Introduction to Fluid Mechanics
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Lecture – 26
Problems and Solutions

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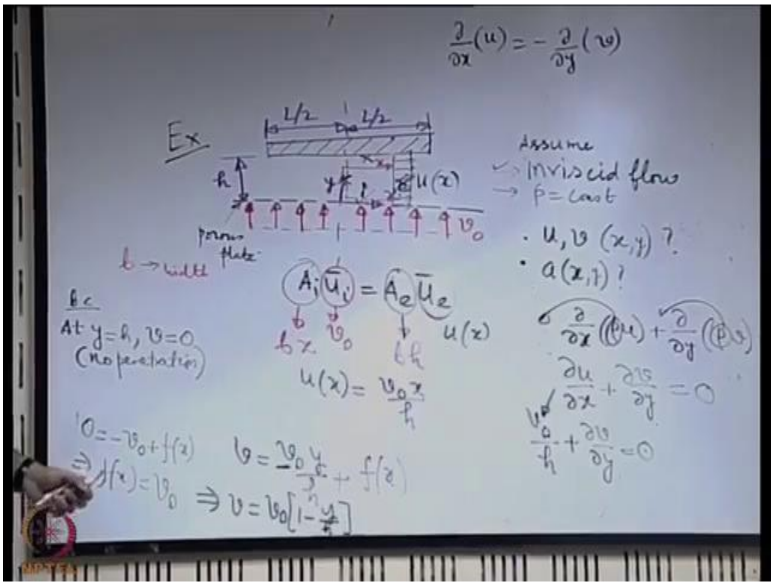

That say the top plate is a rectangular plate with the dotted line representing the axis of symmetry, the bottom plate is a very special plate, it have some holes we call it a porous plate, it has some pores or holes and the idea of keeping these holes is to blow some fluid. We will see that it is not just a mathematically defined idealistic problem. Many times it is it is something which is followed in technology. I can give you one example, let us say that this is a heated electronic chip and you want to cool it.

So, it is possible that you blow air through a porous plate which goes into the chip and tries to keep it cool. So, it is not a very hypothetical type of a situation, but the way we will look into this particular problem is abstracted from that any specific application, but more into the fundamental that what goes behind this. Let us say that this is a uniform velocity with which it enters say we call it v 0. So, this is a porous plate. The gap between these 2 say is h the length maybe this is L by 2 and this is L by 2 and let us say that we have x coordinate like this and y coordinate like this.

We make an assumption that it is an inviscid flow that is given. Assume inviscid flow, what is your objective? Your objective is to find out the velocity components u and v as functions of x that is number 1 and number 2, what is the acceleration of fluid at a given x or maybe at a given x coma y? So, when we say x maybe let us make it generalized say it could be x as well as y. So, let us first physically try to understand that what is happening, whenever you are solving a problem, we can of course, start putting equations, but that is not always necessary first you have to understand. So, what is happening; some fluid is entering.

Now, the fluid cannot leave through the top because of what? Is it because of no slip condition? No because no slip ensure that it has no tangential component, but it simply it is a no penetration condition because it cannot just penetrate through the wall and go out because it is just a fully covered solid wall. So, no matter whether it is slip or no slip you cannot actually penetrate it and go out along y. So, only way this fluid can move in a steady condition is it can move sideways.

So, whatever fluid enters now maybe half enters right and half enters the left. So, what we can say is that we may write gross overall mass balance. So, when you right a gross overall mass balance, what we have to keep in mind that very simple equation just like rho i into A i into u i average is equal to rho e into A e into e v average. So, here let us say that rho is a constant. So, that is another assumption that we make; rho equal to constant. So, when we make the assumption of rho equal to constant; it is we have to just consider the area times the average velocity is same as what enters is same as what leaves.

So, we have to fix up a control volume to write the text machine because you require specific surfaces. So, what control volume we may choose? Let us say that we choose this control volume. See it is symmetrical with respect to the y axis. So, I mean if you consider only one part of the domain half of the domain to the right of y axis the same happens to the left. So, let us say that we consider a control volume like this which is say located its n phase is located at a distance x. So, its local x coordinate is x.

So, with respect to this control volume, what are the phases across which fluid flows can you tell one is the most straight forward is the bottom phase yes through this fluid flows another straight forward is the top phase through which fluid does not flow this one yes or no how many will say yes and how many no think again; think physically see the fluid enters here fluid does not know whether to go to the left or to the right. So, it is equally probable and it is actually equally. So, that may be half will try to go to the right and half will try to go to the left because it is perfectly symmetrical.

