Introduction to Fluid Mechanics

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Lecture - 39
Application of Bernoulli's equation-Part-II

In our previous lecture we were discussing about the Venturimeter, and we will continue with that as an example.

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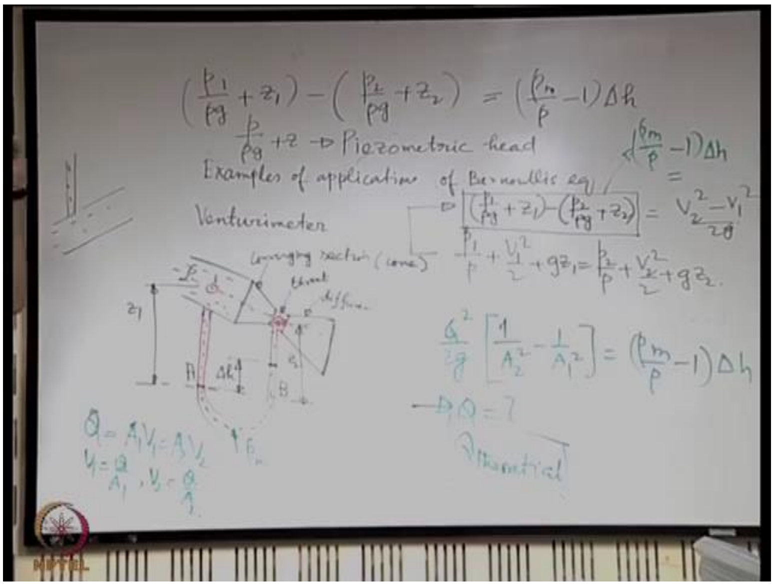

So if you recall; what was the purpose of the Venturimeter; the purpose was to measure the flow rate volume flow rate through a pipe. Now for that we utilize this type of an arrangement where you have a converging section, which is sometimes also called as a converging cone then you have a throat where the area of cross section of the entire arrangement is a minimum and then a diverging section. And the purpose is obvious that this part has to fit with the pipe. So, if you have reduced the cross section area somehow you have to increase it so that it again matches with the pipe.

So, this black coloured portion is like a fitting, which is fitted with the pipe to measure the flow which is occurring through it. The question is why you have such a converging section we have seen that, it gives rise to an accelerated flow. So, it gives rise to a high velocity and therefore, a change in pressure; and we have seen that it is not just the pressure that is important, but the sum of the pressure on the elevation head together that is P by rho g plus Z that term together is something which is changing, because of the change in the kinetic energy head and we gave it a name called as Piezometric head. So, the difference in the Piezometric head is reflected by the difference in reading or difference the reading in the difference in the heights of the liquid columns in the 2 limns, and that delta h therefore, we will be an indicator of the rate of flow which we found out by utilizing the Bernoulli's equation.

The question is that is it a very reliable way of finding out the flow rate. The answer is straight forward in some sense that it is not reliable, the reason is that we have utilized an ideal type of equation this all the assumptions which are there inbuilt with the Bernoulli's equation in steady form or inbuilt with this and therefore, all the ideal idealities which are also inbuilt with this form of equation, those are assume to hold at the ame time we understand that in practice such idealities do not hold true; what is the biggest deviation in practice it is never a friction less flow. So, you have viscous effects and because of the viscous effects something happens or quite a few things happen.

Now 1 of the important things is because of the viscous effects if you have these are the total head or expressed in terms of different units total energy, represented it is a representative of the total energy at section one; and this is a representative of the total energy when we say energy here we only mean the mechanical form of energy at section 2. We assume these 2 are equal because there is no loss because it is a frictionless flow, in practice there is a loss. So, you expect that if you call this as say E 1 and if you call this as E 2 you expect that there is which 1 is more E 1 is more or E 2 is more.

Student: E 1.

