Introduction to Fluid Mechanics

Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 34
Bernoulli's equation-Part-III

Now, you can see that there is some special requirement inviscid and irrotational. There is very important and interesting relationship between these two. Fundamentally we could try to answer these questions if there is an irrotational flow is it true that it has to be inviscid number 1, number 2 if it is an inviscid flow is it true that it has to be irrotational. Remember these are not very simple questions to answer and we will try to look into very basics of looking into these issues.

Let us say that you have an irrotational flow say there is a free stream which is having an irrotational flow; that means, it has null vorticity vector. Now the question is, is there any agent that can make the flow from irrotational to rotational, so will the rotationality be preserved. So, there are certain factors which can create a situation such that an irrotational flow the flow which was originally rotational now becomes rotational. So, what are those factors?

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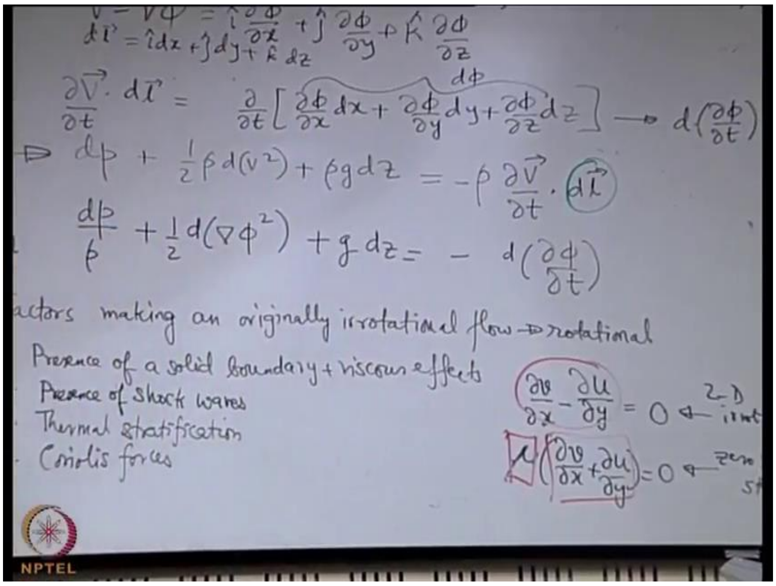

So, the factors making an irrotational flow an originally irrotational flow to a rotational one. One of the important factors is presence of a solid boundary and viscous effects.

Presence of a solid boundary is there in many wall bounded flows and viscous effects are common for fluids with some substantial viscosity. If that is there; that means, even if the flow was originally rotational physically it will not be able to retain its irrotational state; that means, although you may start with an irrotational assumption the viscous flow assumption will not hold that irrotational state physically. We will see that mathematically it will not be able to, will not be able to reflect this directly in such an elementary level it is possible to look into that mathematically, but not in such an elementary level, but physically we have to at least appreciate that if it was irrotational there is no guarantee that eternally it will remain irrotational and the factors which disturb that irrotationality, one of the factors is the viscous effect in the flow presence of wall boundedness.

There are other factors I am just listing those down, not necessary that we will discuss in details. One of the other important factors is presence of shockwaves what are shock waves? Shock waves are created by situations in highly compressible flows when there is a abrupt discontinuity in the fluid properties. So, there is like a wave front across which there is a jump in all the properties of flow and that takes place with a condition that across that there is a change in state from a supersonic to a subsonic flow. So, a Mach number greater than 1 to a Mach number less than 1 now I mean the detailing of how shockwaves take place and all we are not going to discuss here it is entire specialized discussion on compressible flows. But at least we will try to appreciate that these are these are situations where there can be abrupt jump discontinuities in fluid properties and those are the situations where originally irrotational flow may become rotational even if discuss effects are not otherwise important.

Then the third one is say thermal stratification, thermal stratification is like if you have two fluids of different densities and may be that is simulated by a case when you have the same fluid one single fluid, but you are heating it up. So, once you are heating it up the fluid will become lighter and the lighter fluid will occupy the positions which are higher and higher just because of the density gradient. So, the thermal stratification means there is a thermally stratified layer that is being created because the density gradient is being created by the temperature. So, hottest ones are there at the top and cooler and cooler ones are at the further bottom. So, you create a density gradient, but the density gradient is not created by change in pressure, but created by the change in created by the temperature gradients prevailing in the system and that also in a direction oriented against the gravity that is known as thermal stratification.