So, if we tend to go to the right with the velocity say plus u, similarly it will have tendency; some fluid particle located at the same position to go the left with minus u net effect is that add this access of symmetry where no u; u is 0 because it is just like balanced from what. So, whatever enters it has a balancing effect of going to the right and to the left. So, there is no net flow across this. So, when we have there is no net flow across this there is only. So, this one has a net flow. So, when we say i and e may be this is the surface i and this is the surface e.

So, for the surface i what is; so, we can write again. So, here rho is a constant. So, we can write A i u i is equal to A e u e. So, what is A i let us say that the length that the width of the plate perpendicular to the plane of the figure is b that is the width. So, what is A i what is this A i v into x what is this u i average it is v naught because it is uniform. So, average and local everything is same now come to these ones what is A e, yes. So, h into B, so, this is and what is u e average see for that this assumption of inviscid flow is important if you do not consider inviscid flow when you have to know, what is the velocity profile in between you have to integrate that to get the average velocity.

But when you are given that it is inviscid flow your inherent assumption is that it is a uniform velocity profile like this. So, u is not changing with y it is locally changing with x, but along y it is uniform because it is inviscid you can see that with inviscid flow, you cannot impose no slip boundary condition because if it has to be uniform. Suddenly it cannot go 0 go to 0 at the wall. So, it is not no slip here at the wall, but no penetration at the wall that is sufficient for solving this problem. So, no slip is not a necessary condition here and. In fact, it will contradict if you say that it is an inviscid flow you cannot have no slip and in slip and inviscid no slip and inviscid simultaneously.

So, with there is some slip. So, this velocity, it does not vary with y. So, we can say that it is like u just a function of x from the inviscid flow consideration. So, this is like that u which is a function of x. So, no more it is a function of y from the inviscid flow consideration. So, from here you can say what is u as a function of x it is v 0 x by 8 which point inviscid flow how see when you are considering in inviscid flow see what does a viscous flow do see we have earlier discussed always keep this qualitative concept in mind what does viscosity do.

It propagates the effect of a momentum disturbance. So, where here you have momentum disturbance imposed by the wall. So, if the fluid have a viscosity that will propagate from the wall to the inside and it will in effect try to slow down the fluid elements which are close to the wall and as you go more away and more and away from the wall the velocity will be more and more. So, the velocity profile in that case will have like a 0 value at the wall and then increasing away from the wall, but if we have no viscous effect then the effect of wall is not propagated in the fluid; fluid does not know that there is a wall and therefore, it tends to maintain a uniform velocity.

And that is y for an inviscid flow you have such a kind of uniform velocity profile. So, when you have such a kind of a profile. So, you have this like you have u only function of x, but not function of y very idealistic situation, but like simple one to begin with now how do we find out v. So, you know u how do we find out v. So, you have the continuity equation that relates u with v. So, what is the special form of the continuity equation with rho equal to constant and it is a 2 dimensional flow.

So, when rho is a constant, the first term, the time derivative is 0. The next one is if rho is the constant. So, you have like plus this that is equal to 0 for a 2 dimensional flow. So, w is not there rho being a constant it will come out of the derivative. So, it will be this equal to 0 now. So, you know; what is u as a function of x? So, you can write the partial derivative of u with respect to x; what is that v 0 by h? So, now, you can integrate this with respect to y to find out how v varies with y. So, we integrate it how will you what will you get v equal to v 0 y by h plus what plus some may be a some function of x or so; it is a constant of integration, but since it is a partial derivative when you are integrating it you are integrating it partially with respect to y.

With respect to that integral x is like a constant. So, you could have in general a function of x within that there may be a constant it makes itself be a constant, but for generality it is better to write it in this way now you can find it minus yes this is the minus and then this one, now you can find out this with a boundary condition. So, what boundary condition is there at y equal to h you must have v equal to 0 this is no penetration boundary condition not no slip again; I am repeating because it cannot penetrate physically through the boundary.

So, that if you substitute; so, minus v 0 plus f x therefore, f x becomes equal to v c this means that you have v is equal to v 0 into one minus y by h. So, you can see here that u is a function of x v is a function of y only and see look at this equation you can do a mathematical simple mathematical jugglery with it. So, the left hand side u is a function of x. So, this is the function of x only v is a function of y. So, this is expected to be a function of y only and ironically you are getting a situation where left hand side is a function of x only right hand side is a function of y only it is possible only when each is equal to a constant otherwise you do not have a way by which you can cancel x and y from both sides.