E 1 is more because you expect that there may be a loss of energy and the loss of energy will be because of the travel of the fluid from the initial point to the final point. So, when travelling from 1 to 2 it will have a loss so; that means, E 1 will be more so that when you come to the section 2, it is E 2 plus some losses; the losses which have taken between 1 and 2 in 1 of our later chapters we will try to characterise these losses in a more formal way, but we will just keep in mind that there are certain losses because of viscous effects; and these losses will give you a guideline of like what is the direction of flow. So, say you know nothing you are given some E 1 you are given some E 2, if E 2 is greater than E 1 you must be assured that the flow is taking place from 2 to 1 not from 1 to 2. So, it is basically taking place from a high head to a low head, it cannot be the other way because where will the head come from.

Now, here because of this loss what will happen? See this eventually will boil down to a large larger drop in the Piezometric head. So, if it is flowing from 1 to 2 the Piezometric head which is coming into the picture, the Piezometric head drop that is the difference between the 2 terms present there in the bracket, this you expect to be more. Because you expect a more severe drop in pressure, because of the overcoming the frictional resistance effects; that means, whatever delta h you read here it is not the ideal delta h, see in this formula what delta h you put to get the value of ideal Q this delta h is what you experimentally observed. We have to keep one thing in mind what is the basic principle of measurements that we use in experiments, that we have an expression in which we have certain measurable quantities, certain easily measurable quantities and we express some more difficult to measure quantities in terms of the easily measurable quantities.

So, here delta h is something which you measure easily, flow rate you do not measure directly, but use this formula to write flow rate express flow rate in terms of the measure delta h. So, here may be to get Q ideal, it would have been better if you put delta h ideal; but that you cannot do because delta h is what you reading from the practical thing. So, it is giving the delta h which has got manifested because of by considering all practicalities. So, this delta h also considers the practicality that there is some loss of energy to overcome the fluid friction effects ; that means, this delta h is higher than what delta h would have been if it were a friction less flow.

But, you cannot help this is what you read experimentally and it is what you put here; that means, even in terms of theoretical flow rate it is not giving the correct theoretical flow rate it is over estimating that; because you are putting an over estimated delta h the reason is straight forward because experimentally you cannot reveal an idealist picture experimentally you reveal the real picture where the delta h is much more severe than what delta h you could get in a friction less condition.

So, one important thing we realize that if you consider no other effect, this particular effect alone should give you that Q actual should be less than Q theoretical, which you calculate by using this formula, but there are other important reasons also what is the other important reason see when you have written V 1 square by 2, actually we were bothered about the velocity at the point 1.

But the velocity of the point 1 how did we evaluate? We evaluate by using this Q equal to A 1 V 1 V 1 equal to A 2 V 2. So, by that we implicitly presume that V 1 and V 2 are like same as the average velocities over the section, that is possible only when the velocity profiles are uniform over the sections 1 and 2, but because of the viscous effects we have seen that those are not uniform. So, there is a non ideality because of this viscous effect not only in terms of the frictional resistance, but also in terms of putting the V square by 2 term. So, there is also some error in that. So, these 2 errors are very very significant, 1 error is the frictional resistance another error is the like miscalculating the velocity expression or misinterpreting the velocity expression.

So, when you put the velocity here ideally you should have put velocity here in such a way that this could have been represented the kinetic energy across the section one. Again in our later chapter we will see that how to exactly put that, but here we will just appreciate that we have not put it correctly. So, whenever we make a mistake in writing something, the first and foremost thing is to appreciate that we have made a mistake. So, let us appreciate that this is not correct; there is some error in it.

Now incidentally engineers are such classes of people who are happy to get the final result disregarding may be some mistakes which have already been done, and then to adjust that mistake let us say that some adjustment factor is put, let us say we call a new coefficient C d as Q actual y Q theoretical. So, if somehow this coefficient is known to us whatever by magic we will see what magic will tell us this number, but if somehow you get this value of C d then you straight away multiply that with the Q theoretical that you get from here to get what is the Q actual; if it is very close to the final result that you want in many practical engineering applications people are happy.

