Introduction to Fluid Mechanics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 33
Bernoulli's equation-Part-II

We will continue with the Bernoulli's equation about which we were discussing in the previous class.

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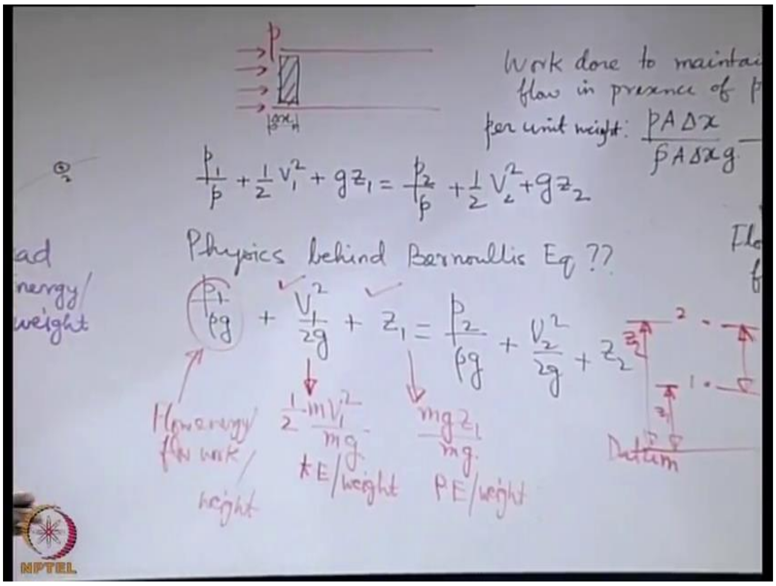

So, the Bernoulli's equation as we have seen, is taking the form. What are the assumptions under which this was derived? Inviscid flow is the most important one I should say, then this version is for steady flow we will see what is the version for unsteady flow subsequently, density is constant, and we derived it along a streamline where it requires no other restriction, but if you want to apply between any 2 points, who are not necessarily located on the same streamline, then it has to be irrotational flow, or maybe a special case, when the cross product of the velocity with the vorticity that is perpendicular to the line element, that is being chosen. So, these are some of the discussions that we made in the previous lecture.

Now, when we come to this form, so let us say that we are considering it along a streamline, there are points 1 and 2, and this equation is valid. So, what does this equation say, that is very important. After all we are not mathematicians, we are not just bothered about that here there is a equation where we can plug in values to get a numerical answer. What is, even more important to us to appreciate that, what is the physics behind the Bernoulli's equation; that is what we will try to learn. Because once we appreciate the physics properly, we will be perhaps able to utilize this equation in certain cases, where this equation may not be exactly valid, but in a somewhat approximate sense.

To understand the physics, it may be better to appreciate the physical consequence of each and every term in the equation. These terms may be written in different ways, this is one standard way of writing it, but in engineering sometimes what we do? We divide all the terms by g, and write it in this form. So, P by rho g is the first term, then we have V square by 2 g the second term and z as the third term. So, this also is one of the forms, and one of the common forms in engineering. This particular form has terms which have dimensions of length, because the third term is a dimension of length, and all terms in the same equation must have the same dimension. So, all other terms are units of length. Now what we will try to see, is that expressed in terms of units of length what do these terms physically represent. We will start with the more obvious terms; that is we will start with these 2 terms. These are more obvious and easy to interpret and in in some way very trivial.

So, you can write say V 1 square by 2 g, how you can write it? You can also write it at like half m d 1 square divided by m g; that means, this in a way represents kinetic energy per unit weight. Similarly this can be written as m g z 1 divided by m g. So, this is potential energy per unit weight. So, you can see that these terms are therefore, representatives of energy per unit weight. Energy per unit weight in fluid mechanics is known as head. So, that is the term given as head, is energy per unit weight.

So, loosely this may be called as kinetic energy head, or may be velocity head in a in a more simple term. So, head is a lengthwise representation of energy, I mean giving the dimension of length to the energy by normalizing it with respect to the weight. Let us look into the P by rho g term. So, this is like kinetic energy per unit weight this is potential energy per unit weight, I guess you have learned this term as pressure energy or something like that, but I do not understand anything called as pressure energy, it is something very absurd terminology which is enforced, may be on, whoever I am you are just habituated in excepting that. So, if you are in general asked that why is P y rho you will say pressure energy, but what is pressure energy. Like we have learnt many types of energies, but I cannot remember. Well I was a very bad student, but still I cannot remember that I have understood something fundamental as pressure energy.

