Introduction to Fluid Mechanics
Prof. Suman Chakraborty
Department of Mechanical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 03
Concept of traction vector

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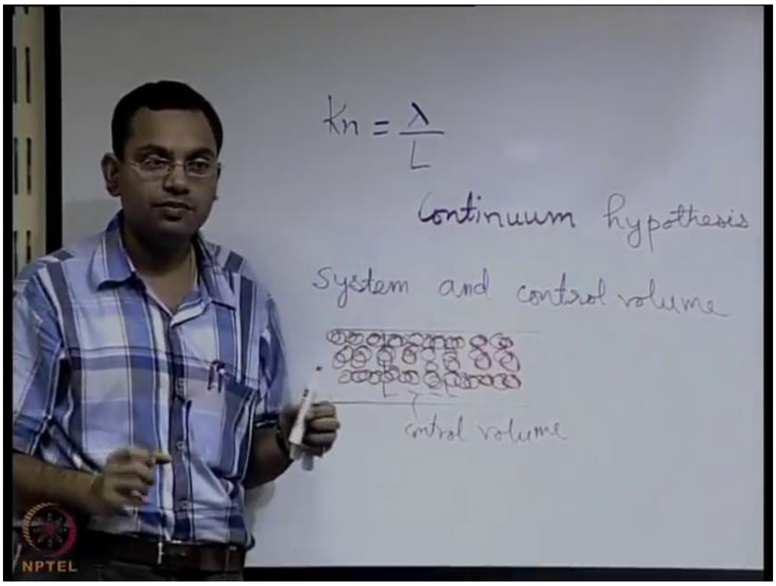

What we left in the last time that is, we define something called as Knudsen number which is the ratio of the molecular mean free path and the characteristic length scale of the system. So, this one may give it a symbolic note for a pipe it may be the diameter of the pipe, for something else it may be some other dimension, but we call it as a characteristic length scale of the system. So, if this ratio is small what does it indicate? It indicates that the mean free path is much smaller than the characteristic length scale of the system, which implicitly tells that it is a sufficiently densely packed system.

On the other hand, if that is not the case, that is if the Knudsen number is large; that means, the mean free path is much it may be even larger than the characteristic length scale of the system. It is if it is very very rarefied, in such cases what happens in such cases you have very very few numbers of molecules and then, there are lots of uncertainties with respect to presence of molecules in individual elemental volumes. So, in those cases the macroscopic point of view is not expected to work so efficiently. So, or the macroscopic way of defining the characteristic or properties of the fluid might not work so efficiently. So, whenever the macroscopic way works, we call it we call the fluid as a continuum and the hypothesis concerned is known as continuum hypothesis.

So, what is the continuum hypothesis? Continuum hypothesis tells that, we treat the fluid medium as a continuous matter disregarding the discontinuities in the system. There are discontinuities if you look in to the molecular level. There are molecules, there are gaps and. So, on, but if those are sufficiently compact then, you may treat it as a continuous matter. Once you can treat as a continuous matter then you may use the well known rules of differential calculus to talk about the changes in properties from one point to the other. So, you can talk about simple gradients second order derivatives and so on.

So, continuum hypothesis works if there are sufficiently large number of molecules. So, that, the Knudsen number that we are talking about is very very small. If the Knudsen number is greater than 0.1, so to say, then it comes to us state where your mean free path threatens to be 10 percent of the characteristic system length scale and when it goes on larger and larger there is a stronger and stronger deviation from the continuum hypothesis. So, if there are situations where when you cannot use continuum hypothesis and one needs to have a different treatment all together.

Throughout this course we will be bothered mostly about situations when continuum hypothesis works so; that means, there are sufficiently large number of molecules in the system. So, that uncertainties with regard to individual molecules do not influence the prediction regarding the fluid flow to the significant extent because, we are not looking from a molecular view point but treating the fluid as a continuous matter. So, keeping that in view what we will do is we will next see that no matter whether we are treating it as a in a microscopic view point or in a macroscopic view point how should we describe the fluid as a system. So, for that we will introduce 2 important concepts system and control volume.