So, we have to see that what is there in the C d which will try to make a more and more happy, and this coefficient is known as coefficient of discharge. So, this takes into account that we have realized that if it is an ideal case all together this would have equal to 1. So, deviation of this form 1 represents the extent of non ideality in the flow, and not only non ideality in the flow in general, but more specifically how that non ideality has got manifested in the prediction of flow rate. So that means, what is the total influence of these frictions in terms of the delta h and what is the influence of the inaccuracy in the velocity distribution that has already got inbuilt in the corresponding expression for energy.

To look into that we will not going to all sorts of detail, but we will just consider one thing that see at the section 1 say we have sort of uniform velocity profile, somehow we have maintained. The again it is we will later on see that it is not easy to maintain that, but fortunately we will be easily maintaining such a situation when the flow is turbulent and when will be discussing turbulent flows we will see that, turbulence is a kind of situation which will create almost a uniform velocity profile over the section. So, that will in some way take care some of our acts of ignorance in writing or describing the current velocity profile, but even then let us try to see that at which section the error will be more severe at section 1 or section 2.

To do that let us say that we consider 2 stream lines which are very close to each other; say you have a stream line like this and another stream line which is very close to each other, both stream lines are connecting the sections contained by 1 and 2. So, here 1 and 2 are points, but let us say that these are sections which are contain the points 1 and 2. So, we have 1 stream line and we have another stream line.

Now, these 2 stream lines are very close. So, close that let us say that here the velocity is u 1 here the velocity is u 2 here the velocity is u 1 plus delta u 1 and here it is u 2 plus delta u 2; where delta is a small change in comparison to the other value. Now if these stream lines are very close to each other what will happen? There will be negligible difference in pressure between these 2 stream lines.

Always remember that there is a difference in pressure between the stream lines because of the curvature effects of the stream lines, but stream if they are very close that effect is negligibility. So, if these 2 are very close there is negligible difference in the pressure head between these 2 stream lines at 1 and at 2, and if they are very close there is negligible difference in the height also that is the Z coordinate. So, what we can say is that the difference in kinetic energy heads between the 2 points, it remains same for the stream line above and the stream line below because the other terms they do not change. So, how we may reflect that in our analysis let us try to do that.

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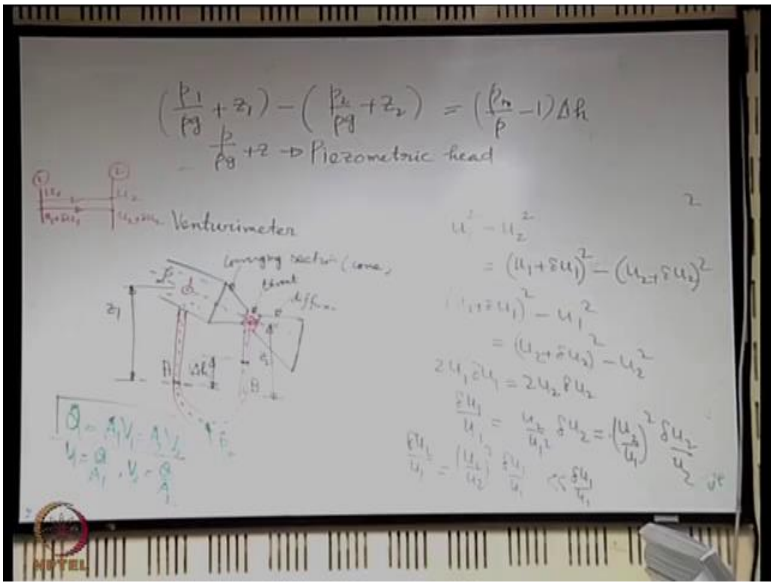

So, what we want intend to write is u 1 square minus u 2 square is equal to u 1 plus delta u 1 square minus u 2 plus delta u 2 square. So, let us try to simplify it keeping in mind that delta u 1 and delta u 2 are much smaller as compared to u 1 and u 2 respectively. So, you are clear that why such an equation has come because other terms like P by rho g and Z those are the same. So, they have cancelled out when you consider the Bernoulli's equation for the stream line above and stream line below; and when you have subtracted that they that effect have gone. So, only these terms remain basically u square by 2 and all those things, but that division by 2 get cancelled out that is how these terms come out.