So, let us try to see that what is that terminology? May be terminology is not so important, we may use some other term also for that, but what does it physically represent. we can understand the kinetic energy and potential energy what do these physically represent, but what does this physically represent, is something which is not very straightforward from this, and if we want to make it very straightforward by calling it as pressure energy, perhaps we are trying to suppress our lack of understanding of it. So, let us not do that, let us expose our lack of understanding and see that what it should be?

So, let us say that you have a pipe, and fluid is trying to enter the pipe, when the fluid is trying to enter the pipe. Let us say that the pressure at this inlet section is p, the fluid is having a particular velocity, it is entering the pipe. Now see this is a flowing system, so there is a pressure in the fluid, this pipe is not that this pipe is like in vacuum. So, you will have a continuous flow going on like this. So, this is filled with say water, and water is continuously entering and leaving like that.

Now, if the water which is entering the pipe has to penetrate over a distance, it has to do that in presence of the pressure, and therefore, it has to do some work. So, what is that what, let us say that it undergoes a displacement of delta x. Why we are considering just a small displacement, because we will consider that this pressure is remaining constant over the displacement, it may be a variable pressure. So, you will consider only a small displacement over which the pressure is supposed to be a constant. The whole idea is based on that we are interested to calculate the work done in presence of pressure.

So, if A is the area of cross section of the pipe. Then what is the work done? To maintain the flow in presence of pressure, in presence of pressure; let us say we want to find it out. So, to do that, we will first consider a small displacement. So, if A is the area of cross section, then because of the presence of this pressure p, what is the work done? P into A into delta x. now just like all the other terms we will also try to express this work per unit weight of the fluid. So, if we want to express it, work done to maintain the flow in presence of pressure, we amplify this as per unit weight. So, then this has to be divided by the weight of the fluid element, that covers up to this, or weight of the fluid rather, that covers up to this delta x.

So, what is the weight of that fluid; rho into A into delta x that is the mass that into g is the weight. So, this becomes P by rho g. So, you can see that that is same as the first term that you can see in this Bernoulli's equation. So, this is what it represents, it represents the work done to maintain the flow in presence of pressure; that is fundamentally what it represents. So, if there is no flow, then this term would have been absent. So, the fluid must possess, this additional, or the fluid must be capable of transferring or transmitting this additional energy through its motion, so that it can overcome whatever pressure is there and still it can maintain the flow. So, this extra energy the fluid must possess to maintain the flow in presence of pressure, this is known as flow energy or flow work. Now, if you give it a name pressure energy, none of us have any problem, it is the name is up to you. So, if you are happy with giving the name you give that yes.

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That is the matter of terminology. Usually we consider it per unit weight and call it flow energy or flow work, but if you have to be more precise, you may call it flow energy per unit weight, it is the sense that is more important. So, if you just write it here, this is flow energy or flow work. This is also expressed per unit weight that one has to remember. This also you can call as pressure energy, there is nothing wrong in it, but one has to understand.

So, this equation is saying that sum total of these 3 forms of energy, is getting conserved along a streamline, under the assumptions that we made. Now the question is. Is this energy being possessed by the fluid or what? To understand that let us consider an example. Say you are there in a airport, and there is a conveyer belt. Now the suitcase is put on the top of the conveyer belt, the suitcase moves from one place to another place, because of the motion of the conveyer belt, and the conveyer belt just as an analogy consider it like a fluid flow.

So, it is like moving the suitcase from one place to another place, is it holding the suitcase anywhere no. So, that suitcase is like something which is put on the flow it is just like some energy. So, you have sum total of these 3 energies that is somehow being there in the flow, and it is getting transmitted from one place to another. So, it is never possessed. Therefore, the Bernoulli's equation essentially says, physically that the sum total of the flow energy kinetic energy and potential energy per unit weight, remains conserved, as it is transmitted from one point to another in the flow field, along the streamline, may be that we are considering in this example.

It is not possessed, it is just transferred. So, the flow is just acting like a medium, which is not holding the energy, but which is transmitting or transferring the energy from one place to another place. This is like a sort of statement of conservation of mechanical energy, but we will see that under restricted cases, it may also take different forms. Not that this is the only form that is there for the conservation of mechanical energy. So, we have now sort of clear picture on the significances of different terms in a Bernoulli's equation.

The next point is that whenever you are talking about energies, in in different terms you must have a reference. Like when you say potential energy, you have a datum, with respect to which you are calculating the height z. So, this z is not in an absolute sense, it it does not make any make any big significance, because eventually in this equation z 1 minus z 2 that is what is important, and that is independent of the choice of the reference. So, if there are 2 points 1 and 2, say this is 1 and this is 2. So, this difference in the height between 1 and 2, vertical height is that what is important. So, if you have this as z 1 and say this as z 2, with respect to some datum, say this is the datum. Then independent of the choice of the datum z 2 minus z 1 remains the same, but still when you want to prescribe it in an absolute sense you require a datum, for on a reference. For the kinetic energy term there is a velocity, and you require a reference for that. So, there should be a reference frame with respect to which, you are prescribing this velocity. Typically this reference frame is a reference frame at rest. So, you are writing here the absolute velocity.