So, we have identify an approach, now we have to identify that what should be that fluid over which we apply that concept or approach. So, when we talk about a system by definitions, system is something a fixed mass and identity. So, something which must have its mass fixed it must have its identity fixed; that means, those are identified for fluids sometimes this is not such simple concept to implement. Let us again take an example of flow through a pipe. So, we have a pipe, there are many molecules or even particles whatever are entering the pipe and leaving, so at particular instance of times. So, these are the molecules which are present.

Now, at a different instant of time, you may have different entities different molecules which are present the reason is quite clear something is entering and something is leaving. So, it is continuously being replenished right. In such a situation, if you want to track the motion of these particles or individual molecules as something of fixed mass and identity becomes difficult and t d s because then you have to put a tag on individual entities and follow it as it is moving. So, this kind of approach or so called particles tracking an approach in mechanics is known as lagrangian approach.

In fluid mechanics, it is not many times convenient to follow that approach. So, what we do instead is like we focus our attention on a fixed region in space. So, let us say that we have focused attention on this identified region. So, what as if we are sitting with a camera, focusing the camera on these zone. What we are observing? We are observing whatever is coming into this zone and leaving that we are only keeping track up to that much. We are ignorant about where from it has come and where it is going. So, rather than focusing attention on individual particles, we are focusing attention on a specified region in space across which matter can flow. So, that region we call as control volume.

So, control volume approach is more convenient for fluid flow because you do not have to track individual particles and fluid is the continuously deforming medium. So, it is very difficult to track individual particles, it is much easier to focus your attention on a specified region and see what happen across that and this particular approach were you use a control volume and analyze what happens across that is also known as Voilarien approach. The just as the Lagrangian approach is following the name of the famous mathematician Lgrange Voilarian is according to the name of Voilar.

So, whatever we will be discussing in this context of fluid flow, no matter whether it is a system approach, no matter whether it is a microscopic approach or it is a macroscopic approach. We will be mostly using the control volume concept for analyzing the flow behavior. With this basic understanding we will now going to the concept of a fluid. So, till now we have loosely talked about fluid, we have not seriously define what is a fluid. Now common sense wise if we are ask that what is the fluid; obviously, we will say something what flows is a fluid right and loosely speaking this is not a bad definition, but there are many things which flow, but those are not so called classical fluids, those may be in a border line in between fluids and solids. So, to say a very formal definition of fluids is like this that fluids are substances which under the continuous deformation even under the action of very small shear force.

So, if you are applying a shear force or there is some shear force acting on the fluid, then even if it is very small it will continuously deform the fluid. For a solid; obviously, if you are applying a shear force, it will not spontaneously deformed it till may be comes to a threshold limit when it will appear to be seriously deforming. Obviously, there are substances which are fluids, but which require a threshold shear to be deformed and therefore, the border line between the fluid and the solid is sometimes not so strict. But, for most of the practical cases this definition is something which we will be keeping in mind and will be classifying fluids or solids according to this behavior.

So, one of the important consequences is that, if there is a fluid which is non deforming. So, non deforming fluid may be fluid at rest. So, if you have a container, within the container you put some water and water is at rest. What is the implication of that? The implication is very straight forward; there is no shear which is acting on it. So, if there is some shear which is acting on a fluid the fluid is deforming converse is also true; that means, if the fluid is deforming there must be some shear which is acting on it. So, when there is a fluid which is there at rest; that means, there is no shear component of force that is acting on it; that means, there is only normal component of force acting on it and that normal component of force per unit area which is acting inwards is called as pressure.

So; obviously, whenever there is a fluid at rest the entire situation of the forcing which is there on a fluid element, may be expressed in in terms of pressure when the fluid is moving it does not mean that they is no pressure. Obviously, pressure is very much there which would have been there if the fluid is at rest. But, there are additional forcing components which come into the picture which are directly related to the deformation of the fluid and this situation all together may be tackled in continuous mechanics that is mechanics of a continuous medium through the concept of stress. So, we will now introduce the concept of stress, which we will be introducing in a context of continuous mechanics. So, it is valid for both solid and liquids.