So, if you have such stratified layers then it is possible that that makes the flow rotational from irrotational. Then other forces like that may be Coriolis forces present. So, coriolis forces or coriolis effects can create a rotationality in the flow if it was originally rotational. So, if you like the earth when it is rotating it has a coriolis effect and if you consider the ocean currents, there are rotationalities in the ocean currents which are predominantly created by the coriolis effects. So, from if even if it was if the earth was stationary that is a hypothetical case to think it might be possible that that was irrotational, but because of the coriolis effects being present that is converted to a rotationality effect. So, there are many factors these things just show that these are very natural factors, these are not any artificially imposed factors on the system and these natural factors have a tendency to create rotationality in the flow.

So, we cannot ensure that if we have a irrotational flow as a reference case or as a undisturbed flow that will remain as irrotational. But if it is inviscid and then if we consider that the effect 2 3 and 4 are not there in a system then if it is inviscid and irrotational originally it will remain irrotational forever. Because let us say that effect 2 3 4 are not present only effect one is present the presence of solid boundary still will not be able to create an create a rotationality if the rotationality was not originally there because the message that the solid boundary is there cannot be propagated through the fluid viscosity is that messenger which propagates the presence of the solid wall into the fluid.

So, if the viscous effects are not there the fluid will be dump in responding to the presence of the wall and then flow is originally rotational will retain its irrotationality that is one of the very important understandings. If you look into these mathematically you see that you may lead to nowhere because if you have irrotationality let us consider a two dimensional flow. So, if you want that irrotationality then you must have the angular velocity in the plane that should be equal to 0 that is irrotationality 2-dimensional irrotationality and inviscid flow what is the requirement the requirement is that effectively requirement boils down to the sheer stress is not there that is the net effect that is there because of the viscous effect. So, for a Newtonian fluid it is mu into this one this is 0 sheer stress.

Now, you can clearly see that there is no relationship between these two, if you ensure that this is 0 this is not ensured to be 0, until or less these two terms are individually 0 we have seen such an example where you have a fluid element which was originally of a particular orientation it does not change anything angularly, it just gets stressed along one direction and reduced in length along the other direction that example we saw in the previous class. But that is a very special case in general if you have an irrotational flow. So, you have terms a minus b equal to 0 that does not ensure that a plus b equal to 0 until and unless a and b are individually 0 that is a very special case. So, you are relying on what? You are relying on having mu identically equal to 0 to have inviscid and irrotational flow.

Physically sometimes it is not a very absurd way of looking into things though mathematically you cannot ensure that see mu is a fluid property. So, if you have irrotational flow still physically it is possible to have a viscous effect because the flow is likely to have a viscosity these term is not equal to 0. So, you can have a viscous effect, but of irrotational flow irrotational flow is also called as a potential flow because velocity potential exists in irrotational flow. So, that type of case when this mu is not 0 this term is not 0, but this term is 0 that is that can be called as a viscous potential flow, it is mathematically very much possible nothing denies that.

But if you just look into it, in bit more physical terms what is the origin of the thought of an irrotational flow, we found that it is a conservative velocity field because the velocity, because the field vector field is conservative we could write it as a gradient of a scalar potential. Now when you have a conservative field physically it means that there are negligible dissipations in the system, just like if you have a conservative field as gravity. So, if you think of a conservative force, force field in a particle mechanics you neglect the effects of friction because friction will no more keep the force field as a conservative one.

Now, if you think of the velocity field velocity field is not exactly like a force field, but you may think it analogously because it is also vector field. So, in a velocity field what could create a disturbance in the conservativeness is the presence of dissipation and that dissipation is through the mechanisms of viscosity. So, if viscous effects are strong then physically it may not help in retaining the flow, flow field as a conservative field. So, physically it might be very common that if it is irrotational, if it remains, if it wants to remain irrotational it has to be inviscid because viscous effects will create sort of dissipations in the flow just like what friction does in a force field. So, that is one important conceptual thing that we need to keep in mind.