So, if you now write it like this. So, you will. So, what you have here. So, you can write this as I mean u 1 plus delta u 1 square that you can bring in one side, u 1 plus delta u 1 square minus u 1 square is equal to u 2 plus delta u 2 square minus u 2 square. So, you can write it as 2 u 1 into delta u 1. So, when you write this like a square minus b square formula when you write a plus b it is 2 u 1 plus delta u 1, delta u 1 is much smaller as compared to 2 u 1. So, only 2 u 1, and the difference only delta u 1 is there, so that is equal to 2 u 2 delta u 2. We are interested to express delta u 1 by u 1 in terms of delta u 2 by u 2; to see that what is the relative error in relative change between the velocities into adjacent stream lines. So, you will have delta u 1 by u 1, as if you had dividing both sides by u 1 square. So, you write as u 2 by u 1 square delta u 2. So, here you can write this as u 2s by u 1 whole square into delta u 2 by u 2 right.

That means delta u 2 by u 2 is equal to u 1 by u 2 whole square, into delta u 1 by u 1. Now which velocity you expect to be more u 1 or u 2. Look at the sections 1 and 2 here the area is large, so the velocity will be less. So, u 1 by u 2 more is this reduction in area u 1 by u 2 will be lesser and lesser. Square of that will be small so from this our conclusion is that this delta u 2 by u 2 is expected to be much much less than delta u 1 by u 1, provided there is a greater reduction in section; that means, if it is approximately uniform at section 1 it will be even better at section 2. Because the non-uniformity is much less, what this represents a non uniformity when you go to a different stream line along the same section, you expect the velocity to be different and that difference give rise to a non uniformity.

So, when you have this non uniformity, but again see this an estimation, because for estimating the non uniformity we have again utilised the ideal equation which is like the Bernoulli's equation, but what we have consider that even for a non ideal case this is not very very invalid, because whatever is the frictional effect that also has cancelled out when you subtracted the Q equation; assuming that the frictional effects are also same as the fluid flows from 1 to 2, along the 2 stream lines above and below.

So, even if frictional effects are considered and they these the Bernoulli's equation for the 2 or the modified Bernoulli's equations considering the frictional effects they are cancelled, or they are subtracted one from the other that effect will cancelled. So, this is not a bad estimation. So, this estimation shows that if the if the velocity is such that you are going towards a cross section of reduce size, if at the bigger cross section the velocity was more or less uniform, the smaller section it is expected to even more uniform. The reason is quite clear that if there were stream lines like these, stream lines will more converge to each other because they are now confined to be there within a very small space as compared to how they were earlier. So, if the stream lines were are quite large distances apart. So, if the stream lines were like this.

Now, when the stream lines are confined; so what will happen all the stream lines will try to converge. So, when the stream lines try to converge, you see the distance between the stream lines corresponding stream lines become smaller and smaller; and eventually different stream lines represent the sort of like different states of flow. So, if you have them quite close to each other and almost parallel to each other, that non uniformity in the velocity is almost like it is not like totally nullified, but it becomes a better situation.

So, by having a section 2 like this which is like a convergence sections it is not bad, it sort of eliminates one non ideality. The other non-ideality because of negligibility friction that may be reduced to some extent by what; by minimizing the length travelled between 1 and 2 because the frictional resistance will be related to how much length the fluid has travelled against the viscous effect.