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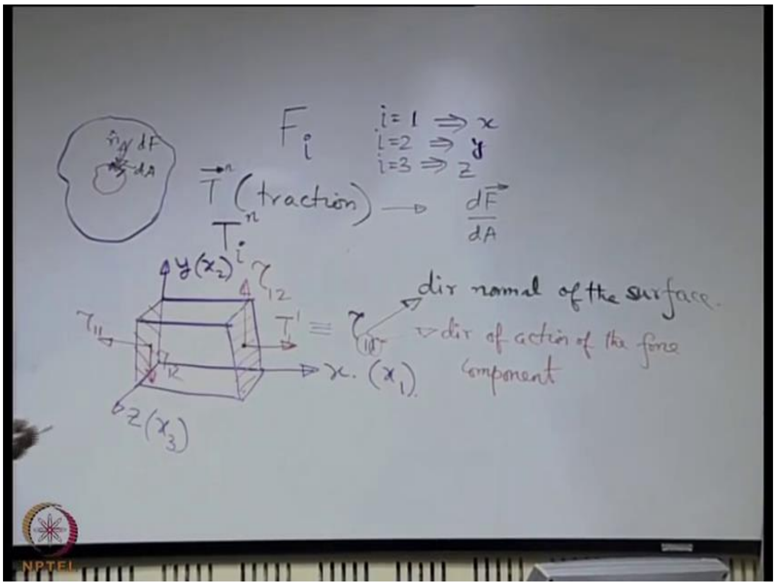

Now, let us say that we have an element, when we say we have an element it may be an element of a solid fluid whatever we are not particular about it and we are identifying a small chunk from that. On that small chunk we are taking a small area say dA what we are interested is to see that whenever we are taking out this small chunk the other part of the body will exert some force on this just by Newton’s third law. So, we are interested to identify that force and let us say that that force is directed like this it is absolutely arbitrary. So, it depends on many situations.

So, let us say that the forces dF. So, if we want to define some force per unit area then we are just giving it a name we are calling it T or traction, we will call it a vector because it is having the nature of the force it will be implicitly determined by this one where F is the force, but we have to remember that it is not unique until and unless we specify the area what it means let us say that centered around the same point we take a different elemental area same dA, but differently oriented.

So, if we take the differently oriented area now if we find out the result in force on that it is likely to be different; that means, given the point around which we take that differentially small area as fixed that is the location fixed given the magnitude of the so called elemental area as fixed still this ratio is going to be different. So, this strongly depends on not just the dA, but how the dA is oriented. So, it is important to give a kind of superscript or subscript to this. So, let us give us superscript n. So, this n denotes that we are talking about an area which is having its outward normal in the direction of the n vector.

So, whenever we denote the orientation of area it is customary to denote by the unit vector in the outward normal direction let us say that n is such a vector, even if it is not a unit vector there is no problem because we can always normalize it in the form of a unit vector. The direction is what is important if n is changed; that means, the orientation of the area is changed; obviously, this traction vector will change. Now, this traction vector therefore, is not denoting something which is ordinarily like any other vector. So, if you are talking about say a force. So, whenever you are writing the component of a force say f is a force you are using an index I to denote the component of the force. So, this index i equal to 1 will mean the x component, i equal to 2 will mean the y component and i equal to 3 will mean the z component. Therefore, by using one index i and varying it from 1 to 3 we may denote the components of a vector.

But, when we are trying to denote this traction vector yes it has a component because, it is it is like a force unit per area. So, base based on the direction of the force it has its own direction it has its own components. But, its specification depends on also another sort of index n which denotes the choice of the area orientation that has been employed to calculate this T. So, it is something more general than an ordinary vector, how general it is to understand that we will take some special examples. What are the special examples? let us say that, we take an element of a fluid which is of a rectangular shape like this we may orient axis like x y z in terms of the index we call this as x 1, x 2, x 3, index using the index is a very convenient way because just by varying the index you can vary the directions.