So, as we were discussing that it is not very straightforward to give the answer to this question that if it is irrotational and inviscid then are there relationships between these two, but I hope you have now some kind of physical picture on this understanding.

We will now workout may be one problem which will be based on the concept of say the Bernoulli's equation that we have discussed there are many application applications of the Bernoulli's equation and we will look into some of the important applications in today's class and then may be in the next couple of lectures. But before that we work out a problem which is not based on a very common application, but it is still not a bad example.

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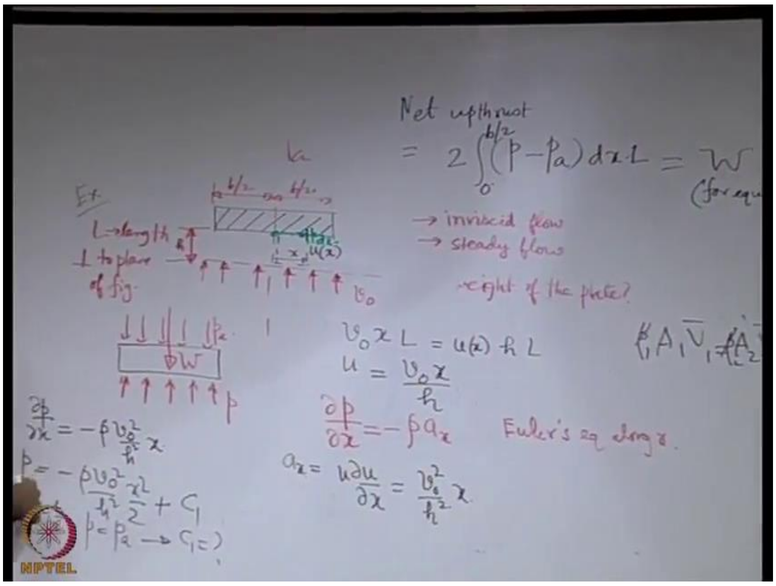

We refer to this example as such to work out a problem in context of fluid kinematics. So, there is something like plate a rectangular plate and there is a bottom plate there are holes in the bottom plate through which fluid say air is blown like this, if we recall we worked out such a problem and the velocity will continue with that problem and try to work out a different problem from this. So, we have a uniform velocity say v naught with respect to which air is entering through this pores and let us have a coordinate system like this where this is symmetrically located with respect to the plates. So, let this be b by 2 and this is b by 2 which are the half of the dimension.

Let us say that L is the length of this plate perpendicular to the plane of the board. So, L is length perpendicular to the plane of the figure. The gap between these two, let us say the gap is h and our assumption is that it is inviscid flow and let us say steady flow. We are interested to find out what should be the weight of this plate to keep it in such a position; weight of the plate what? The density of the fluid is given, classical design example we have discussed about this, but just to iterate say this is an electronic chip you want to cool it by blowing air because it has become hot with heat generation because of the electrical effects.

Now, you are blowing air there is a because of this because of the air velocity it has come to a floatational state and it will come to an equilibrium height where it will remain stable based on its weight. So, if this is the height then what is the weight of the plate here you consider as a chip, so what is the weight of that. It is like it is not a very absurd question I mean it might appear to be a very absurd question that flow fields etcetera these are given now what is the weight of the plate or weight of the chip, but once we look into it carefully we will find that it is not very absurd it should follow from the basic considerations.

So, when we do that the first thing is we need to find out how the velocity varies because for any one any type of calculation that we have seen involving the kinematics or even the dynamics of flow the velocity field is very important. So, from this given consideration we have to find out what is the velocity field. So, if you recall that we earlier considered like at a distance say x from one n and we found that what is the rate of flow entering and rate of flow leaving this control volume which is marked by the dotted lines. So, the rate of flow that was entering is v naught into x now the length perpendicular to the plane of the board is L this is the volume flow rate and what it leaves here let us say u is the function of x because of the assumption of inviscid flow u does not vary with y.

