Introduction to Fluid Mechanics
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Lecture - 35
Euler's equation in streamline coordinates

We are now going to discuss the Euler's equation of motion in streamline coordinate system. So, let us consider a streamline coordinate system as we were discussing in the previous class.

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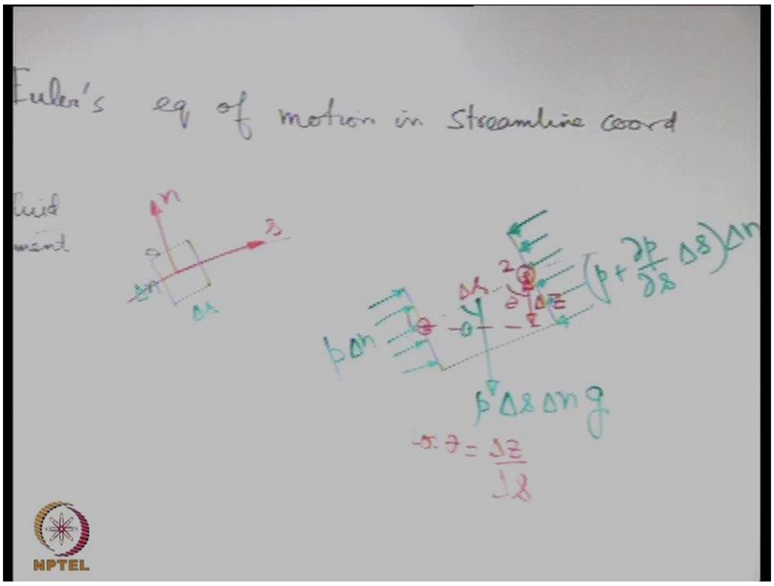

Euler's equation of motion in stream wise coordinates or streamline coordinates. So, we consider that there is a streamline and we consider a coordinate system such that you have a coordinate system s n where s is given by the stream wise coordinate fundamentally it is not exactly the tangential coordinate, but it is oriented just along with the streamline, but effectively it is just like a tangential coordinate the tangential coordinate is a coordinate along a direction which is given by a slope and this is along a direction given by the curve. So, it moves, it is a coordinate system that is allying with the streamline itself, but locally it is as good as like a tangential coordinate and n is the normal coordinate.

So, with in this coordinate system let us say that we have a small element of fluid like this is a fluid element, but this fluid element has a specialty we have now considered this fluid element to be sort of coaxial with the streamline at a given location let us try to identify all the forces which are acting on the fluid element we separately draw it for clarity. So, let us say that we have a fluid element like this and let us consider that the center line of the fluid element is such that it is representing the streamline locally now we are going to identify all the forces which are acting on this fluid element. So, what are the forces which are acting on this fluid element again there are forces which may be resolved in the coordinate directions s and n we will first write the equation of motion along s. So, we will only identify the forces which have components along s. So, we consider the left face where you have a pressure distribution and what is the resultant force on this if p is the pressure let us identify the element by its dimensions along n and s say delta s and delta n.

So, p into delta n into 1 is the width perpendicular to the plane of the figure then there will be a pressure distribution here p plus this into delta n, remember whenever we are talking about the Euler’s equation the most fundamental assumption that we are making that is an in viscid flow. So, no question of any shear component of a force. Then on the other faces there are forces which are acting along the n direction. So, those will have no components along s, but something else will have a component along s. What is that? The weight of the fluid element should have a component along s. So, let us identify that let us say this is the weight of the fluid element and let us say that it makes an angle theta with the direction of s. So, what is the weight of the fluid element let us say rho is the density of the fluid element. So, rho into delta s into delta n into one is like the volume of the delta s into delta n into 1 is the volume of the fluid element rho into that is the mass and that into g is the weight.