Now, let us try to see that what are the special cases of traction vectors on the phases of these cuboid. So, these has 6 phases and this phases that special surfaces, why this are special surfaces? This have their normal directions either along x or y or z this is absolutely an arbitrary area and what we are interested to do is to figure out what happens for an arbitrary area by referring to special areas which are either oriented along x y or z. So, that is the motivation of taking such an element.

So, once we have taken this element, what we are going to see if we are going to write such expressions for different phases. So, when we come along this phase we are interested to write the components of the traction vector. So, let us write that. So, components of the traction vector there is a component along x in terms of this notation what should be the subscript?

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One, what should be the superscript?

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See what does it represent? It represents.

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The unit vector outwards normal of the surface chosen. So, here the surface chosen has unit vector along x 1. So, this should be 1. Alternatively we use a notation equivalent to this is tau 11. So, 2 indices are there what are this indices representing - the first index is representing the direction normal of the surface and the second index is representing the direction of action of the force component.

So, in general it is like tau i j, where i represents the directional normal of the surface which we have chosen and j represents the direction of action of the force component itself. So, let us take a second example, let us say we write the y component on this surface. So, this one, so tau what should be the subscripts?

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First one is 1 and second one is 2, right. So, this is very simple and you can do it for all the surfaces. So, what I will advise you to do is that you repeat the same thing for all the 6 surfaces to get a feel that you understand the notation, this like notation grammar, if you want to learn say classical music, say you require to know the notation. So, we are starting with the notation the notation is important because that will help us in developing the basic equations in a elegant manner.

Interestingly let us look into the opposite phase of this one. So, for this phase the outward normal is along negative x right. So, we will develop a sign convention that if the outward normal of the surface is along negative i, we will have the sign conventions such that positive tau i j is along negative j; that means, here the. So, this one we will call as positive tau 11 for this surface y because the first one is actually along the negative x the first index therefore, the positive sense of the j which is the second one is along negative x.

So, on this surface if you want to draw positive sense of tau 12. So, that should be downwards, this are sign conventions. So, if it actually is the other ways it will come as minus of this number just like in free body diagram you draw a force the force might have come in the negative; that means, it is actually in the opposite sense in the and what is drawn in the figure. So, this are also like that. So, as if you are drawing the free body diagram of a chunk or an element. So, we are establishing sign conventions for that. So, what we have learnt as the sign convention is if the direction i is along the negative of one of the coordinate axis then the tau i j positive sign will be oriented along the negative j direction and if it is positive it is the other way.

So, this tau i j, so when you write tau i j this i may vary from 1 to 3 and j may vary from 1 to 3. So, these are certain quantities. In general you may have 3 into 3 9tau i j components. We will later on see that actually out of this 9 you have 6 which are independent and utilizing those independent 6 components which are called as components of a stress tensor we can actually find out the state of stress for any arbitrary plane which is neither oriented along x nor y nor z. So, this particular quantity is which are like called as components of a stress tensor we may understand that these are not like vectors. So, what are the differences between this and the vectors? So, very logically you can see a vector requires a single index for its specification i, this requires 2 indices for a specification, what are the special indices? One index is just like making it act like a vector, but other index is specifying the direction normal chosen to calculate that quantity.

So, it is something more general than a vector this actually is called as a second order tensor. We will not be defining in general what is the tensor because it is an involved mathematical concept and there is not enough scope here that we discuss about that, but at least from common sense you can appreciate that the order of tensor in this Cartesian notation is like the number of indices that you are requiring to specify it. So, vector is also a tensor it is a tensor of order 1. Scalar does not require any index for specification. So, it is like a tensor or order 0.

So, we have very easily come across 3 different orders of tensors. Tensor of order 0 which is a scalar, tensor of order 1 which is a vector and tensor of order 2 example is a stress and we will see examples where we will be having tensor of order 4 as of course, there may be nth order tensor in general, but we will see that there are certain important tensors in the context of mechanics in a continuum or a fluid. So, a fourth order tensor is one such example which will come across later on in this course.