So, you can take just u as the function of x into h into L. So, it is as good as writing A 1 V 2 equal to A 2 V 2. If you write A 1 V 2 equal to A 2 V 2 what are the assumptions under which that is valid? I am going to hammer this on you again and again and again because many times you have used this without keeping in mind the assumptions. So, let us write A 1 V 2 equal to A 2 V 2 what are the assumptions in which these are valid.

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So, 1 and 2 are the two sections that we are looking for over which we are having

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Equivalent constant velocities V 1 and V 2, so if they are not constant these have to be replaced by the average velocities over the sections. But there are even more important assumptions which are inbuilt here, what are those?

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If we put say rho 1 A 1 V 1 equal to rho 2 A 2 V 2 where rho 1 and rho 2 are the densities at the sections 1 and 2, let us say these are average densities over the sections then what is the requirement under which this will be valid. See you have to keep in mind why we do derivations in the class; you have to keep in mind how this was derived. This was derived by dropping the unsteady term in the continuity equation and integrating the remaining terms in the continuity equation; that means, the only assumption was it was unsteady flow steady flow. So, unsteady term goes away.

So, when it is steady flow it need not be constant density. So, you can write it still in this form if rho 1 and rho 2 are the same then it becomes A 1 V 2 equal to A 2 V 2. So, it has two assumptions - one is the steady flow another is rho is a constant because you have cancelled or may be whatever function of rho is there in the left hand side same function is there on the right hand side, say rho is a function of something else say time. So, left hand side and right hand side it is the same function, they are cancelled out, but rho cannot be a function of time because you already considered a steady flow. So, it cannot be a function of time. So, it is just like a constant which is same in the left hand and in the right hand side that is how these two got cancelled.

And when we say rho equal to constant other thing again I am going to hammer on you keep in mind rho equal to constant is a special case of incompressible flow, but incompressible flow does not require rho to be a constant. This is often like even in some of the best of the text books this confusion is retained. So, we will see that when assumptions are written for a problem it is written that incompressible flow where incompressible flow can be handled without requiring rho to be a constant. So, whenever we consider rho to be a constant we specially specifically we will say that rho is a constant that is what is our assumption, incompressibility is not good enough to ensure that rho is a constant, but if rho is a constant it has to be incompressible.

So, this is something that we have already derived and let us write the velocity u as the function of x, so that is h sorry v 0 x by h. When you have this as v 0 x by h then it is possible to find out the acceleration which we earlier found out, but we have to see what is what is that that we want to find out. So, we have to make a strategy for this for solving this problem. We know what is u and you also know what is v, we have found out v by using the continuity equation that also we did in the previous example of similar type that we worked out, but we will not concentrate on finding out v we will concentrate on finding out a strategy for solving the problem.

See what are the forces which are acting on this. So, there is, if you consider the sort of free body diagram for the plate or if you want to think it as a chip. So, there is a pressure distribution from the bottom, there is a pressure distribution from the top which is because of the atmospheric pressure, let us say that it is entirely surrounded in a uniform atmospheric pressure which say is p atmospheric which is along all the sides except the inside point. Now because of this difference in pressure let us say this is p. So, if p is greater than p atmospheric pressure there will be a up thrust only and that should be balanced by the weight to keep it in equilibrium. That means, if we find out what is the resultant force due to pressure on this chip that will give us a insight on what is the weight because then we can use the conditions for equilibrium. To do that what we will do? We will find out how pressure varies with x.

So, how pressure varies with x we have the Euler's equation of motion along x. So, what is the Euler's equation of motion? That was the Euler's equation of motion along x, Euler's equation along x and acceleration along x, acceleration along x is only one term will be there, the other terms will be 0 because u is a function of x only.

So, you can write. So, let us say that we want to integrate this expression from x equal to 0 to let us say x equal to b by 2 and it will be similar in both the sides, so 2 into that will be the total integral to get the force. So, let us say that at a distance x we take a small strip of width dx and we are interested to find out what is the pressure on this strip. So, if we integrate along this. So, we are integrating with keeping y equal to h which is a constant. So, the other component of velocity has no effect because of no penetration v is 0 here.