One important thing is although this is a curvilinear coordinate system remember this is not a rectangular coordinate system this is a curvilinear coordinate system, but when you take small elements this almost behaves like a rectangle although it is a curvilinear coordinate system that is one of the advantages of taking the small element. Now we can write this component of this force as you can see from this figure the component of the force along the direction of s as related to cosine of the angle theta and that may be described in terms of the difference in vertical elevation between the points one and 2 which are located say at the centers of the 2 faces the left face and the right face let us say that these points are located at a vertical elevation of delta z. So, delta z is what delta z is the difference in height between the points 1 and 2 which are the centers of the faces. So, you can clearly see that this angle will also be theta and you can therefore, write that cosine of the angle theta is given by delta z divided by delta s. Because the other length is delta s that is the axial length of the element.

Now we can write the Newton's second law of motion for the fluid element because we have identified all the forces which are having some components along the s direction.

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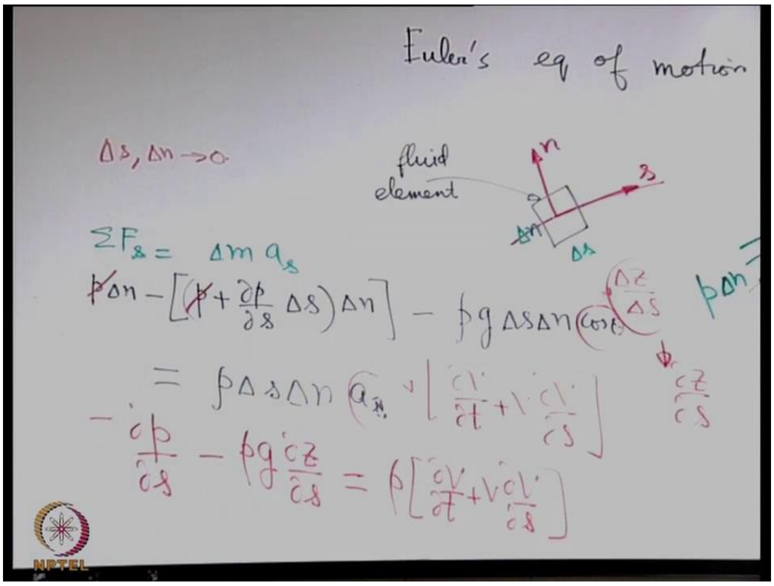

So, resultant force along s should be the mass of the fluid element times acceleration along s let us simplify this. So, you can write p into delta n minus the other term then the weight component should have minus rho g delta s delta n into cos of the angle theta is equal to rho delta s delta n into the acceleration along n sorry acceleration along s. Now you can simplify this equation by considering the previous discussion that we have regarding the description of the cos theta. First you can cancel certain terms next delta s into delta n both this product gets cancelled out from both the sides we have to remember we are considering delta s delta n small tending to 0, but definitely not equal to 0 we can express cos theta as delta z by delta s and we can express acceleration by what should be the acceleration along s see the expression should be similar as we had acceleration along x y or z.

So, it should be this one see what is the advantage of using a streamline coordinate the advantage of using a streamline coordinate is that seems the flow is always tangential to the streamline with respect to the streamline it is locally like a one dimensional case because you are not having any cross stream wise velocity components. So, this is like a one dimensional case of course, the velocity may be function of time, but otherwise in terms of special variation it is only a function of s because flow is always tangential to the streamline that is how the streamline is defined and for further simplification we have to keep in mind that in the limit as delta s tending to 0 this is as good as. So, it becomes the derivative in place of the differences because in the limit as delta s tends to 0 this will become the derivative just from the basic definition of the derivative. So, what we can say here that minus partial derivative of p with respect to s minus this one is equal to you will, you can clearly see that this if you integrate with respect to s this will become the Euler's equation of motion in the streamlines coordinates we have derived that from a different point of view by considering the equation of motion along x y and z and then using that to find out the net change in pressure.

So, if you want to find out the change in pressure as the across between 2 points located along the same streamline you can integrate this and the integration is; obviously, along a streamline because you are using a stream streamline coordinate system. So, integration is along that line only and it will follow exactly the same form as that of the other form that we have developed by this time you can clearly see that this v d v d s type of term you can write d d s of d square by 2. So, you have all the terms like d d s of something integrate with respect to s this will become like change in pressure this will become like rho g d z this will become rho d v d t multiplied by d s and this will become d of v square by 2. So, then the partial derivative and d will become similar because v is a function of s only provided v is not a function of time, but otherwise we have to keep in mind that if v is the function of time also this partial derivative nature has to be maintained. So, for unsteady case one has to retain it like a partial derivative.