So, only way is you may get it as a constant. So, there are I mean these are just ways of views of looking into a problem, it is not just a matter of solving a problem there are nice interesting concepts and remarks that you can get from a problem now the next is. So, we have got u and v acceleration is very straight forward. So, how do you calculate the acceleration? Let us do that.

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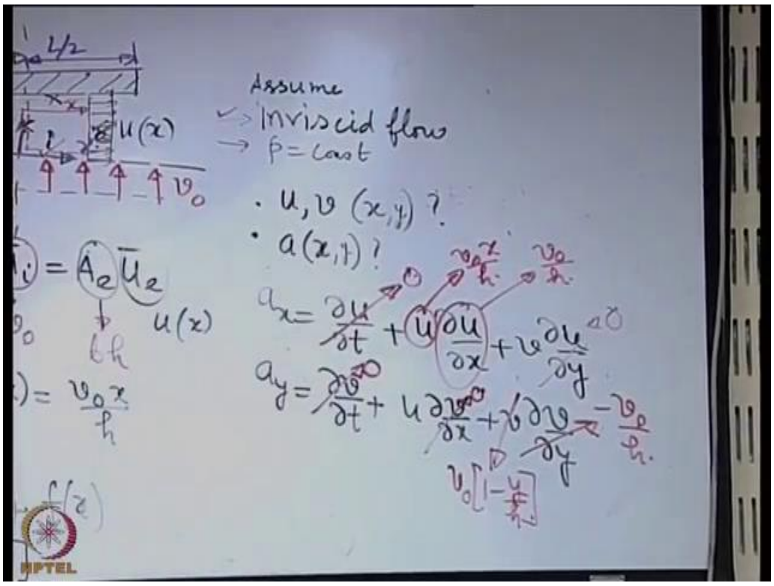

So, what is acceleration along x? This is acceleration along x, this we derived earlier. So, this not a 3 dimensional case, the w term is not there. So, first of all its u v is not a function of time.

So, this is 0 then you can just substitute what is u here v 0 x by h and so this will become v 0 by h. The other term will be 0 because u does not vary with y. Similarly let us write the y component of acceleration again because v is not a function of time this is 0 then v is not a function of x. So, this is 0 and here you have. So, v as a function of y, what is v as a function of y, you have v 0 into 1 minus y by h and what is d v d y minus v 0 by h? So, you have a final concrete expression for acceleration components along x and y.

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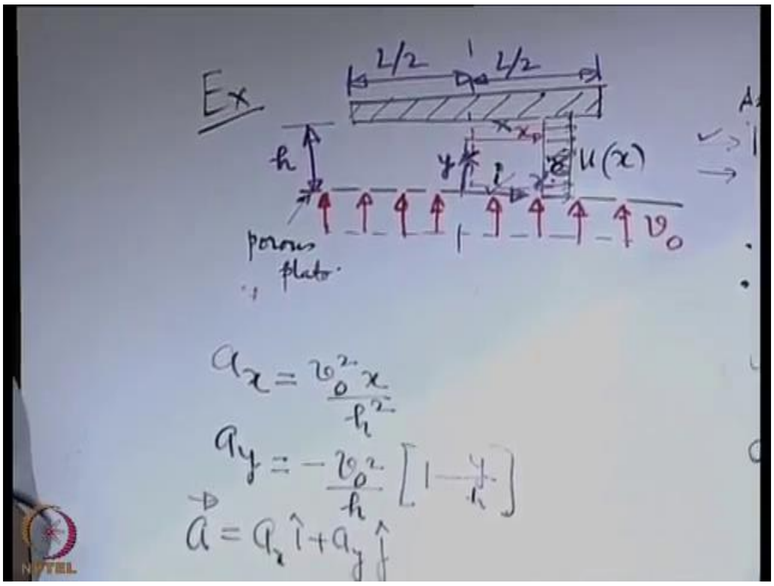

So, you have acceleration component along x as v 0 square x by h square acceleration along y minus v 0 square by h into one minus y by h and the acceleration vector is a x i plus a y j which is a function of both x and y. So, this is the acceleration at a point. So, if there is a fluid particle located at some x coma y that will locally be subjected to that acceleration because locally the fluid particle and the flow behavior is identical. So, you can clearly see that although the velocity components are not functions of time, still you get acceleration. That is what is the important implication that we get from an Eulerian approach because the entire acceleration component has arisen; because of the convective component of acceleration because of the variation of velocity with respect to position. It is not because of change in velocity due to change in time.

So, we stop here today we will continue in the next class.