider the part within the pipe line and another pipe line within the tank. So, what is this, you have at each and every point you are locally finding this time derivative of velocity and integrating this over this entire length.

So, how you are doing it? You are doing it by considering may be this part and this part clearly for the part within the tank the velocity is much less than the part within the pipe. So, this may be approximated to be as good as the path within this length L and because the area of cross section is not changing here like d is not changing with the length. So, this is approximately same as like d V to d t into L.

So, this is like 1one by g into d V to d t into l. So, from this consideration you can you can integrate this by considering at time equal to 0, V 2 equal to 0 because time equal to 0 is the time at which the valve is suddenly kept open and then like you can say for at variables and integrate to find out how we do varies with time it is a very simple integration. So, the whole idea is that how you utilize this unsteady term properly to find out an estimate.

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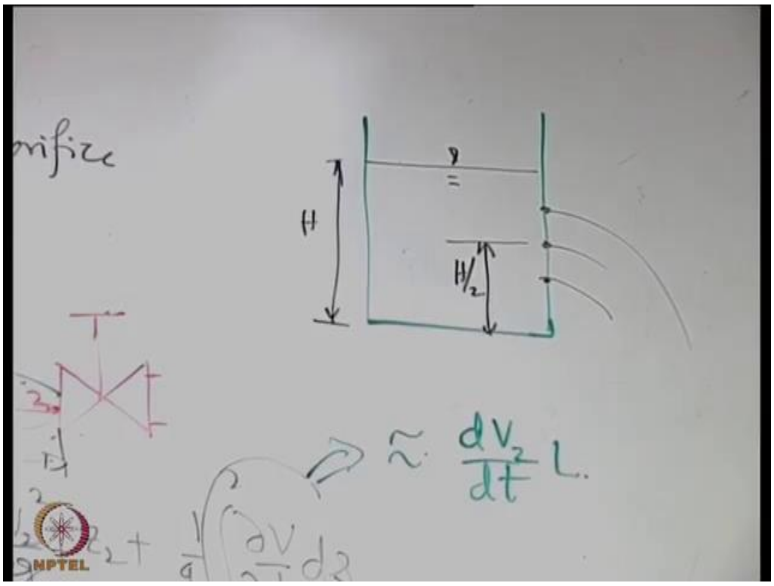

So, I will end up the discussion of this class by giving you a very simple exercise again like junior class level problem. So, you have a tank like this, you have 3 holes in the tank, if this height is h, the central hole is at h by 2 and the others are symmetrically located one at the top and one at the bottom. So, when water jets are ejected like this which one will traverse the greatest distance, this is like your entrance examination problem you have solved it many times.

Now, you try to figure it out. Can you tell from your ay maybe memory or whatever which one should be the most?

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Yes.

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Not the upper one not the lower one, but the middle one, and now with all these background that we have developed your objective will be to find out yes the middle one will be like that, second is what are the approximations or assumptions under which that analysis will hold true. So, I hope that you will complete that exercise. So, with that we stop our discussion today. In the next class we will start with a new chapter the conservation equations for control volumes.

Thank you.