So, we can integrate along this surface. So, we will have p is equal to minus rho v 0 square h square into x square by 2 plus let us say some constant, the constant in general could be function of y because it is a partial integration with respect to x, but we have already fixed y as y equal to h. So, let us say that it is a constant c 1. How can we find out this we know that at x equal to b by 2 p is equal to p atmosphere. So, at x equal to b by 2 p is p atmosphere. So, from here you can find out what is c 1 by putting this boundary condition

So, then you know p as a function of x. So, what is the net up thrust? That is p upwards from the bottom p atmosphere from the top. So, p minus p atmosphere dx that is the total force acting on the element of thickness dx and length perpendicular to the plane of the figure l, so that is the area on which it is acting. 0 to b by 2 is half of that, 2 into that is the total force right. So, this must be equal to the weight or equilibrium. So, remaining exercise is very straightforward. See eventually when you find out c one you will get it in terms of p atmosphere, you will get a explicit expression of p minus p atmosphere from this equation which you substitute here and integrate in the simple polynomial integration. So, that will give you the weight of the chip or the plane.

So, this is a simple illustration of the use of the Euler’s equation, in the next class we will use the Bernoulli’s equation for solving some other problems, but before that in the next class what we will do we will just create a small introduction for that we will write the Euler’s and Bernoulli’s equation in terms of a different coordinate system that is a streamline coordinate system. So, why we are interested to write it in a streamline coordinate system because we know that along a streamline under certain conditions these equations are valid. So, if we write it for two points along a streamline it may be very convenient if we use the streamline coordinates we will just briefly see that what are the streamline coordinates and what how they are related with the other coordinate systems.

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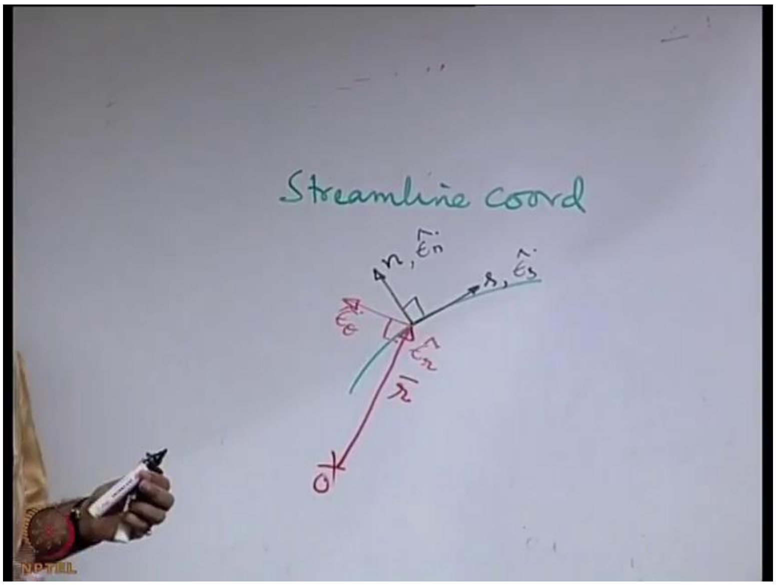

So, let us just consider the streamline coordinates. So, the streamline coordinates are like this. So, if you have a streamline we consider tangential to the streamline as s and normal to the streamline as n. Many times there is confusion between this coordinate system and the coordinate system that is used in a cylindrical polar coordinate system so we will try to avoid that confusion from the very beginning, say if you are using a polar coordinate system.

So, if you have this as the origin or the pole then what how the coordinate is represented, it is represented by 1 radius r this is the radial direction and perpendicular to that theta direction. So, we have unit vectors along this as epsilon r and epsilon theta and unit vectors along these as epsilon s and epsilon n. Both of these are orthogonal systems, but you have to keep in mind that they are not the same many times there is confusion. Many times I have seen students calling these as the radial direction and this as the tangential direction. No, you can clearly see that this is not a tangential direction only for a circular geometry normal to the radius is the tangent, but not for all type of curves.

So, this is fundamentally called as radial and cross radial direction. So, epsilon r is the unit vector along the radial direction, epsilon theta is the unit vector in the cross radial direction which is perpendicular to that or orthogonal to that whereas, these are sort of tangential and normal directions. So, you should not confuse between these two coordinate systems and we will write the Euler's equation of motion in the streamline coordinate system specifically, that we will do in the next class. Let us stop here today.

Thank you.