So, how do you reduce that? One of the ways is like is you have this angle this cone angle quite large. So, that it converges quickly to a small section, in practice this angle is like typically kept as like 20 degree or so. These are like designed considerations of this device, it is not that it is 20 degree is a magic number, and it is always kept like that I am just giving you a rough idea of what is the range in which it is kept in practice.

Now, there are different issues like you cannot make it as large angle as you like there are issues of manufacturing the device and so on. It is not that whatever angle you want and you propose one has to also fabricate it and put it in practice. One particular aspect on which one may not make a compromise which is like by putting by locating this section 1, where you are having this manometer leem. It should be preferable somewhat away from the place where the reduction has started. So, that this disturbance is not influencing the velocity at this point significantly, and that is why it is kept little bit away from this one and roughly it may be a if the diameter of the pipe is d it is roughly like distance d away I mean it is it is again a rough estimate there are more accurate estimate for each device.

So, the connections of the pressure tappings are also very important, that is where are they to be put. So, one is here then it is roughly like 20 degree and this creates a good accelerated flow if you if you achieve it in a very small or a short distance it is good, you have less frictional resistance; and smaller the cross section you expect that more will be your resolution in terms of these delta h. So, the experimental objective is the delta h is if the delta h is more it is better because that is you reading. If it is very small your error in resolution will be will be effecting your result significantly, but if the reliability of this is good.

Then the error corresponding error is less that is why you are trying to trying desperately to reduce the cross section area, so that there is a change in the kinetic energy head very severely which is manifested in terms of this delta h. Now after this section has come and then what you have to do, then you have to revert back to the pipe diameter again. So, you have diffuser which is like a diverging section. So, you have a converging cone you have the throat where you have the minimum area and then you again have a diffuser. The question is what should be the angle of this diffuser? I mean do not get confused with the sketch that I have drawn in the board, it is just because of lack of space that I have drawn it not to scale. So, this does not this angle does not represent what is there in reality it just represents the shape, but not really the sense of the angle. So, what should be this angle?

Now, again there are 2 conflicting requirements: engineering is such an area where when you want to design something there are 2 aspects that you have to keep in mind, one is it should satisfy the fundamental scientific requirements, so that the device is based on a thorough scientific principle. The other important thing is that it must optimally satisfy the performance requirements. So, what are the corresponding in influencing parameters always you will see that there will be opposing parameters. So, opposing parameter means if you increase this angle then something good will happen and something bad will also happen. So, let us see that if we increase this angle of the diverging section, first let us see that what good thing will happen that is very obvious.

So, if we increase the angle of the diverging section what good thing will happen? Yes if you increase the angle of this what is the good effect of that? The portion will decrease in length. So, in a relatively short length this device will merge with the pipe. So, the loss due to frictional resistances will be less. So, just like I mean what would have been a good effect of making this angle large the same logic hold there also, but one of the logics that does not hold is that there is a great difference between and accelerating and decelerating section this is an accelerating section, but this is a decelerating section why this is a decelerating section.

So, if you see the area of cross section is increasing. So, you expect the kinetic energy head to reduce; that means, if you expect the kinetic energy head to reduce; that means, P by rho g plus Z that Piezometric head will increase to compensate for that. So, if you say let us say that you are having a horizontal venturimeter. So, if you having horizontal venturimeter Z 1 and Z 2 are the same or may be 2 and 3 here you consider another point 3 Z 2 and Z 3 are the same.

So, then if you go from a point 2 to say a point 3, you expect what. If these 2 are located at the same height, then what you expect you expect that pressure will increase or decrease pressure will increase. So, when pressure increases; that means if you consider the direction of the flow as x. So, you can write the d P d x as the rate of change of pressure with respect to x. In the converging section d p d x was what less than 0, but here in this particular section d p d x is will be greater than 0 because pressure is increasing with x, what is the consequence? The consequence is see you expect that if the pressure is decreasing we take that is fine, because then a higher pressure is creating a drive for you. That if you if pressure is decreasing with x; that means, P 1 is greater than P 2, and that is the in some way it is trying to create a driving force for you.