And this reference frame for pressure is also very important. We have already discussed when we were discussing the statics of fluids, that you can prescribe pressure also with reference to something, if we prescribe with reference to the atmospheric pressure; we call it a gauge pressure. So, you can as well prescribe the pressures in in the different terms, in terms of a reference pressure, and then you can substitute at gauge pressure. Important thing is whatever reference you use for postulating the different terms, it should be same in the left hand side and right hand side; that is very common and obvious conclusion. Now, we will see that what could be the other variants of this Bernoulli's equation, or more fundamentally the Euler's equation. So, we will next see the unsteady case. Till now we had discussed about the steady case, but let us take the example 2. In the example one we considered a steady flow.

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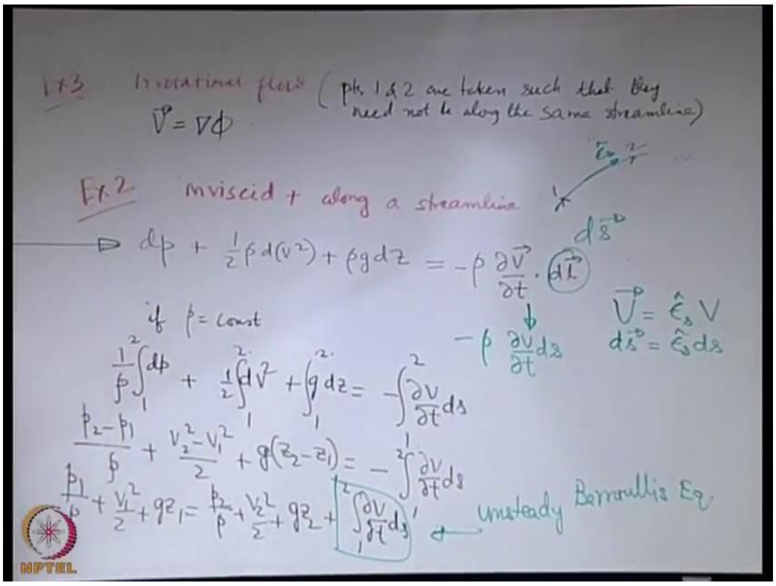

So, in the example 2, we will keep all our previous assumptions as valid; that is inviscid flow and flow along a streamline, but we will not consider it to be steady apriori. So, we will consider inviscid plus along a streamline; rho constant we will take in a later step, we will start with the Euler's equation form. So, Euler's equation form let us try to write that d P plus half rho d V square plus rho g d z. These are the first 3 terms, this was equal to now 2 extra terms were there which we dropped off, by consideration of steady flow and along a streamline. So, now, the term which is there along a streamline that is dropped, but the term which is there because of unsteadiness that now will not be dropped because, now we are considering the unsteady flow. So, what will be that term minus rho dot d l?

Now, we have considered along a streamline. So, our d l is d s, where s is the streamline coordinate. We will subsequently write special forms of the Bernoulli's equation using the streamline coordinates, but for the time being, let us say that we are considering this as the streamline direction, so we call it d s. Now when we are considering the streamline V is already oriented along the streamline, because that is the definition of the streamline, tangent to the streamline represents the direction of the velocity vector, at each and every point.

So, if you write in terms of the streamline coordinates. So, V will be let us say that epsilon s is a unit vector in a stream wise direction. So, V you can write the magnitude of V times the unit vector in a stream wise direction. So, if this is the streamline, may be this is the stream wise direction. And d s, again you can write epsilon s into the magnitude of the length of the element. So, when you take a dot product of these 2, you can just write it in a simple scalar form for along a streamline. So, you can write this one, where these are just magnitudes keeping in mind that we have written this along a streamline.

Now, next what we can do. We can take rho equal to constant. So, this is Euler's equation of motion, for a general case, where the density may be a variable, density need not be a constant, but if you take the density as a constant, then you have d P by rho plus half d V square plus g d z is equal to minus. Let us integrate it from point 1 to 2 along a streamline, and if rho is constant we will take rho out of the integral when we are evaluating the integrals. So, 1 by rho will come out, take the integral from 1 to 2. So, we can write P 2 minus P 1 by rho plus V 2 square minus V 1 square by 2 plus g into z 2 minus z 1 is equal to this one, you can of course, rearrange the terms and write P 1 by rho plus V 1 square by 2 plus g z 1, just like the standard Bernoulli form P 2 by rho plus P 2 square by 2 plus g z 2 plus 1 extra term.