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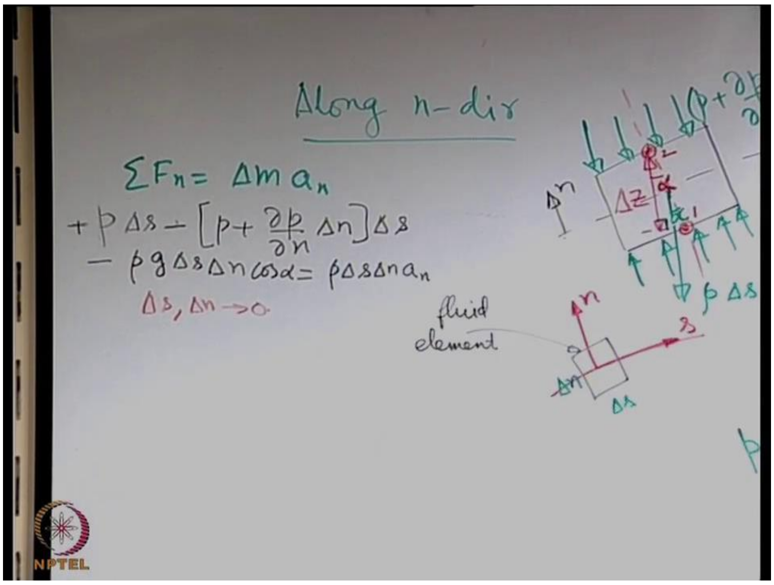

Now you can write similar expressions for the n direction. So, for the n direction if you write let us see that how it should look like. So, equation of motion, along n direction usually we are happy to write it along the s direction because we feel that that gives us enough information because flow is along that s direction, but along the n direction something interesting also is possible and let us try to look into such possibilities. So, the same element let us draw it may be again and we are interested now to capture what happens along the n direction. So, we will now write all the force components with an understanding that we are interested for the forces along the n direction.

So, let us represent those forces in the n direction now you have from this side you have what is the force p into delta s and from this side p plus the partial derivative of p with respect to n into delta n that into delta s again this has its own weight. So, one has to consider the weight of the fluid element and when you consider the weight of the fluid element now we are not bothered about the left and the right face we are bothered about the bottom and the top face let us say again that the centers of this faces are located at a vertical elevation difference. So, this is say one this is 2 and there is a difference in vertical elevation or the height between the points 1 and 2; say that is given by delta z.

So, delta z is what delta z symbolically in either case represents the vertical elevation between the 2 faces which are contributing to the forces along that direction and again let us say that this makes an angle theta with the vertical direction is making an angle theta with the n axis of the element now if we write or if we describe the weight of the fluid element in the same diagram that is oriented along z minus z; obviously, and it is again rho delta s delta n g. So, it has a component of again its component along the direction of n is this into cos theta remember this theta and this theta are not the same this is a different figure differently defined theta. So, whatever was defined theta here it is actually 90 degrees minus that. So, if you want to avoid confusion may be let us give it a different name. So, that later on when you look into it you are not in confusion. So, let us say this is alpha which is like 90 degree minus theta.

So, you can see that the cos alpha component of this is what contributes to the force along the n direction and cos alpha is given by what is cos alpha delta z divided by delta n. Just like what we got from a right angled triangle in this figure, similarly here also we can. So, let us write the equation of motion for this case, resultant force along n is equal to the mass of the fluid element times acceleration along n the left hand side will be very similar to what was along s except the s's are being replaced by n. So, let us write the left hand side. So, p into first minus p into delta s into delta sorry minus p into delta s then plus p plus partial derivative of p with respect to n into delta n into delta s then again your positive end direction is outwards and the weight component is in the negative of that direction. So, minus rho g delta s delta n into cos of the angle alpha is equal to rho delta s delta n into a n.