On the other hand here from 2 to 3 the pressure is opposing you, because as you are moving from 2 to 3 you are experiencing higher and higher pressure so; that means, it is a sort of effect that tries to inhibit the motion of the flow. So, that is why this type of pressure gradient is called as adverse pressure gradient. So, this type pressure gradient is called as a favourable pressure gradient.

So, favourable and adverse the English names are quiet clear, favourable means which favours the flow and adverse means which is not good for the flow. So, when you have an adverse pressure gradient which is like this d p d x is greater than 0 what happens? The flow has a tendency to be decelerated because of that kind of a pressure gradient. So, if you try to sketch that what happens to the stream line in such a case? So, the flow tries to move like this, but because of the deceleration effect and the deceleration effects are more severe close to the wall.

Why because viscous effects propagates from the wall. So, at those locations what will happen is the flow may not be capable enough of being dragged with the main or the core flow. Because it is slowed down so severely that it just creates a local rotation, but it does not contribute to the main flow. So, that type of thing is called as a flow separation; that means, you have a main flow like this, now the flow or the fluid particles this poor guys close to the wall they are so severely disturbed because of the adverse special gradient which are acting on them, that they really cannot maintain the flow and they might even reverse their direction of flow. So, local vertices are created close to the wall.

How do these vertices contribute? They contribute in a sort of negative way, see these vertices by virtue of rotation have some energy, but that energy is not contributed to the main flow. The main flow is like this which is moving; now here this energy which is there because of the rotation of this vertices because of flow separation, that does not contribute. So, effectively as if some energy is taken away from the main flow to sustain the rotation of these vertices. So, effectively there is a kind of loss of energy of the main flow, and that loss has been created because of this flow separation; and this flow separation effect is stronger more is this angle of diffuse diffuser. The reason is more is this more severe will be the adverse special gradient, because more severe will be the pressure increase over a given length the length becomes smaller. So, this is a conflict with the requirement of the frictional resistance.

So, we have seen that if we increase the length of this one or may be reduce this angle then this effect will be less. So, the adverse pressure gradient effect will be less if you make this angle quite small. So, that this length is large, but if this length is large the direct frictional resistance will be more. So, these 2 are 2 conflicting parameters in the design, that is where you have to come to an optimal design where you cannot keep this angle may be as large as this, and the common optimization is that this is typically like 5 degrees, 6 degrees like; much less than the angle of the converging section.

So, this is something we have to understand very carefully that why in the practical design the diffuser angle is much much less than the converging section angle. When you have a converging section you do not have such a case of flow separation. So, only the frictional resistance is because of the length is the only important resistance, because flow separation will be there when the flow is decelerated, but in the converging section the flow is accelerated.

So, it does not suffer from a resistance because of adverse pressure gradient. In fact, the pressure gradient here is favourable which makes it move in a much more convenient manner. So, the design aspects are quite clear that why you should have different angles for converging and diverging sections, and what are the parameters which should decide the range of these angles.

And keeping these things in mind if one may designs this device quite well by minimizing the losses, then the coefficient of discharge which is the ratio between the actual flow rate and the ideal flow rate, it is actually very close to 1; 0.98 - 0.97 like that. So, somehow if the device is very cleverly designed, some of the non idealities are taken care of in some way not that it becomes ideal, but our ignorance about non ideality does not get manifested so much. The reason is that one is you are using a like a continuously converging section in this way, and the diffusing section is also say properly well designed.

Now this venturimeter is therefore a very common device which may be used in a pipe line to measure the flow rate. At the same time this is not a very inexpensive device. It is not very highly expensive, but at the same time for very routine applications one might look for some cheaper devices which are broadly following the similar principles. And let us see one such device that device we call as orifice meter.