So, the extra term has now appeared, because of relaxing the requirement of steadiness. So, now, it also can be an unsteady flow. This is known as unsteady version of the Bernoulli's equation. We will work out some problems subsequently to illustrate the use of this one. So, what is clear, is that whatever term we have brought, because of steadiness now that term has appeared and it is just creating an extra effect, and the meaning of this term is quite clear, because it gives a gradiation of the, effect of the gradiation of the velocity with respect to time. Now, we will take the third example when we consider irrotational flow.

So, let us take the third example; example 3, irrotational flow. So, when we take the example of irrotational flow, let us see that what happens to the equation. So, when you have the irrotational flow, the thing is that d l. We are not writing as d s, because when you are writing irrotational flow, we are keeping in mind that points 1 and 2 are taken; such that they need not be along the same streamline.

So, if they need not be along the same streamline what is the consequence? The consequence is that the equation is just like this equation what is there in this example 2, but we will not substitute d l with d s. we will just keep d l as it is t. When it is an irrotational flow we know that the other term, where there was a vorticity vector that term will become zero. Not only that you can write V as the gradient of a scalar potential, which is the velocity potential that we discussed. So, we can write. So, this is like what, this is. Now what is d l? So, let us try to write what is this expression for partial derivative of V with respect to t dot with d l. Now we have to keep in mind that d l need not be along d s, but that is ok because we have considered an arbitrary d l with its x y and z components. It is not necessary it has to be along a streamline.

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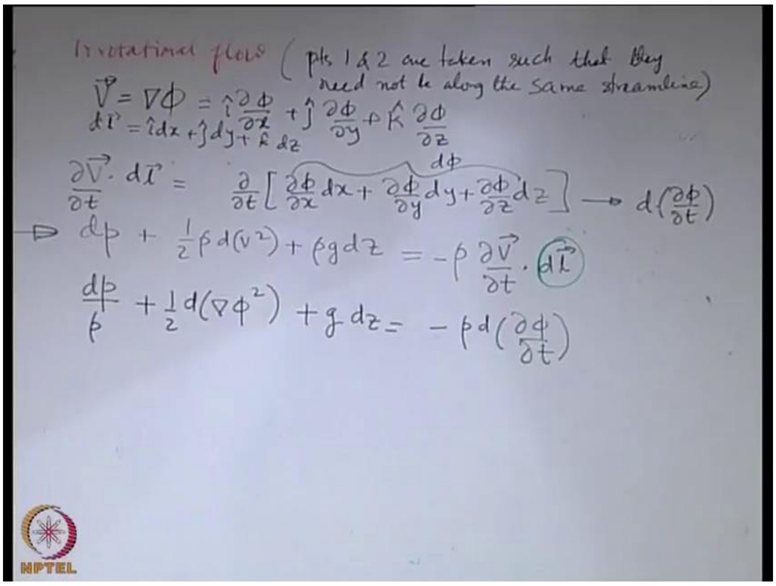

So, V dot with our partial derivative of V with respect to t dot with d l. What will be that? So, we can see that it is a sum of the 3 partial derivative, terms for variations along x y and z. So, what is this? This is the total change in the velocity potential. You can also write it as, sorry, d of the partial derivative of phi with respect to t, because you have to remember that d and this del operators they are interchangeable, mathematically. So, it is just possible to consider this in place of this in the other way. So, you can write this, as the exact differential of the partial derivative of the velocity potential with respect to time; that is one observation.

The other observation is that you can also express V directly as a gradient of the scalar potential, and therefore, you can express this Bernoulli's equation, solely in terms of the velocity potential. So, you can write as this is not the Bernoulli's, this is actually the Euler's equation form the step prior to the Euler's equation. You have to keep in mind that Euler's equation is the more general form, when substituted rho equal to constant and integrated that gives the Bernoulli's equation. So, this is the Euler equation form, even the Euler equation form can be simplified with that. This is the Euler's equation in terms of the velocity potential valid under what assumptions.

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inviscid flow and irrotational flow. So, this is valid for inviscid plus irrotational. There is no other assumption, because it does not require to be along the same streamline, and till this stage it is not necessary to make rho equal to constant. If you make rho equal to constant and then try to integrate it, then that rho equal to constant will come as the.

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Sorry, R H S it should be divided by rho, correct. So, it does not require rho to be a constant in this stage, but if you integrate it by taking rho equal to constant, that it will give a equivalent Bernoulli type of form. This is Euler equation type of form.