Student: (Refer Time: 18:25).

Which one, this is the first one is plus and this one is minus. Now we will also consider the limits as delta n delta s all are tending to 0.

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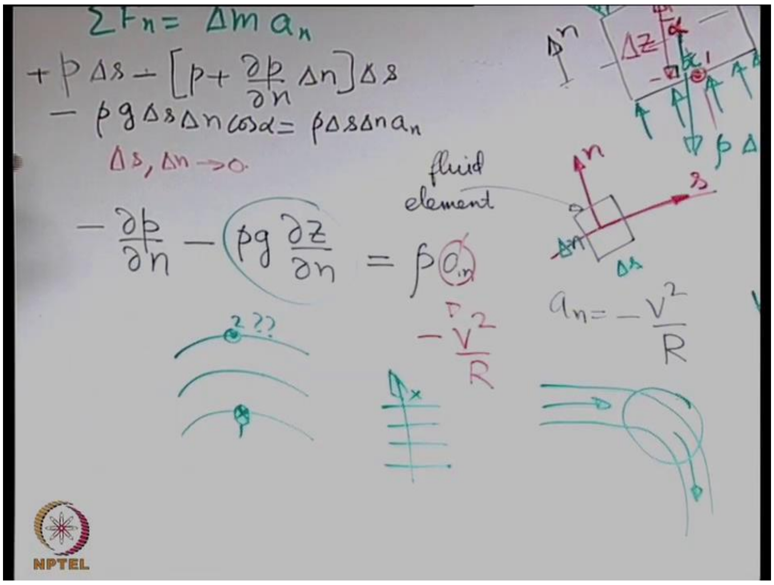

And then what follows: you have minus. So, delta s into delta n will cancel from both the sides minus what will be there rho g this one is equal to rho a n important thing now will be what is a n. So, you have to keep in mind that you are considering a fluid element in the limit as this size tends to 0 it becomes like a fluid particle. So, you can think of a particle moving along a curve, the basic particle mechanics when the particle is moving along a curve. So, what is the normal component of acceleration if the particle has a tangential component of velocity as v, what is a n?

Student: v square (Refer Time: 20:09).

V square by r, but not in the positive end direction, but in the negative end direction. So, minus v square by r what is r? R is the local radius of curvature of the streamline at that point where we are considering the fluid element. So, the very important distinction between the tangential and the normal components comes through this description of the acceleration. So, this will now become minus v square by r. So, let us consider a very simple case to illustrate the use of this.

So, what does it say it says that if there are curve streamlines there will be a pressure gradient across the streamlines? So, let us say that you have curve streamlines like this when such curve streamlines are possible let us say that you have a pipe bend. So, there is a pipe which was carrying fluid like this now the fluid is changing its direction it is moving in a different direction its very much possible and very very common pipes are bends to change the direction of flow. So, in the pipe bend if you consider a streamline streamlines are like oriented along the direction of the flow. So, you have curve streamlines.

Let us say that this entire bend is in the horizontal plane. So, that there is no change in the vertical elevation say there is a pipe which is when in a horizontal plane, then just for simplicity so that this term is not there. So, when that term is not there you can clearly see that the pressure gradient along the normal direction.

Student: Should be given.

Should be given by v square by r, if the streamline have radius of some radius of curvature and if you know the pressure at say this point one you should be able to predict what should be the pressure at this point 2, if you know the radius of curvature of these streamlines if the streamlines are such that they are parallel to each other then the radius of curvature tends to infinity; that means, there is no cross wise pressure gradient because of the curvature. So, this is only the curvature effect. So, because of curvature of the streamline there is no cross wise pressure gradient there might be cross wise pressure gradient because of a change in vertical elevation, but here we are considering it in a horizontal plane. So, that is not coming into the picture. So, this n component of the equation of motion is very important because it gives the pressure gradient because of the curvature of the streamlines in the cross stream direction which you cannot get from the equation of motion from the s direction.